INTRODUCTION 1

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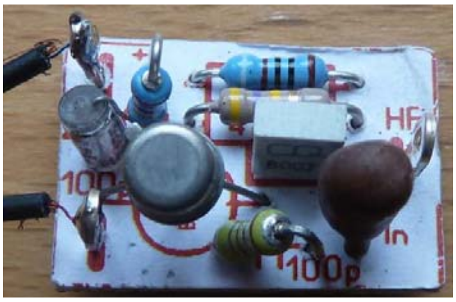
The design of amplifiers has changed from the beginning as the technology grew.
The initial standard way to design was connecting transistor by transistor, resistor by resistor, etc.; namely brick by brick.



We can imagine the cost and, more important, the reliability of such a way to proceed, if we consider that an amplifier to work, needs at least a few tens of transistors, every transistor has 3 legs to connect, and so on.
Space occupation is another very important issue.

Now we have to consider that:

- Reliability is inversely proportional to the number of contacts in the circuit;
- The test of the circuit, done automatically on large series, takes a time duration that is proportional to the measurement nodes.

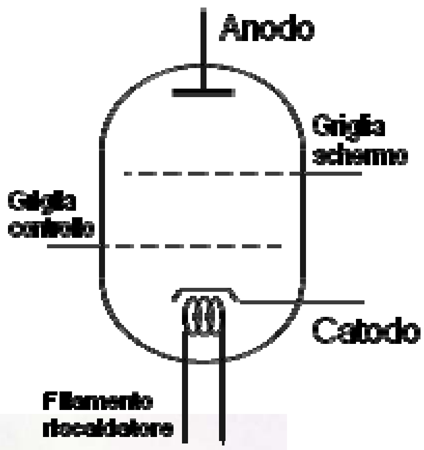
In conclusion,
the development time rises with the number of components to be used and efforts have been made so far in trying to improve this.

INTRODUCTION 2

We are now living the age of complex integrated circuits. These devices have a large number of connecting pins and are composed of several transistors, or bricks, already connected to form circuits with specific functions. A few external components can then be added from outside to personalize several of their functions.
Two categories of such circuits exist: digital circuits (micro-controllers, micro-processors, etc. capable to contain millions of bricks per square mm) and analogue circuits that, strategically, contain a few hundred of transistors as their functionality are limited.

It is interesting here a brief historical review of the technological evolution of the solid state electronics.
The first form of transistors were the valves (components in 3D) that worked on a different operating principle: thermionic emission of charge from metal.
Valves generate heat by default and are totally incompatible with the modern electronics.

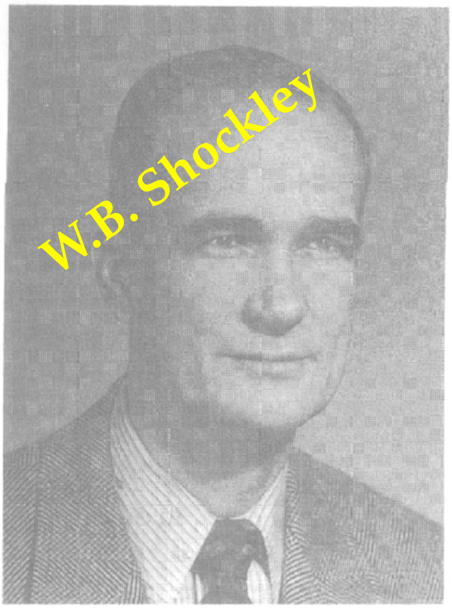
The valve

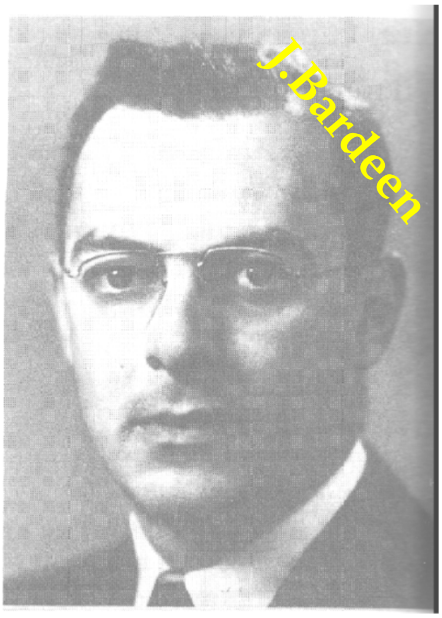


To give the order of magnitude: the implementation of the microprocessor of our smartphones with valves takes probability the volume of this building.

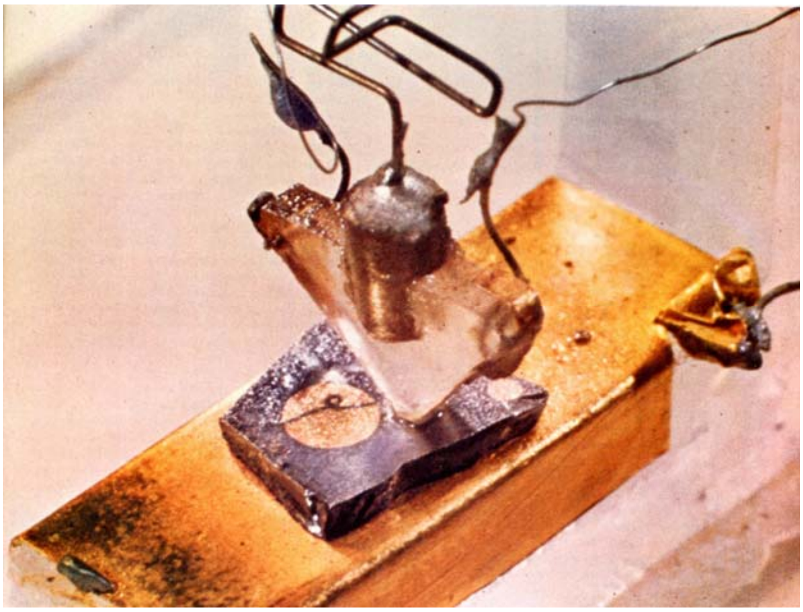
INTRODUCTION 3

At the end of the '40 of the last century it started the study of semiconductors, mainly Germanium and Silicon,

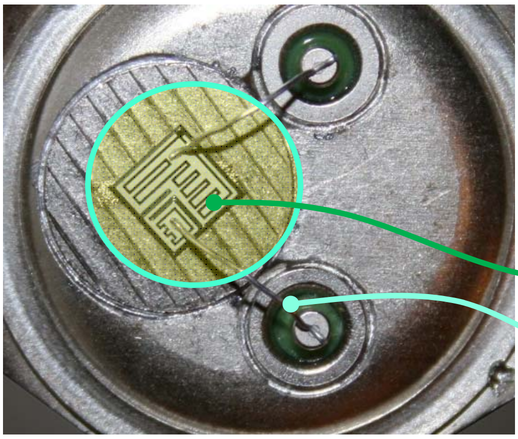




Two researchers that played a big role in this business were William B. Shockley and John Bardeen



The first working transistor, a bipolar transistor, were in Ge and was developed at the Bell Labs.



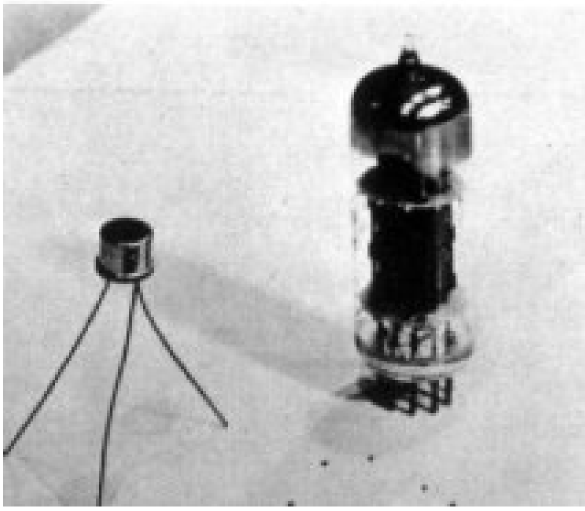
Since then transistor manufacturing became planar, namely 2D and Si was the standard substrate for it.

Power transistor (~mm^2)

Bonding wire

INTRODUCTION 4

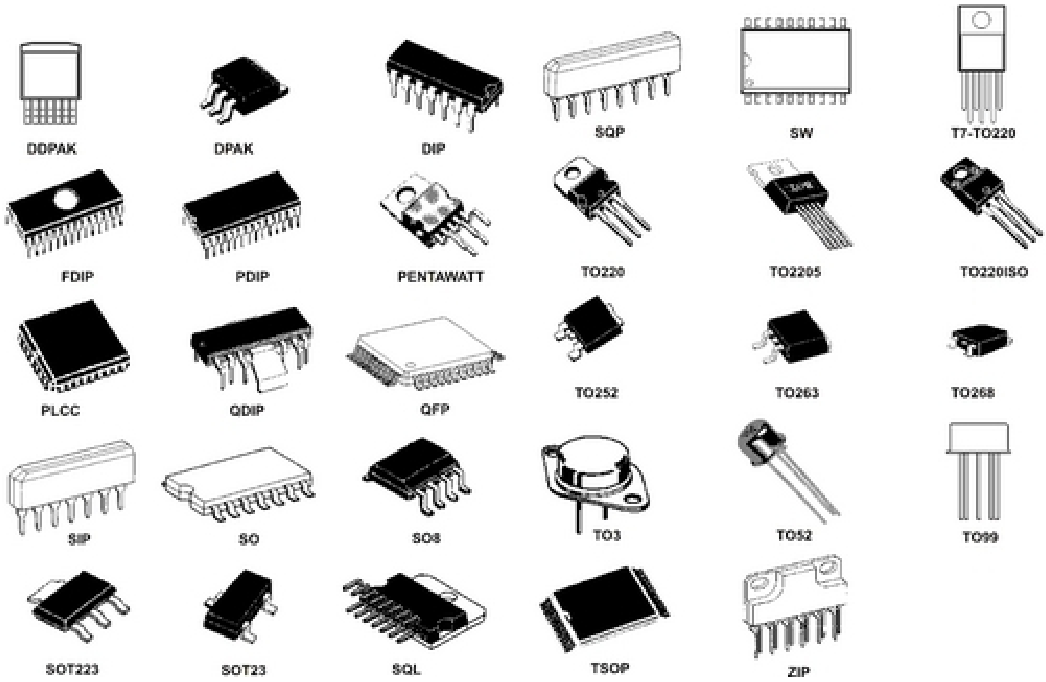
Pretty soon the transistor evolved and the valves retired.
The change was dramatic: the space occupation was reduced by order of magnitude (circuits changed from 3D to 2D…) and, more important, the power consumption became negligible in comparison to valve since the operating principle of transistors is not thermionic.



One of the first commercial transistor in its metallic package.

The technology evolution was not only in the semiconductors, but also in packaging and the managing of the semiconductor-chip inside.

Several are the constraints to be followed: compactness, stress resistance, power dissipation, hence thermal conductance, …



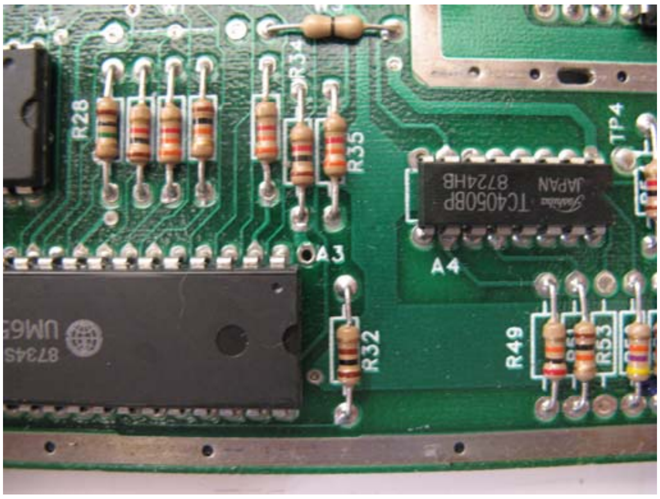
Having the transistors available, amplifiers can be implemented combining several of them with resistors, capacitors and inductances.

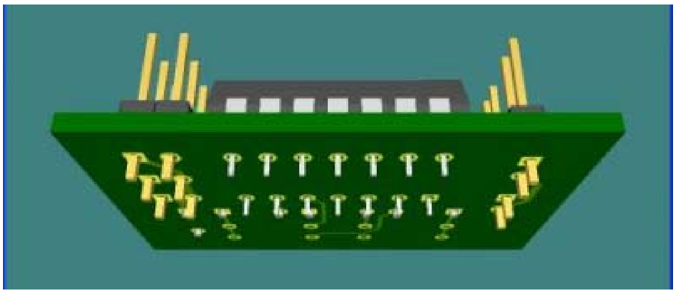
At this moment of our study we do not know which way an amplifier is composed and we can consider it as a mess of components for the moment: several transistors are needed to compose an amplifier just as many bricks are needed to construct buildings.

Having available only one transistor per package, the only way to design amplifiers was by connecting packages to packages. This way to proceed is adopted also nowadays, but a new approach made available by the evolution of the technology was possible.

For quite a long time one package included only one transistor and there was an evolution in packaging from metal can, through-hole (TO) to surface mount devices having much smaller dimension.

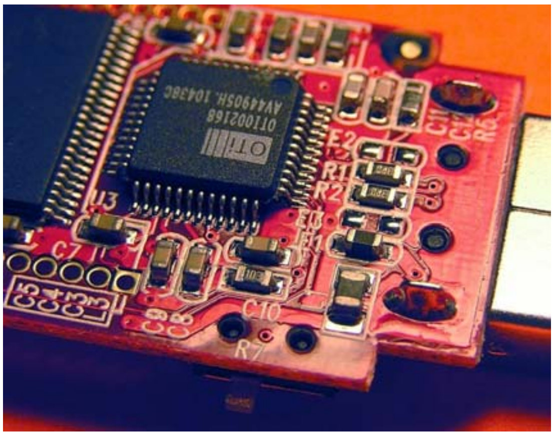
Through-hole means that the connecting pins of components are inserted in corresponding holes and soldered on the other side of the Printed Circuit Board, PCB, the substrate support where copper tracks are "printed" for electrically connecting pins of the components:





There is a limit in the size of and the distance between holes, so the compactness of this circuits is limited.

Surface mount devices, or SMD devices, are soldered on the same side where the devices are located and take advantage from the fact the soldering pads can be closer and smaller: these circuits are much more compact.



The assembling of SMD circuits is more expensive compared to through-hole circuits and this is a parameter of decision about which technology to use: if space is not an issue through-hole is the choice.

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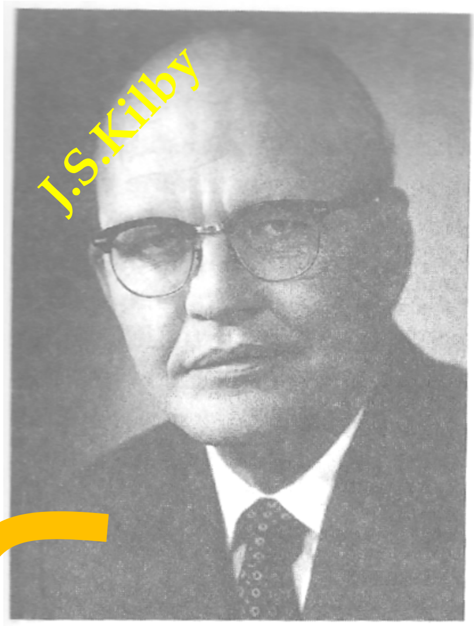
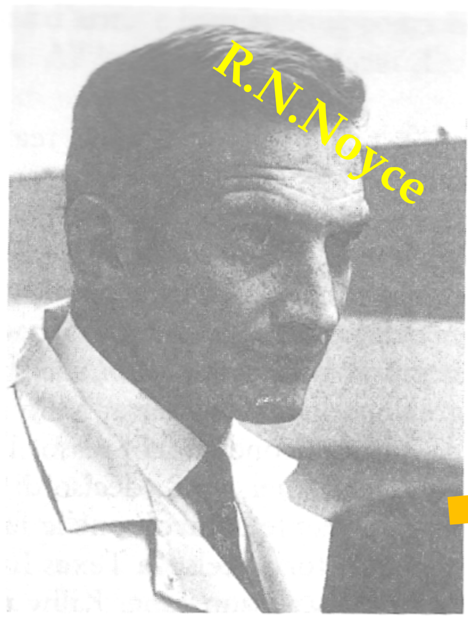
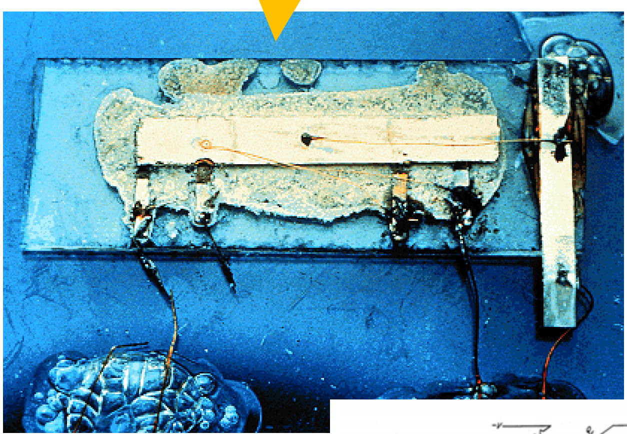
It could be considered strange, but the idea of trying of implementing more than one transistor in a package was not immediate (a similar, much longer, evolution was the introduction of the “zero” in numbers…).

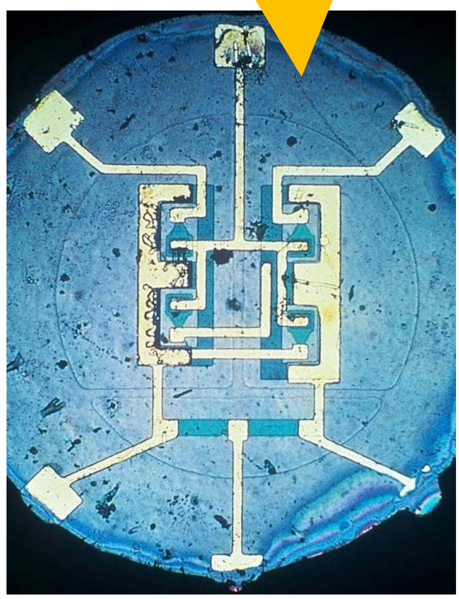
At the end of the ‘50 of the last century the first integrated circuits as we know now came. Its idea and implementation were developed in parallel at Texas Instruments and Fairchild semiconductor mainly by Jack S. Kilby and Robert N. Noyce

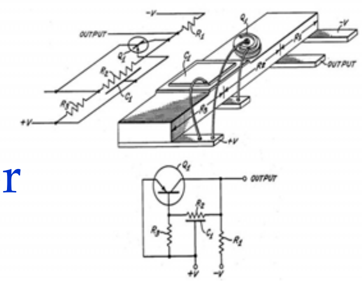






For the first time resistors, capacitors and transistors share space on the same substrate (Ge).

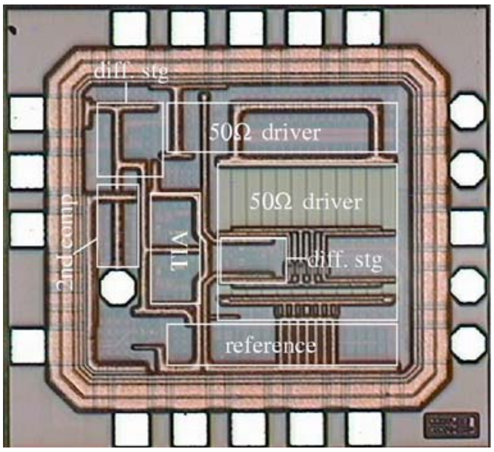




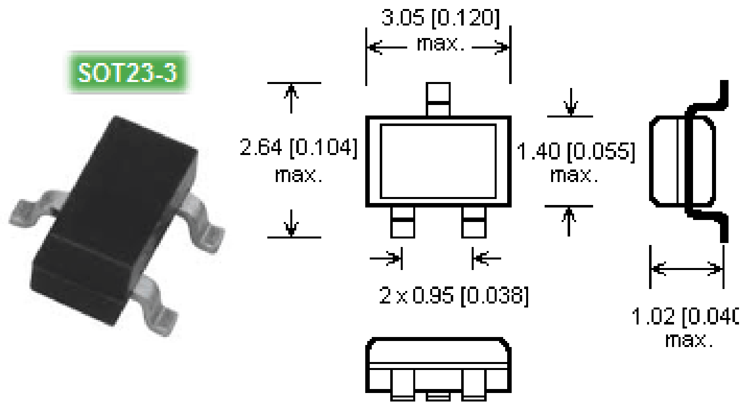
An oscillator

A flip-flop (memory)

Monolithic technology grew very fast since the first prototypes, and now the number of transistors that can be integrated on a singe piece of Si and housed in a single package have practically no limit, or the limit is approaching that of atom volume:



Concerning our interest about analogue circuits, there is a very very wide choice of the so called Operational Amplifiers, OA, available with a large choice of different characteristics, therefore able able to satisfy a very wide range of applications.



The space occupied by the package is not always related to the dimension of the Si on its inside. This is a single transistor in SOT23 package.

And this is an amplifier with a few tens of transistors contained in its inside: same package above, but with 2 additional pin connections. The name is called SOT23-5.

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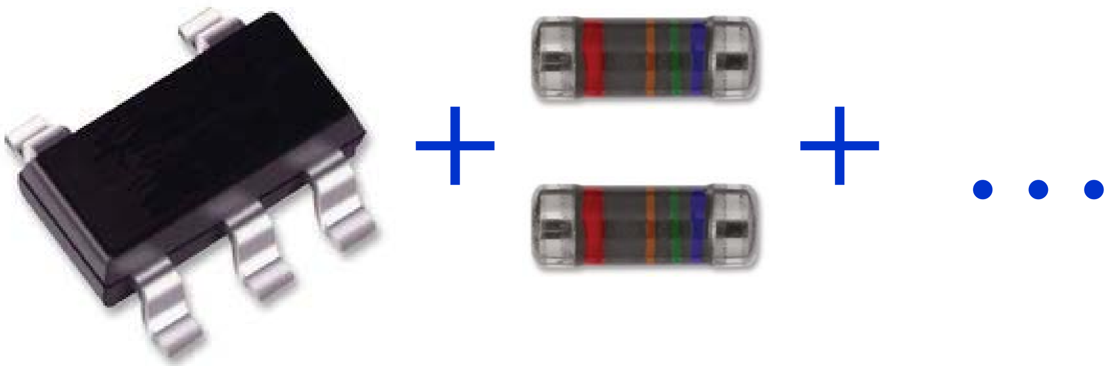
Now, this very large number of different kind of monolithic amplifiers are not intended to satisfy any possible application. Namely, things do not work such that an amplifier with X gain, Y bandwidth, etc., needed for our application, can be found of-the-shelf.

Operational Amplifiers are sold "rough" and, with the help of a few components added externally, the final needs can be pursued, generally with great precision:



This extended range of obtainable possibilities are all based on the exploitation of the negative feedback, which needs to be managed with care to obtain the final goal.

We will study deeply the theory of negative feedback with operational amplifiers, as this technique does not reduces merely to the addition of a few components around an operational amplifier, since other constraints must be met, such as dynamic, frequency, noise. etc.

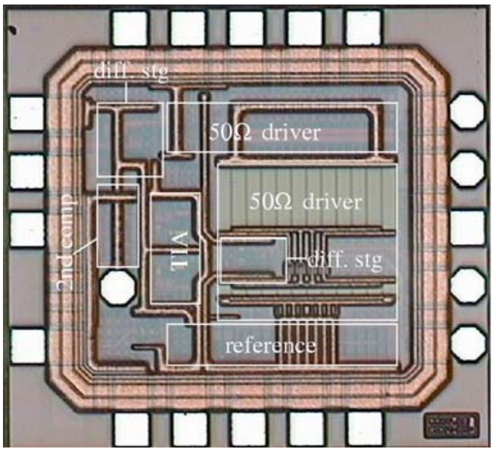
Then, the physical principle of operation of transistors will be also part of our study. But, because of transistors are themselves feed-backed amplifiers, they will be considered a class of OAs and several of the rules valid for operational amplifiers will maintain also for them.

Let's briefly summarize our next steps in our study.

First we will study the use of amplifiers as black-boxes, namely, we assume that the Operational Amplifier, OA, is available and we can manage it by adding some components around:

OA as a black box, a mathematical tool.

Then, we will enter the black box to try to understand the OA, or Amplifier structure:



This is the inside of the black box above.

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We start first with the introduction of a few simple electrical properties that will be necessary afterwards.

Then, we will enter deeply in the study of amplifiers with feedback. The amplifier will be modeled as a device with some characteristics.

The last step will be the introduction of transistors and their use in the implementation of amplifiers.

This way to proceed is in some sense not standard, as normally transistors are introduced first with the aim of their use in the realization of amplifiers.
Experience shows that facing 2 new concepts at the same time, transistors and the design of amplifiers with transistors, leads to some difficulties in the comprehension from one side, and does not follow the way circuits are designed nowadays for which, at the startup of a new design, the answer to the following question is at the basis:
Does it exist an OA suitable to the application?

1) If yes (almost 95% of cases), use it with a handful of passive components (resistors and capacitors);
2) If not, the amplifier is designed with the use of transistors.

Actually there is a further in-between solution consisting in merging OAs with a few transistors. This is often necessary when noise is an issue (we will see this).

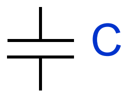
THE BASIC PASSIVE ELECTRICAL ELEMENTS TO KNOW

RESISTORS:

$$v = Ri$$

The current is linearly dependent on voltage through a resistor. If the current is zero the voltage is zero: this means that the energy given to resistor is promptly dissipated into heat.

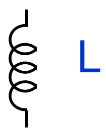
CAPACITOR OR CAPACITANCE:



$$v(t) = \frac{1}{C}\int_0^t i(t)$$

Current and voltage are not linearly dependent. If a short current pulse, namely a charge, is loaded on a capacitor a voltage drop is created that remains indefinitely: the energy spent for that is not dissipated, but stored in the capacitance.

INDUCTOR OR INDUCTANCE:

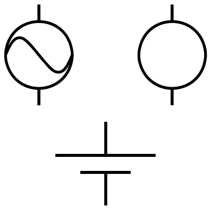


$$v(t) = L\frac{di}{dt}$$

Current and voltage are not linearly dependent. If a short voltage pulse, namely a magnetic field, is applied to an inductor, a current is created that circulates indefinitely: the energy spent for that is not dissipated, but stored in the inductor.

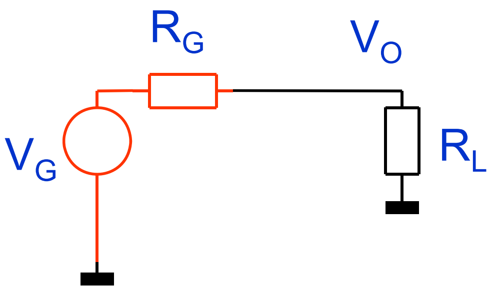
THE BASIC ELECTRICAL ELEMENTS TO KNOW 1

IDEAL VOLTAGE SOURCE:



An ideal voltage source is able to maintain a dropout (constant or variable with time) that does not depend on what it is connected across its terminals. When this is the case we say that it has zero-series impedance and is ideal: we will show why in a while.

NON-IDEAL VOLTAGE SOURCE:



A non-ideal voltage source is is modelled by an ideal voltage source having a resistor R_G in series.

The resulting signal at the load is then:

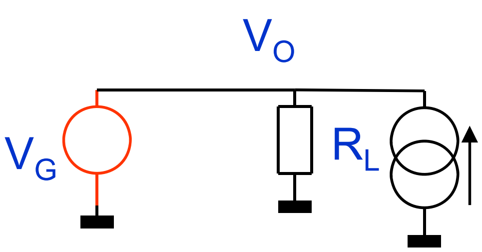
$$V_O = \frac{R_L}{R_L + R_G} V_G \xrightarrow{R_G \to 0} V_G$$

(see next page for evaluation)

According to the model above, a non-ideal voltage source tries to be close to an ideal voltage source as soon as its series impedance tends to zero, or it is << smaller than the load impedance.

ZERO-VOLTAGE SOURCE:

To null a voltage source means that the source itself generates a zero-amplitude voltage.
A voltage source with zero-amplitude can be modelled with a short circuit.



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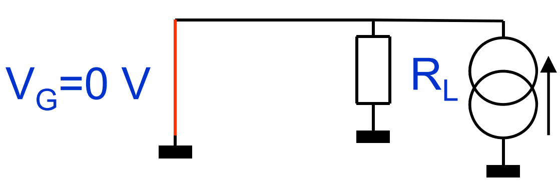
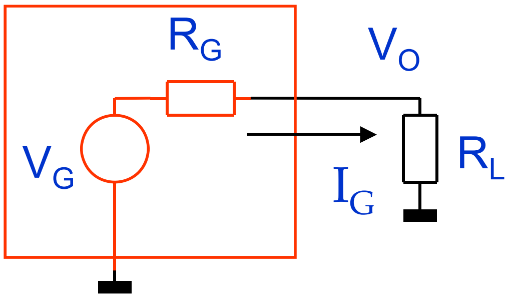




Let's evaluate the output voltage:

The current I_G flows through the series of the 2 resistors:

$$I_G = \frac{V_G}{R_G + R_L}$$

Now V_O is the voltage drop across R_L:

$$V_O = I_G R_L$$

Therefore:

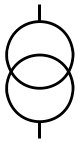
$$V_O = \frac{R_L}{R_G + R_L} V_G$$

We can arrive at the same result by considering the voltage V_G is linearly shared between both resistors, while the current is the same:

$$V_G : (R_L + R_G) = V_O : R_L$$

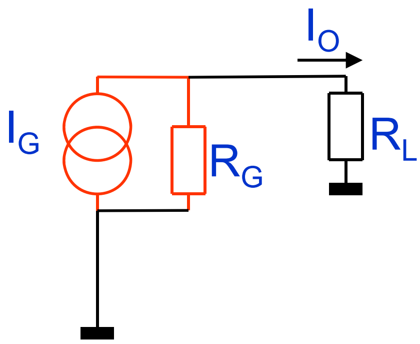
THE BASIC ELECTRICAL ELEMENTS TO KNOW 3

IDEAL CURRENT GENERATOR



An ideal current source is able to drive a current (constant or variable with time) that does not depend on what it is connected across its terminals. When this is the case we say that it has an infinite-parallel impedance and is ideal: we will show why in a while.

NON-IDEAL CURRENT GENERATOR



A non-ideal current source is modelled by an ideal current source having a resistor R_G in parallel.

The resulting signal at the load is then:

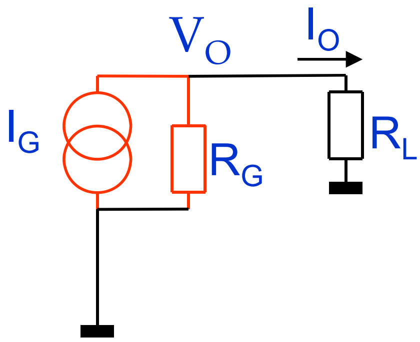
$$I_O = \frac{R_G}{R_L + R_G} I_G \xrightarrow{R_G \to \infty} I_G$$

(see next page for evaluation)

According to the model above, a non-ideal current source tries to be close to an ideal current source as soon as its parallel-impedance tends to infinite, or it is >> larger than the load impedance.

ZERO-CURRENT SOURCE:

To null a current source means that the source itself generates a zero-amplitude current.
A current source with zero-amplitude can be modelled with an open circuit.



Let's evaluate the output current:

Voltage V_O is common to both resistors:

$$I_G = \frac{V_G}{R_G} + \frac{V_G}{R_L}$$

Hence:

$$V_G = \frac{R_G R_L}{R_G + R_L} I_G$$

The current through R_L is:

$$I_O = \frac{V_G}{R_L}$$

Finally:

$$I_O = \frac{R_G R_L}{R_G + R_L} I_G$$

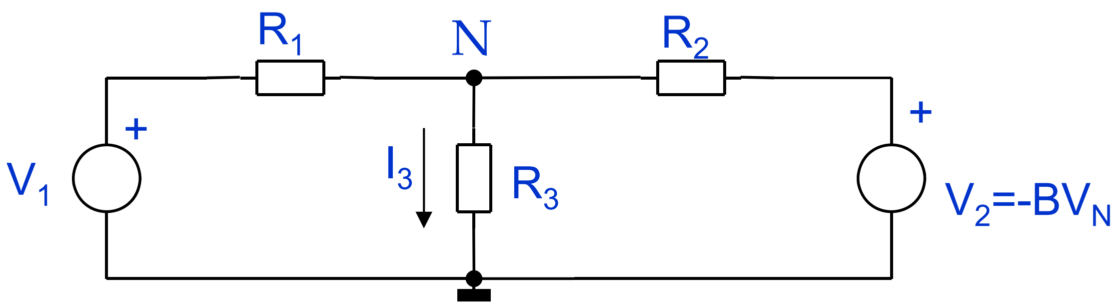
We can arrive at the same result by considering the current I_G is linearly shared between both resistors, while the voltage is the same:

$$I_G : \left(\frac{1}{R_G} + \frac{1}{R_L}\right) = I_O : \frac{1}{R_L}$$

CURRENT-CONTROLLED/VOLTAGE-CONTROLLED SOURCE:

There is a class of voltage and current generators that are at the basis of the modelling of amplifiers and transistors: the voltage-controlled and current-controlled sources.

These sources generate a voltage (current) that is dependent on the voltage or current of a node or branch of the circuit. Here an example:



As it can be seen voltage generator V_2 generates a voltage that is linearly proportional, with coefficient "-B", to that at node N.

SUPERIMPOSITION PRINCIPLE 1

THEO: The behaviour of a linear network containing independent voltage and/or current sources can be calculated by solving a number of networks equal to the number of sources, where each network differs from the other for having all the sources nulled but one (dependent sources are considered independent during calculation, their dependence being considered at the end of the process, once the nodes to which dependences are appended are highlight); then the algebraic sum of every individual elaboration is the final answer.

NOTE: this theorem is very useful when dealing with noise networks.

Do Not forget: a nulled ideal voltage is replaced by a short circuit; a nulled ideal current source is replaced by an open circuit.

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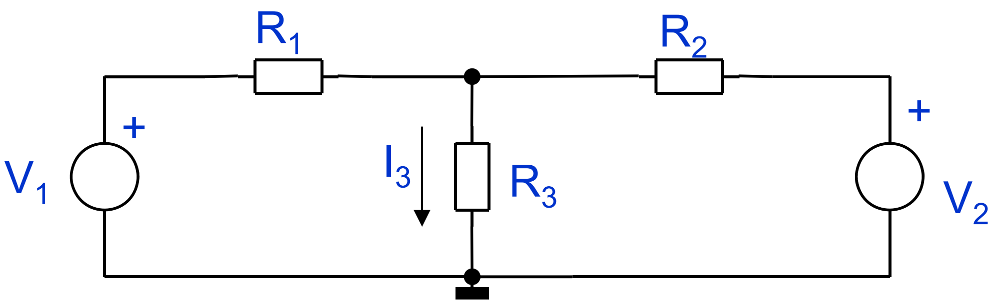
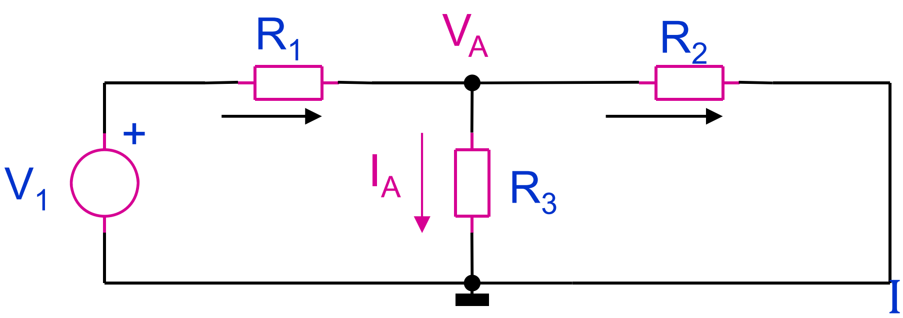


V_1=6 V
V_2=-14 V
R_1=1K Ω
R_2 =9K Ω
R_3 =2K Ω

I_3=????

CASE a) let's null V_2:



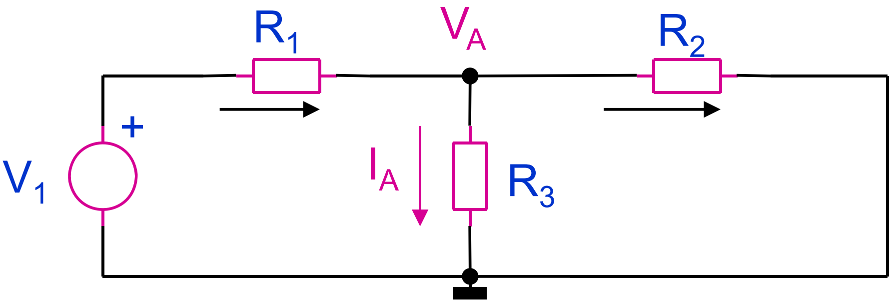
(see next page for evaluation)

This first result is:

$$I_A = \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3} V_1$$

SUPERIMPOSITION PRINCIPLE 2

Let's write the solving eq for the network:



It is convenient to start from one side and stop at the first node. Then write the balance of the currents at the node and verify if the eq has only one unknown.
If no, then we must proceed to the following node for adding a new equation. This process will end when the number of written equations equals the unknown.

For the network above the balance at node V_A is:

$$\frac{V_1 - V_A}{R_1} = \frac{V_A}{R_3} + \frac{V_A}{R_2}$$

With only one unknown and:

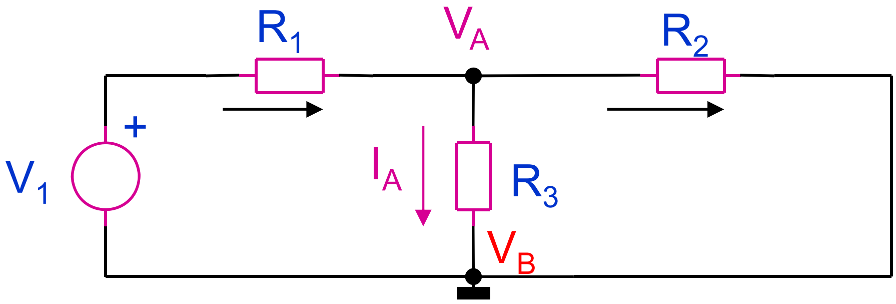
$$I_A = \frac{V_A}{R_3}$$

SUPERIMPOSITION PRINCIPLE 3

CONSIDERATION:

Let's see why the voltage, common to all, at potential
Can be assumed equal to 0 V.

Let's call it V_B and see what happens:



$$\frac{V_1 + V_B - V_A}{R_1} = \frac{V_A - V_B}{R_3} + \frac{V_A - V_B}{R_2}$$

Now, if we call:

$$V_X = V_A - V_B$$

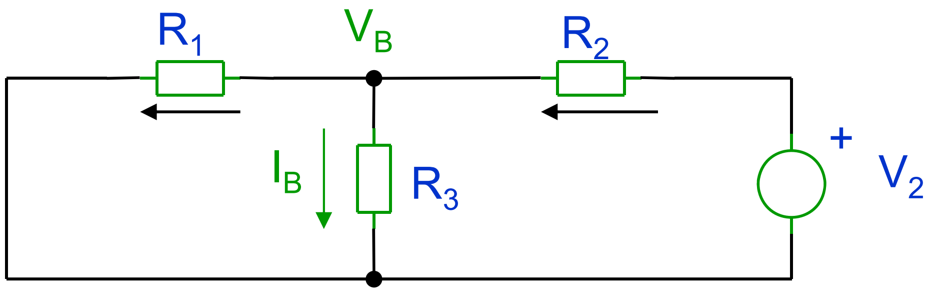
We obtain:

$$\frac{V_1 + V_X}{R_1} = \frac{V_X}{R_3} + \frac{V_X}{R_2}$$

Which is the same equation of the previous page.

SUPERIMPOSITION PRINCIPLE 4

CASE b) let's null V_1:



The second result is:

$$I_B = \frac{R_1}{R_1R_2 + R_1R_3 + R_2R_3} V_2$$

Finally (please, take care at the arrows that indicate the direction of the current flows: currents having the same direction must be summed with the same sign opposite to that of the currents having opposite direction):

$$I_3 = I_A + I_B$$

$$= \frac{R_2V_1 + R_1V_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$= \frac{9K \cdot 6 + 1K \cdot (-14)}{9K \cdot 2K + 1K \cdot 2K + 1K \cdot 9K} = 1.379 \text{ mA}$$

THEVENIN THEOREM 1

With respect to a pair of its terminals, a netwtork can be modelled by an ideal voltage source, V_{TH}, having in series a resistor, $R_{TH.}$ The voltage V_{TH} is the voltage measured at the terminals when the load impedance is ∞, while R_{TH} is the impedance that can be measured once that all the independent sources of the network are nulled.

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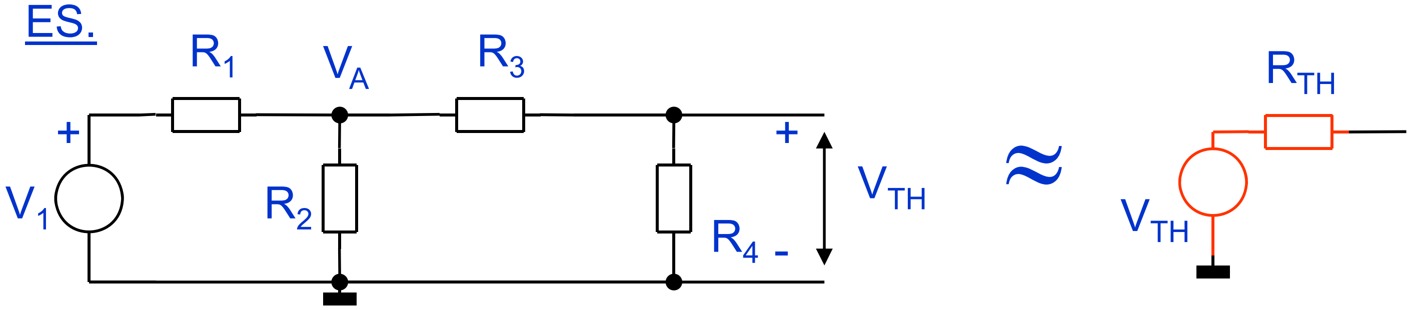
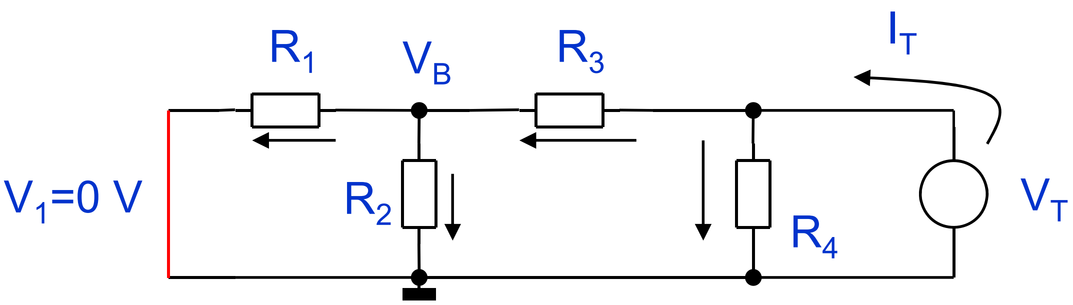


With respect to the 2 terminals above V_{TH} results in:

$$V_{TH} = \frac{R_2 R_4}{(R_1 + R_2)(R_3 + R_4) + R_1 R_2} V_1$$

(see next page for evaluation)

To calculate R_{TH} let's proceed by applying a test voltage, V_T, at the nodes of interest and measuring the resulting current, just as if we were using a measuring instrument. The ratio of the applied voltage and the driving current being the wanted impedance R_{TH}. Before doing this all the independent voltage sources must be nulled:

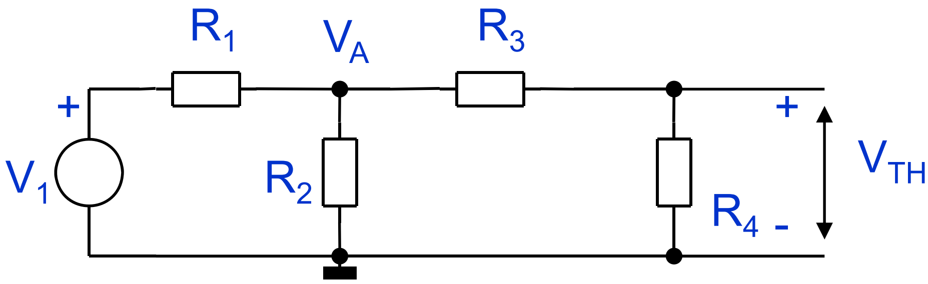


The result is:

$$R_{TH} = \frac{V_T}{I_T} = \frac{R_4\left(R_2 R_3 + R_1 R_3 + R_1 R_2\right)}{R_4\left(R_1 + R_2\right) + R_2 R_3 + R_1 R_3 + R_1 R_2}$$

THEVENIN THEOREM 2

Network evaluation:



Starting from the left we encounter the first node V_A:

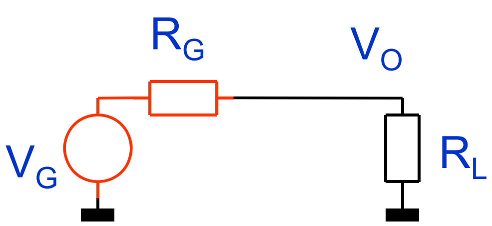
$$\frac{V_1 - V_A}{R_1} = \frac{V_A}{R_2} + \frac{V_A - V_{TH}}{R_3}$$

We see now that the above eq has 2 unknown, V_A and V_{TH}, and we need to add an eq.
For that we move further on the right:

$$\frac{V_A - V_{TH}}{R_3} = \frac{V_{TH}}{R_4}$$

This new eq does not add any new unknown and the 2 eq above allow to solve the network.

CONSIDERATION: Why the voltage V_{TH} is the voltage measured at the given terminals is soon understood if we remember that the output voltage V_O of the network below equal V_G only and if only R_L equals ∞ every time R_G has a finite value.

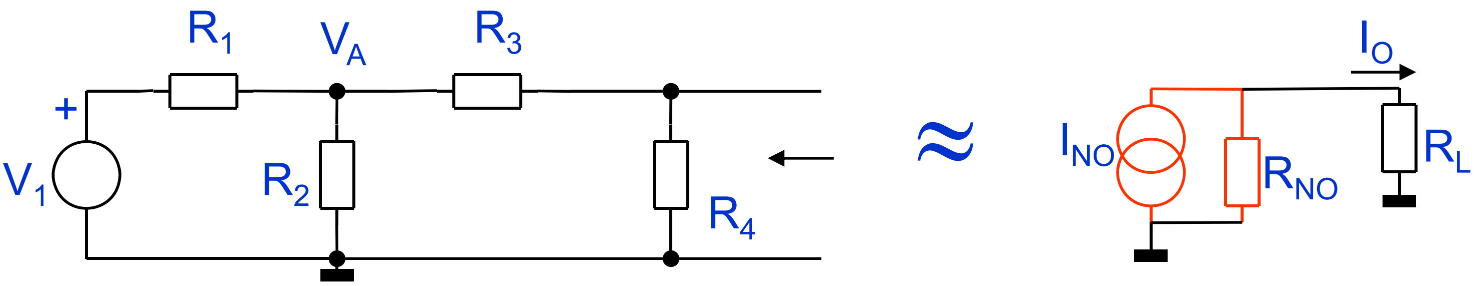


$$V_O = \frac{R_L}{R_L + R_G} V_G \xrightarrow{R_L \to \infty} V_G$$

NORTON THEOREM 1

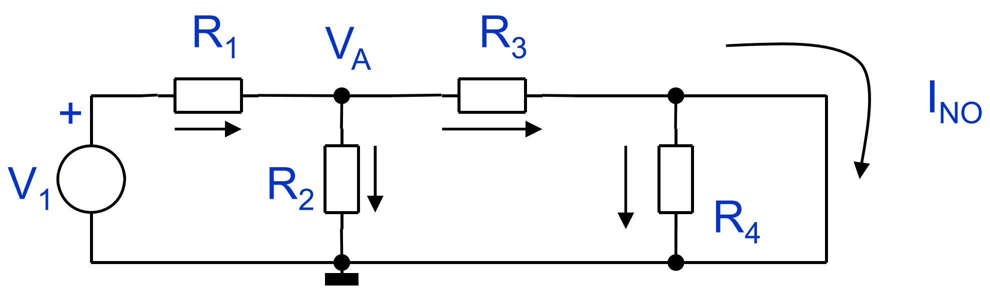
With respect to a pair of its terminals, a netwtork can be modelled by an ideal current source, I_{TH}, having in parallel a resistor, $R_{TH.}$ The current I_{TH} is the current measured at the terminals of interest when the load impedance is 0, a short, while R_{TH} is the impedance that can be measured once that all the independent sources of the network are nulled.

ES.:



The impedance seen at the nodes of interest is the same as that of the previous example (the network is the same).

To evaluate the current I_{NO} we consider the following:



From which we obtain:

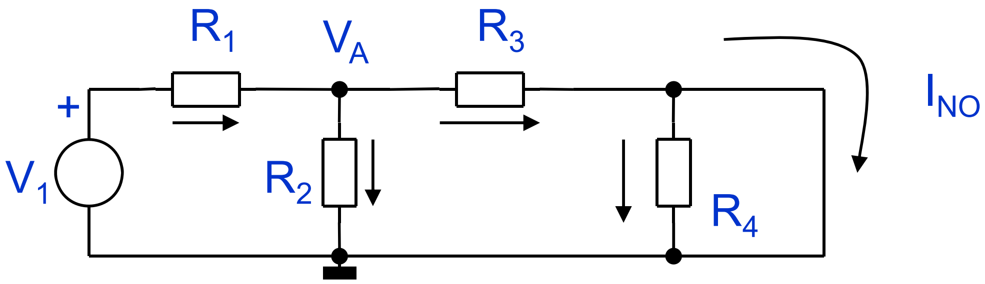
$$I_{NO} = \frac{V_A}{R_3} = \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3} V_1$$

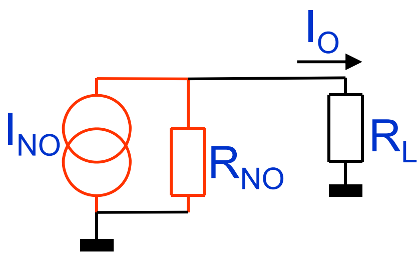
IMPORTANT: if we apply the Norton Theo to a network to which we have applied the Thevenin Theo, or vice versa, it can be shown that:

$$V_{Th} = I_{No}R_{No}, \qquad R_{Th} = R_{No}$$

NORTON THEOREM 2

Let' try to understand why the Norton current I_{NO} is evaluated this way:





That is done this way because we know that current I_O equals I_{NO} if and only if R_L is zero if R_{NO} has a finite value:

$$I_O = \frac{R_{NO}}{R_L + R_{NO}} I_G \xrightarrow{R_L \to \infty} I_G$$

A note on the text colour of equations

In the following a formula coloured like this:

$$\frac{1}{R_{of}} = \frac{1}{R_L} + \frac{1}{R_{i\beta A}} + \frac{1}{R_o}\left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right)$$

Represents a standard result of an evaluation.

Equations coloured like this:

$$\frac{T}{R_{ool}} = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta}} \frac{1}{R_o}$$

represents an intermediate passage of an evaluation and can temporarily skipped.

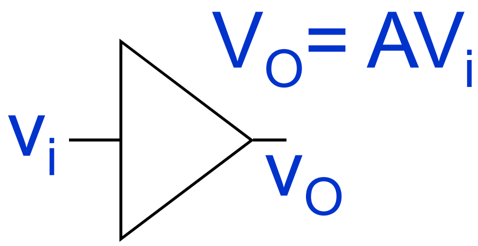
Equations coloured like this:

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta}} \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o}$$

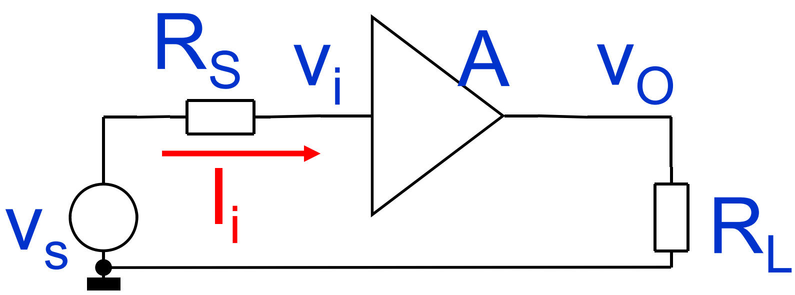
Indicates a result that is widely used around.

AMPLIFIERS: SYMBOLS 1

The simplest form for representing a voltage amplifier is the following:



The above is an ideal amplifier since there is no indication of the characteristics of its input and output and its frequency behaviour. Dealing with an ideal model we can introduce some useful properties when driving it with a non-ideal voltage source:

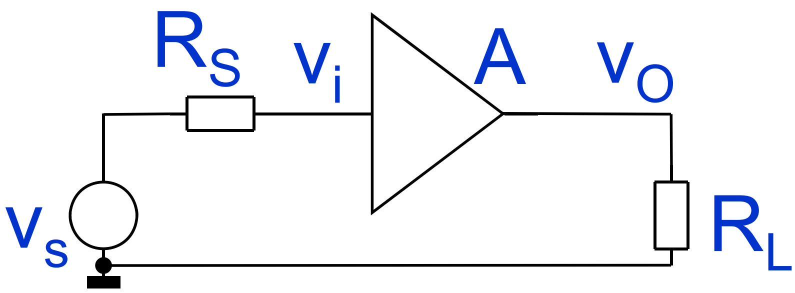


The amplifier, being ideal, has no current that enter its input, so there is no current that flow through R_S. The consequence is that the voltage V_i is equal to V_S.
Being ideal, the output voltage does not depends on the load impedance R_L. As a consequence:

$$V_O = AV_S$$

The result is independent from the source resistor R_S and load resistor R_L.

AMPLIFIERS: SYMBOLS 2



$$V_O = AV_S$$

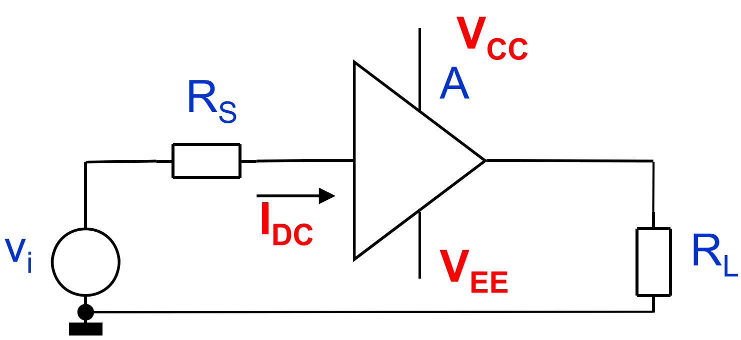
Moreover, there is another important consequence:
The input power, or energy, is negligible, being:

$$P_i = V_S I_i \sim 0$$

The output power is instead:

$$P_o = \frac{V_o^2}{R_L} \gg P_i$$

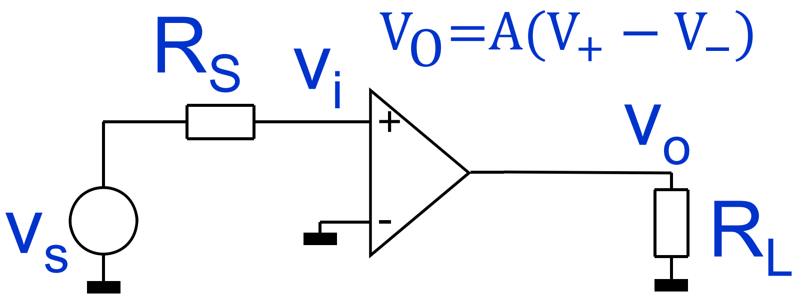
The amplifier is a transducer able to bring a weak signal, from a weak source, amplifying it for generating a strong signal across a load; the output delivered energy being larger than the input energy.
The amplifier is not able to generate energy by itself and, for taking into account the actual balance of energy, we must consider that the amplifier is not isolated and some voltage sources are needed to provide the energy:



The amplifier merely takes energy from the supply voltages, bringing it to the load. In this respect the output cannot exceed the supply rail.

AMPLIFIERS: SYMBOLS 3

There is another common, actually the most widely diffused, amplifier configuration: the differential amplifier or Operational Amplifier, OA:



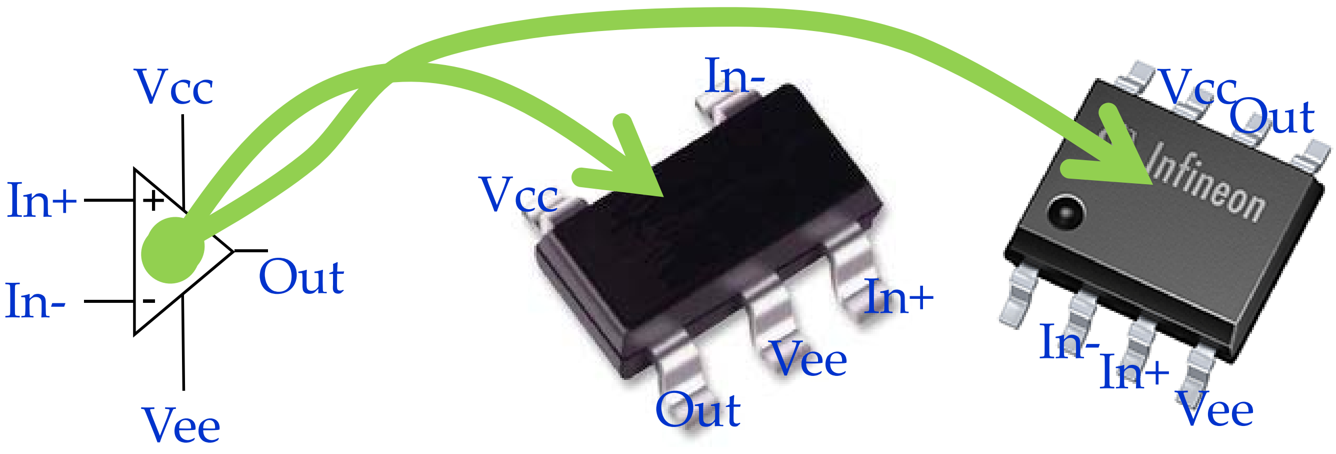
The advantage of such a configuration will be soon evident in connection with feedback, but also in several other applications. OAs are able to amplify signals floating from a common potential, in contrary to single ended input amplifiers.
OAs have only a disadvantage in some situations concerning noise.

The concept of the OA as it is shown above is very useful and widely exploited in modern electronic design.

We will start from the above ideal model and, as soon as we will acquire confidence, we will complete it with its several actual characteristics.

AMPLIFIERS: SYMBOLS 4

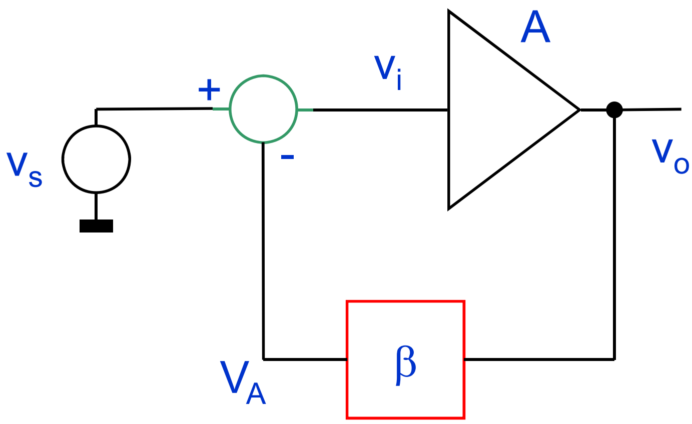
The model, actual or ideal, is not an abstraction as OAs exist and are widely used.
The know-how we will acquire will find practical applications.



We can consider our OA as a black box having some specifications which links its output to the difference of its inputs with a certain proportionality, or gain, A that is a more or less complicated function of frequency or time.
The factor A we are referring is generic, but must assume different values, which depend on the applications and we should find the method to personalize it.

The negative feedback concept (1)

Our aim is to try to find a way of obtaining an amplifier suitable for an application, whatever it is, starting from a generic standard amplifier.
Let's start from an ideal situation for better understand the concept.



In a feed-backed system a fraction of the output is taken and subtracted or summed to the input. In the first case we deal with negative feedback, in the second case positive or re-generative feedback.

We are interested in negative feedback for analogue systems, while re-generative feedback is exploited in digital applications.

The solution of the above network is the following:

$$V_o = AV_i$$

$$V_i = V_s - V_A$$

$$V_A = \beta V_o$$

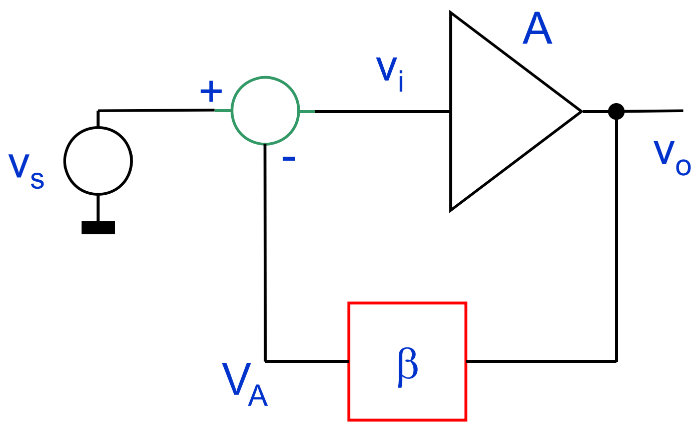
$$V_o = A(V_s - \beta V_o)$$

$$AV_s = V_o + A\beta V_o$$

$$AV_s = V_o(1 + A\beta)$$

$$V_o = \frac{A}{1 + A\beta} V_s$$

The negative feedback concept (2)



Let's concentrate on 2 of the relations obtained so far:

$$1) \qquad V_o = A(V_s - \beta V_o) \quad \Rightarrow \quad V_o = AV_s - A\beta V_o$$

$$2) \qquad V_o = \frac{A}{1 + A\beta} V_s$$

The first eq says that the output is given by the algebraic sum of a term proportional to the input excitation, AV_S, and the quantity $-A\beta V_O$, the fed backed quantity.

The parameter **T= -Aβ** is told ***loop gain***, because its path starts from the output and arrives at the output itself, after having travelled around the feedback path.

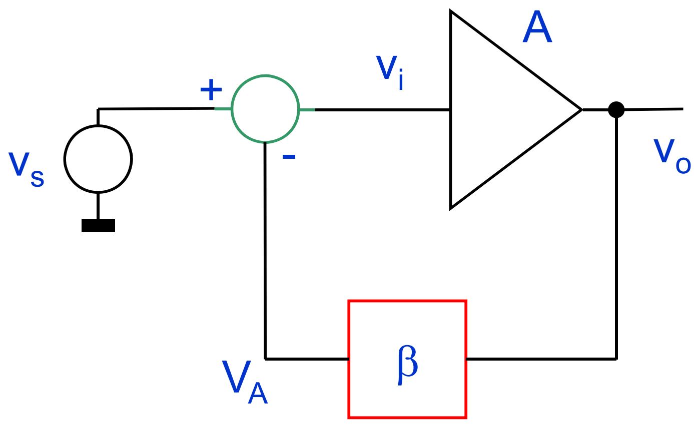
Eq. (2) can be re-written:

$$2) \qquad V_o = \frac{A}{1 - T} V_s$$

At this point the meaning of negative or re-generative feedback becomes clear:

- Negative feedback has T<0 and the denominator of (2) is always different from 0;
- Re-generative feedback (that is not this situation now) has T>0 and, therefore, the denominator of (2) can be zero, leading to divergence.

The negative feedback concept (3)



$$V_o = \frac{A}{1 - T} V_s \qquad T = -A\beta$$

We can recognize the negative feedback mechanism:

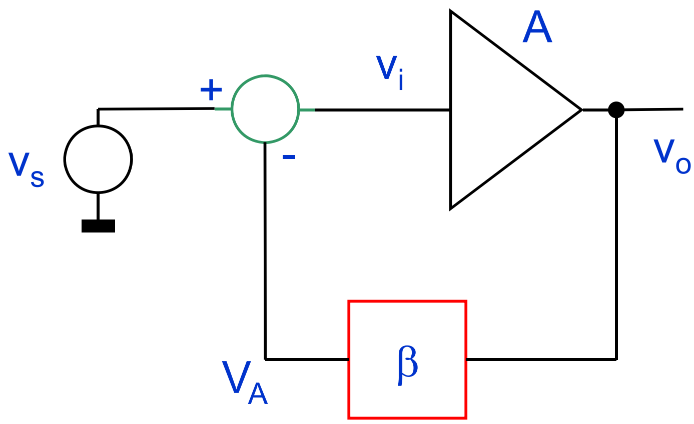
- A fraction of the output is subtracted from the input if A>0;
- A fraction of the output is summed to the input if A<0.

Conceptually, the aim is to reach an equilibrium condition.

The typical negative feedback example is when we drive a car at constant speed on an autobahn: we monitor the odometer (the output) and regulate the speed by pressing/leaving the gas-pedal.

Positive feedback on an autobahn is set by increase the pushing of the gas-pedal whenever we see that the speed on the odometer rises, resulting in a crash.

The negative feedback concept (4)



So far we have made some mathematical investigations and found a consistency.
But we would like to exploit this property for adapting the amplifier gain to our application.

That is soon understood if we elaborate a bit our result:

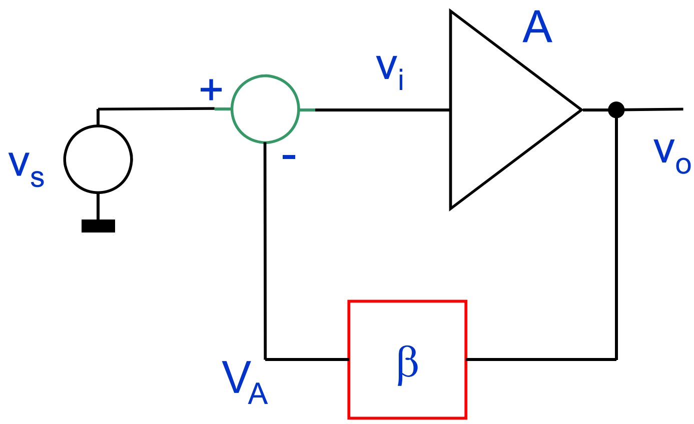
$$V_o = \frac{A}{1 + A\beta} V_s = \frac{1}{\beta} \frac{A\beta}{1 + A\beta} V_s = \frac{1}{\beta} \frac{-T}{1 - T} V_s$$

First: one would try not to be tied to the value of the amplification factor A. This is easily obtained if A is very very large because in this case we obtain:

$$V_o = \frac{1}{\beta} \frac{A\beta}{1 + A\beta} V_s \xrightarrow[A>>1]{} \frac{1}{\beta} V_s$$

This is very important and practical: if A is very very large the output signal is independent from A.
This implies an important property: there is no need that A has a precise value, we only ask it to have a very very large value.
Indeed, the design of the operational amplifiers, as they are found off-the-shelf, follows this strategy: an amplifier with a very very large and not precise gain.

The negative feedback concept (5)



$$V_o = \frac{1}{\beta}\frac{A\beta}{1+A\beta}V_s \xrightarrow[A>>1]{} \frac{1}{\beta}V_s$$

Second: once that A is very very large and the approximation above is valid, the output is proportional to the input by the value 1/β and, if β is smaller than 1, the output has an absolute value larger than the input: the output signal is an amplification of the input signal with an amplifying factor that does not depend on the amplifier gain A.

That is a very good and useful result since a factor β smaller than 1 is easily obtainable, we will soon see how, with passive components, i.e. resistors, capacitors, etc. and the values and precision of these components can be easily found off-the-shelf.

The negative feedback concept (6)

At a first glance it could be that the precision of the final gain would not be good, as the gain A changes with the environment (temperature, humidity, …) and from sample to sample.

We can verify if this is true or nature is kindness with us.
Let's start from:

$$A_f = \frac{v_o}{v_s} = \frac{1}{\beta}\frac{A\beta}{1+\beta A}$$

…and calculate:

$$\Delta A_f = \frac{1}{\beta}\,\frac{\beta(1+\beta A) - \beta^2 A}{(1+\beta A)^2}\,\Delta A$$

$$\Delta A_f = \frac{1}{\beta}\,\frac{\beta}{(1+\beta A)^2}\,\Delta A$$

$$\Delta A_f = \frac{1}{\beta}\,\frac{\beta A}{(1+\beta A)^2}\,\frac{\Delta A}{A}$$

$$\Delta A_f = \frac{1}{\beta}\,\frac{A\beta}{1+\beta A}\frac{1}{1+\beta A}\frac{\Delta A}{A}$$

$$A_f = A_f\frac{1}{1+\beta A}\,\frac{\Delta A}{A}$$

$$\Rightarrow \qquad \frac{\Delta A_f}{A_f} = \frac{1}{1+\beta A}\frac{\Delta A}{A}$$

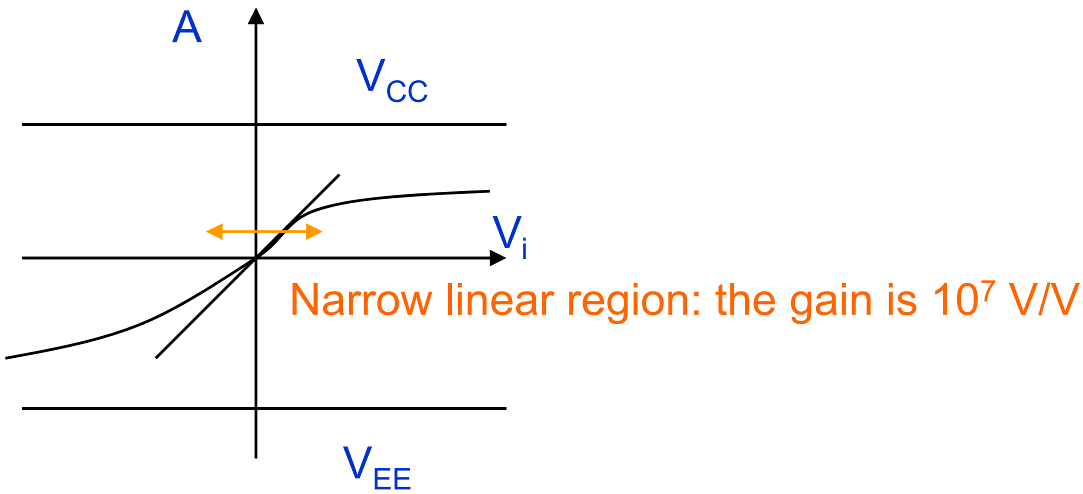
The above result says that the percentage of change of the final gain, A_f, equals the percentage of change of the gain A, but attenuated by the factor $1+\beta A$.

The negative feedback concept (6)

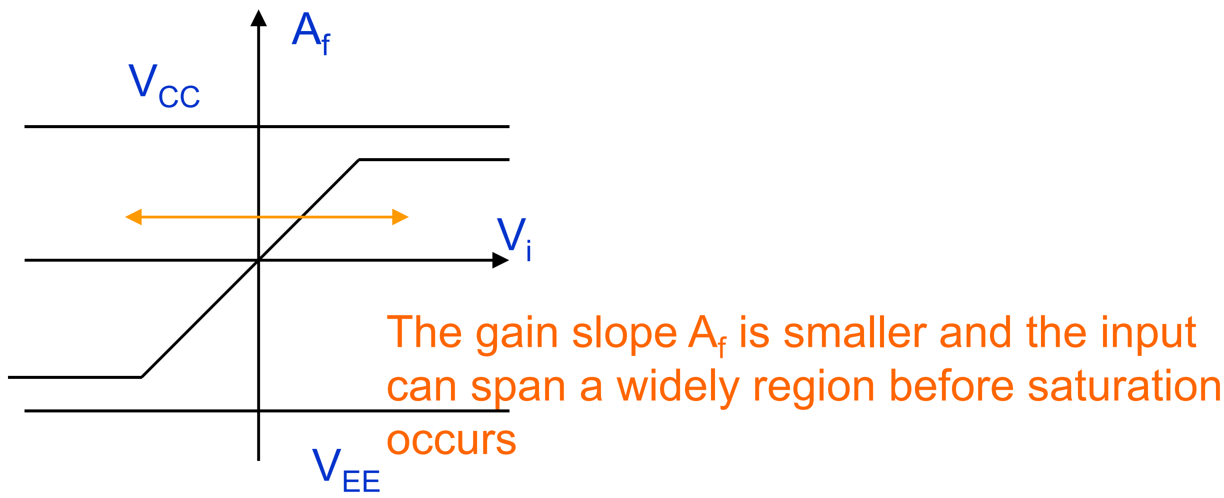
$$\frac{\Delta A_f}{A_f} = \frac{1}{1+\beta A}\frac{\Delta A}{A}$$

Just for having an idea. If β=0.01, or the gain 1/β is 100, and $A=10^7$ V/V, the factor βA results in 10^5, and even a 100% of change of the gain A results in only 10^{-3}% of change in A_f.

This effect can be interpreted graphically. The response to the input signal of the open-loop OA, namely, the OA with no feedback, or with β=0 is very steeply and gets saturation towards the rails as soon as the input move away from zero:



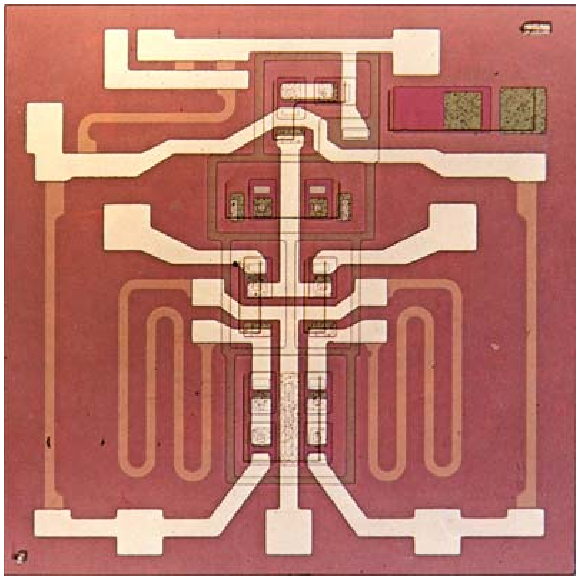
The gain A_f is much smaller than A and the input voltage can spans larger values before the saturation toward the rails occurs:



... a further bit of history

The first high gain amplifiers introduced were not monolithic, but based on valves. They were the K2-W and 12AX7 in the '40 - '50 of the last century.

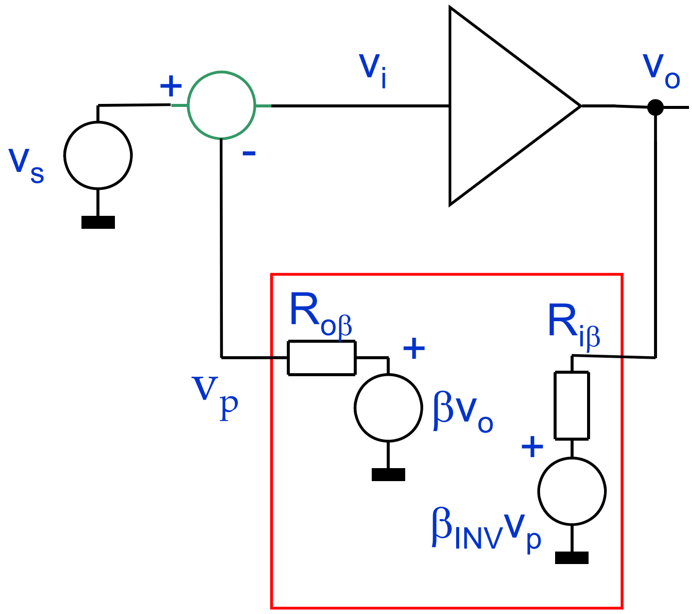
...and then it came Bob Widlar with the µA700 in 1963:



The negative feedback concept (7)

Our study must move towards the real word and we need to start to consider we are not facing ideal elements.

Our feedback will be composed of passive components, in general, and to model it the feedback block is expanded this way:



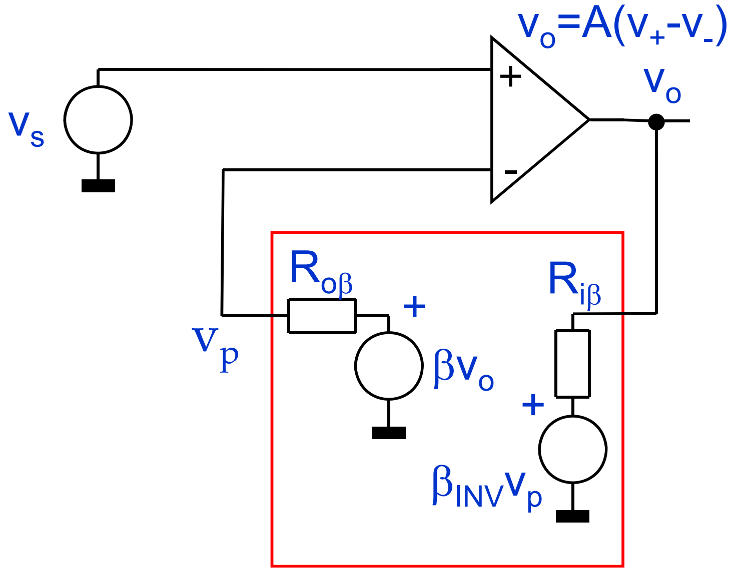
The block β is expanded in a 2 ports network. Its output is modelled with a real voltage source, but also its input is modeled the same way. The arguments seen in the previous pages remains valid: at most the loop gain is changed, as it will be evident soon.

The block is modeled bi-directional, typical of a passive network, for which signals can go back and forth.

The model above is very easy, but in a practical case is not obvious the passage from the actual network to the two-port network above and we will try to skip this passage, trying to adopt an analytical/physical method.

The negative feedback concept (8)

Before to continue we simplify the system by using an OA for which the subtracting block is missed and the non-inverting input is exploited instead:



Let's introduce now the way to solve the network by successive approximation.
We expect now that (we will verify this):

$$V_o = \frac{1}{\beta}\frac{A\beta f(\beta)}{1 + A\beta f(\beta)}V_s \xrightarrow[A>>1]{} \frac{1}{\beta}V_s$$

The term $f(\beta)$ is now introduced to take into account that the feedback network (and the OA, see later) is not ideal and some effects are introduced. Normally, for a passive feedback network $|f(\beta)| \leq 1$.
The first approximation we consider is A=∞, even if this is not true.
In this case we expect that:

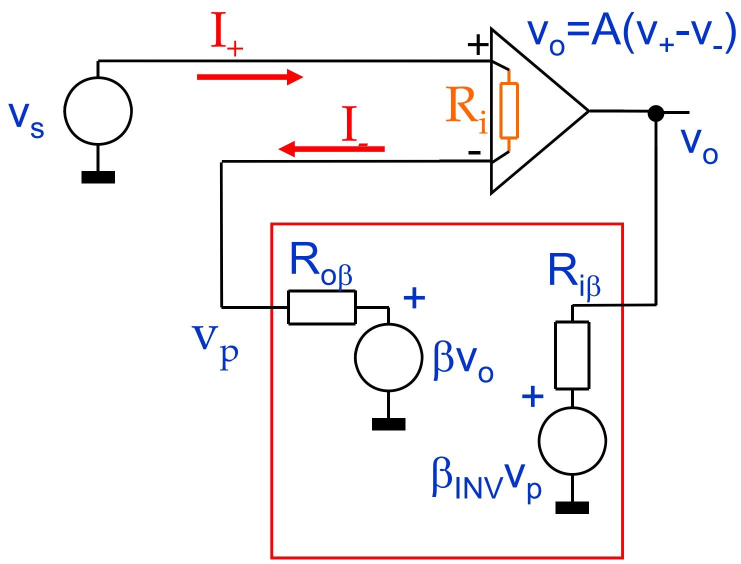
$$V_o = \frac{1}{\beta}V_s$$

With an error of the order of:

$$\frac{A\beta f(\beta)}{1 + A\beta f(\beta)}, \qquad T = -A\beta f(\beta)$$

Non-ideality means that the loop gain T becomes smaller, but still large if the system is well designed

The negative feedback concept (9)



The double assumption:

$$(1)\,A = \infty \quad \Rightarrow \quad (2)\,V_o = \frac{1}{\beta}V_s$$

is very useful from the point of view of the solution of the network in the actual case.

Condition (2) says that the output V_o has a finite value if V_s has a finite value too (within obvious limits) .

If V_o has a finite value and A=∞, then, being respected that $V_o=A(V_+-V_-)$, we expect that:

$$(3) \qquad V_+ \approx V_-$$

…not only. The OA above has been made a bit less ideal considering the presence of an input impedance **R_i**.

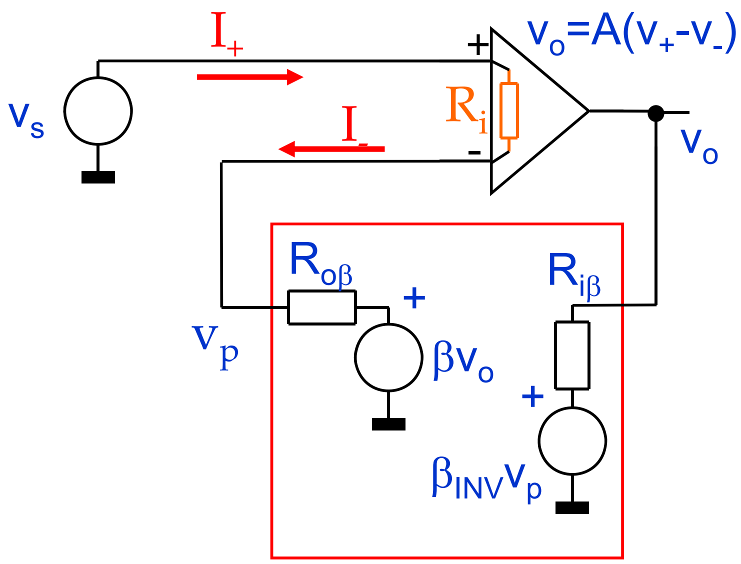
From (3) we have that:

$$I_+ = I_- = \frac{V_+ - V_-}{R_i} = \frac{0}{R_i}$$

So we can add this further result:

$$1) \quad A = \infty \Rightarrow (2)\;V_o = \frac{1}{\beta}V_s, \quad (3)\;V_+ \approx V_-, \quad (4)\;I_+ = I_- = 0$$

The negative feedback concept (10)



$$1)\quad A=\infty \Rightarrow (2)\ V_O=\frac{1}{\beta}V_S,\quad (3)\ V_+\approx V_-,\quad (4)\ I_+=I_-=0$$

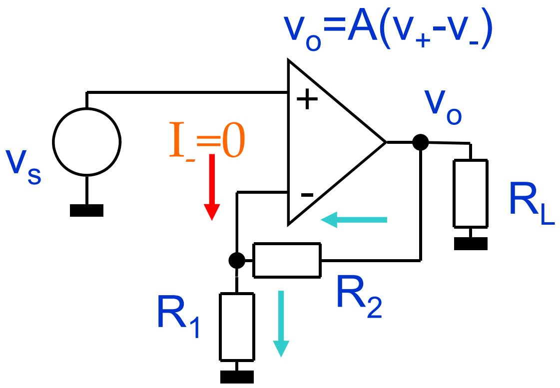
Applying (4) above we obtain that $V_-=\beta V_o$,

then, now with (3): $V_s=\beta V_o$, namely: $V_o=V_s/\beta$, irrespectively of the non ideal feedback.

So far, as soon as the gain is very very large the feedback works the same way in both ideal and non-ideal conditions.

We will see in the following what does it happen with the loop gain T.

The negative feedback circuit examples (1)



Let's apply the condition:

$$1)\ A = \infty \Rightarrow (2)\ V_O = \frac{1}{\beta} V_S,$$

$$3)\ V_+ \approx V_-, \qquad (4)\ I_+ = I_- = 0$$

According to (1) and (2):

From (1):

$$v_+ = v_s \approx v_-$$

Therefore currents in R_1 and R_2 satisfies , according to (4):

$$\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1}$$

$$\frac{v_o}{R_2} = \frac{R_1 + R_2}{R_1 R_2} v_-$$

$$\Rightarrow \quad v_o = \frac{R_1 + R_2}{R_1} v_-$$

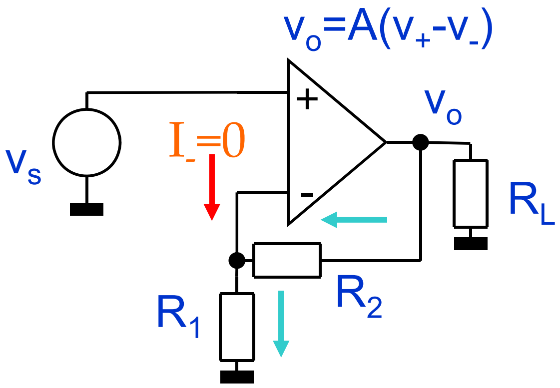
The final result has been obtained very quickly without the need of modelling the feedback by mean of a 2-ports network.

Finally, from (2):

$$v_O = \frac{R_1 + R_2}{R_1} v_S \quad \text{hence:} \quad \beta = \frac{R_1}{R_1 + R_2}$$

(See also Appendix A)

The negative feedback circuit examples (2)



$$v_O = \frac{R_1 + R_2}{R_1} v_S \;\; \text{hence:} \;\; \beta = \frac{R_1}{R_1 + R_2}$$

In obtaining the above result we did not apply any considerations about the input/output impedance of the fed backed OA, nor the direct/indirect transmission across the feedback.

Moreover, under the approximation we did, the result is independent from the load impedance R_L.

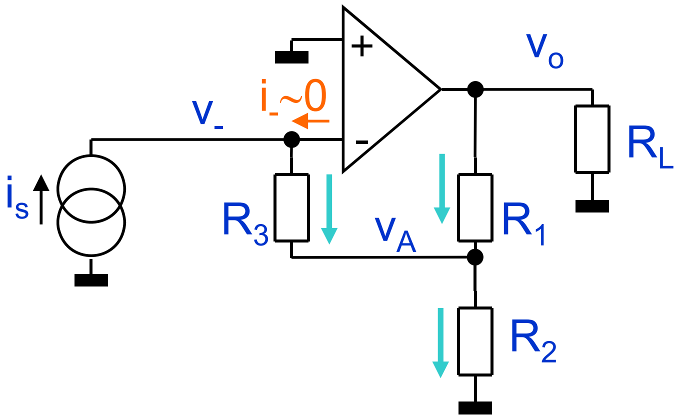
In solving the network we have considered the OA ideal. This assumption does not affect the result as soon as the gain A is assumed ∞.

We are dealing with negative feedback: a fraction of the output is taken to the inverting input, namely, is subtracted from the input.

We will see soon that when the OA is assumed not ideal and all the elements are taken into consideration, the above result remains valid and some multiplicative terms are added, or summed.

The negative feedback circuit examples (3)

Another example:



1) $A = \infty \;\Rightarrow\; (2)\; V_O = \frac{1}{\beta} V_S$

3) $V_+ \approx V_-$, (4) $I_+ = I_- = 0$

Again a fraction of the output is taken at the inverting input, where it is subtracted: once again the feedback is negative.

$v_- \approx v_+ = 0$ Apply (1) and take care: here v_- results 0 just because v_+ is connected to ground.

$$i_S = \frac{v_- - v_A}{R_3} = \frac{0 - v_A}{R_3} = -\frac{v_A}{R_3}$$ Apply (4)

$$\frac{v_o - v_A}{R_1} - \frac{v_A}{R_3} = \frac{v_A}{R_2} \qquad \frac{v_o - v_A}{R_1} + i_s = \frac{v_A}{R_2}$$

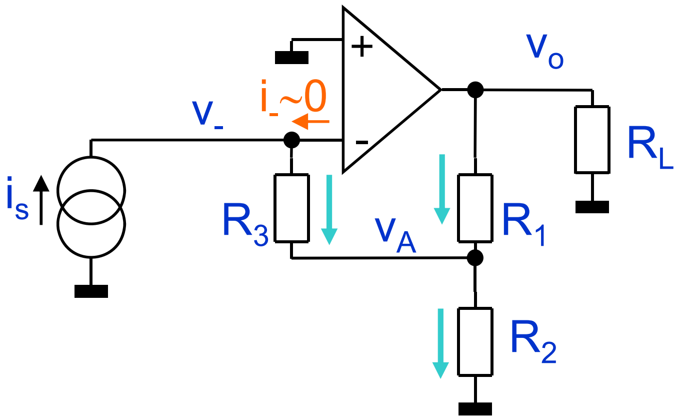
$$\frac{v_o}{R_1} = -i_s + \frac{R_1 + R_2}{R_1 R_2} v_A$$

$$\frac{v_o}{R_1} = -i_s - \frac{R_1 + R_2}{R_1 R_2} R_3 i_s$$

Again we have obtained the link between the output and input.

$$v_o = -\frac{R_1 R_2 + R_3 (R_1 + R_2)}{R_2} i_s$$

The negative feedback circuit examples (4)



$$1)\, A = \infty \Rightarrow (2)\; V_O = \frac{1}{\beta} V_S,$$

$$3)\, V_+ \approx V_-, \qquad (4)\; I_+ = I_- = 0$$

$$v_o = -\frac{R_1R_2 + R_3(R_1 + R_2)}{R_2} i_s$$

Again, (2) is valid, therefore:

$$\beta = -\frac{R_2}{R_1R_2 + R_3(R_1 + R_2)}$$

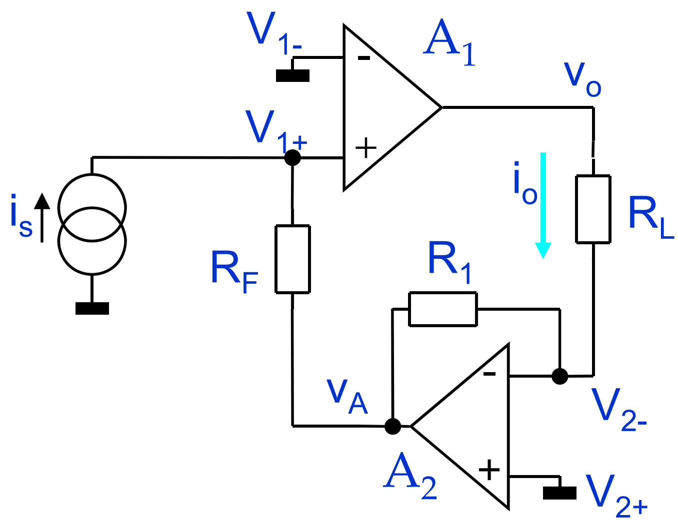
Here we have found other 2 properties:

The output is a voltage, while the input is a current: when to disentangle between the input/output variable type?

The consequence of the difference in output and input variables type is that the gain 1/β results dimensional. This seems to conflict to the fact found that βA must be dimensionless.

The answers to both of the above questions will be soon given.

The negative feedback circuit examples (5)



Here we have a first OA A_1, for which across its feedback path there is another feed-backed OA, A_2.
Note that since A_2 is inverting the feedback at the input of A_1 is taken at the non-inverting node, for negative feedback.
Again:

$$1)\ A = \infty \ \Rightarrow \ (2)\ V_O = \frac{1}{\beta} V_S,$$

$$3)\ V_+ \approx V_-, \qquad (4)\ I_+ = I_- = 0$$

At the input node:

$$V_{1+} \approx V_{1-} = 0$$ From (3)

$$I_s = \frac{V_+ - V_A}{R_F} = -\frac{V_A}{R_F}$$ From (4)

Now from the output, merging the input:

$$V_{2+} \approx V_{2-} = 0$$

$$\left(i_o = \frac{v_o - v_{2-}}{R_L} = \frac{v_o}{R_L} \right)$$

$$I_o = \frac{V_{2-} - V_A}{R_1} = -\frac{V_A}{R_1}$$

We see that input and output have the same dimension, but are currents.

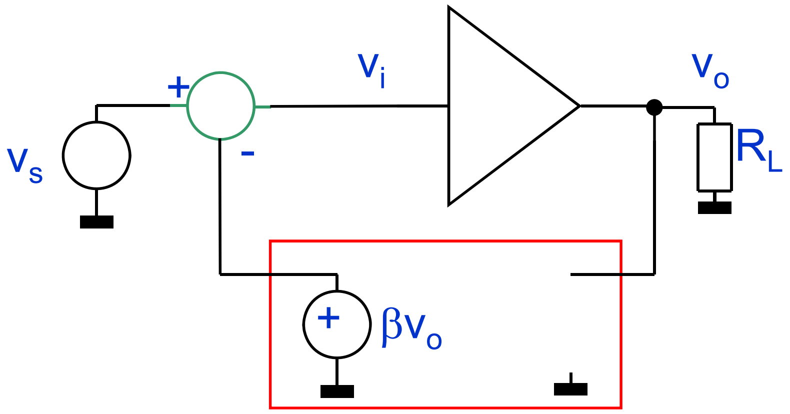
$$I_o = -\frac{-I_s R_F}{R_1} = \frac{R_F}{R_1} I_s$$

From (2): $\beta = \frac{R_1}{R_F}$ Parameter β is dimensionless.

So far we have became familiar in solving fed-backed networks in case the OA gain A is close to ∞.
In doing this, some question arose.

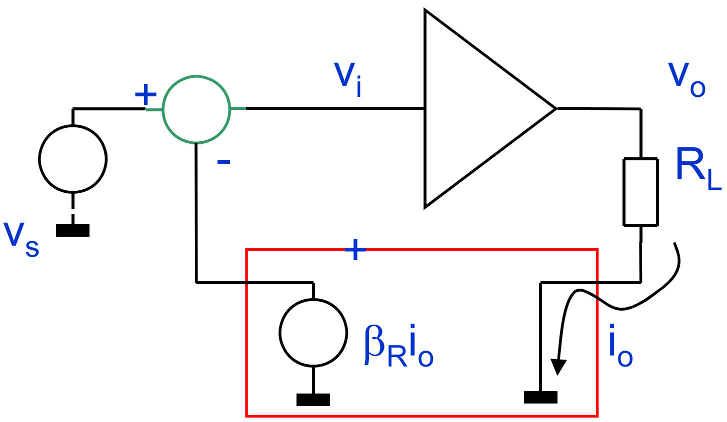
One of the results obtained is that the output and the input signal can differ in type. Actually this is the case and any combination of the output over the input can be obtained. Here the list of all cases.

Voltage output over voltage input:



The output voltage is read, and a fraction of it is subtracted from the input.

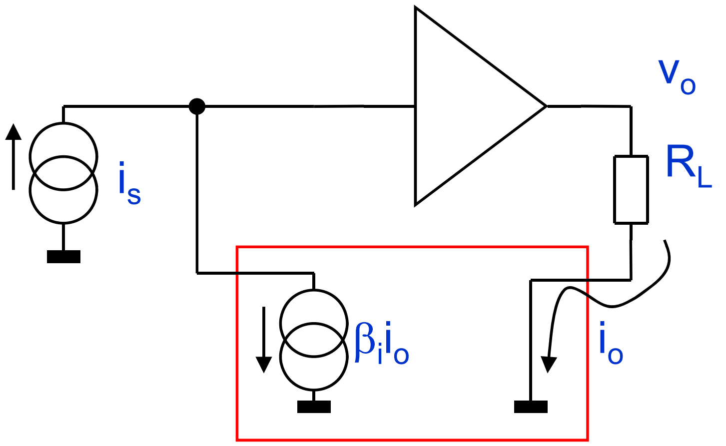
Current output over voltage input



The output current is read, and a fraction of it is multiplied by a coefficient having proper dimension and subtracted from the input.

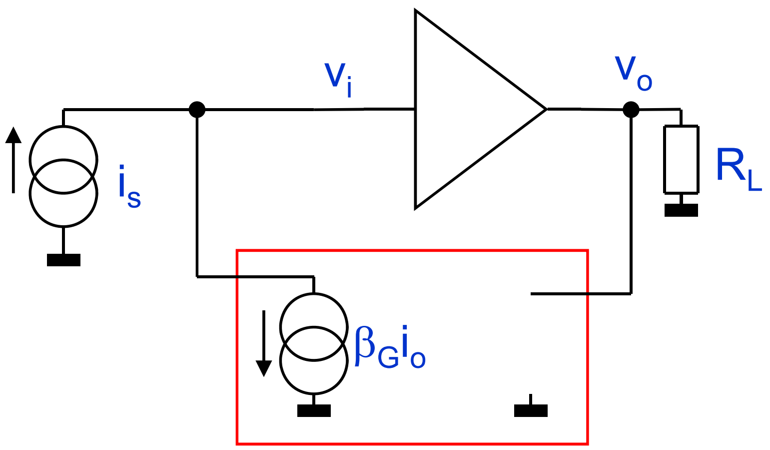
The negative feedback concept (11)

Current output over current input



The output current is read, and a fraction of it is subtracted from the input.

Voltage output over current input



The output voltage is read, and a fraction of it is, after being multiplied by a coefficient having proper dimension, subtracted from the input.

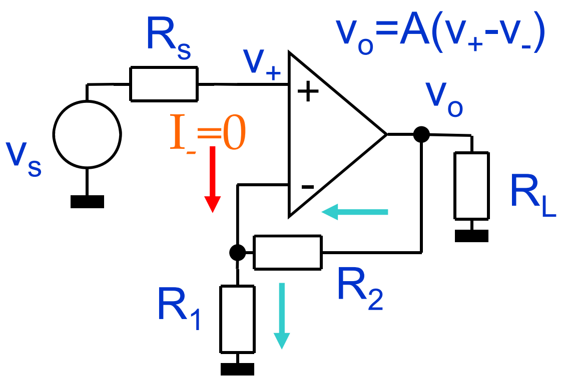
The negative feedback concept (12)

A further consideration is how to understand which is the variable, output or input, worked out by the feedback.

The answer is tied on what is our interest: we would like that the amplified signal is as much as possible independent from the source and load impedances, so approaching the ideal situation.

In this respect, rather than thinking at a variable that is managed we can think at our final aim.

Let's see the practical cases:



We have found that:

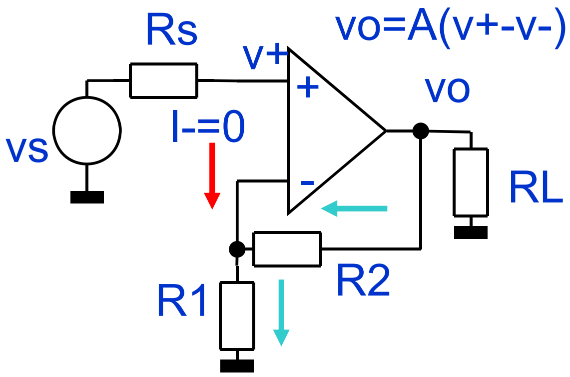
$$v_O = \frac{R_1 + R_2}{R_1} v_S$$

The output voltage is independent from the load impedance R_L, under the assumption that the gain is very very large, whereas the output current is:

$$i_O = \frac{v_O}{R_L} = \frac{R_1 + R_2}{R_L R_1} v_S$$

We can see that the output current is fully dependent on the load impedance: the output variable in this configuration is therefore the voltage.

The negative feedback concept (13)



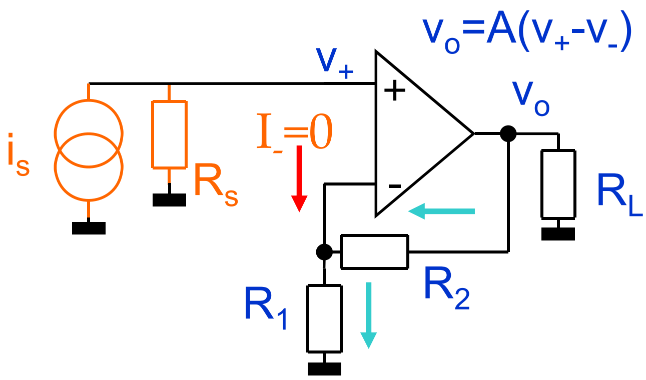
From our approximation we know that the input current is negligible; the consequence is that the voltage drop across R_s is negligible and $V_s = V_+$.

Indeed our result is:

$$v_O = \frac{R_1 + R_2}{R_1} v_S$$

So the final result does not depends on the input source resistor R_s and we can consider the voltage the input variable.

Let's consider the case the input is a current generator with its source parallel resistor R_s.



We have now that:

$$v_+ = R_S I_s \text{ and } v_+ = v_S$$

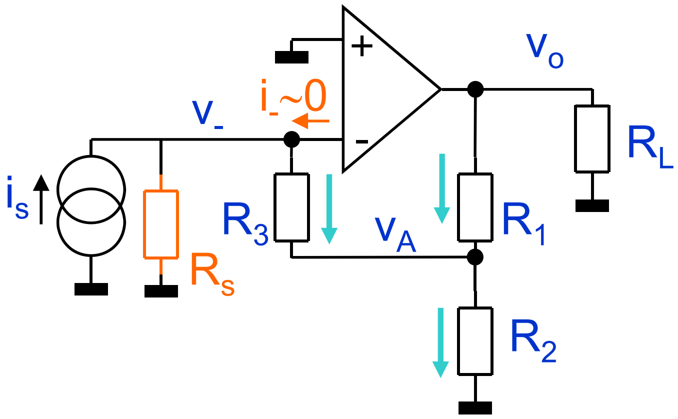
Therefore:

$$v_O = \frac{R_1 + R_2}{R_1} R_S I_s$$

We can see that the output voltage is fully dependent on the source impedance in this latter case: the input variable in this configuration is therefore the voltage.

The negative feedback concept (14)

Let's consider the second of our examples.



We have found that the gain is:

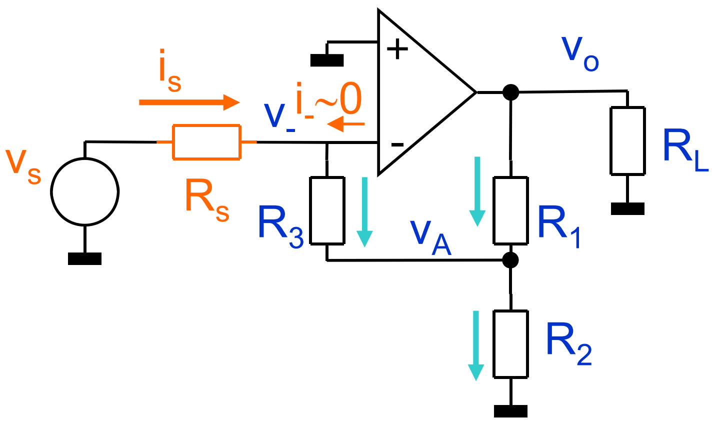
$$v_o = -\frac{R_1R_2 + R_3(R_1 + R_2)}{R_2} i_s$$

That does not depends on the load R_L: the output variable is therefore a voltage.

We now consider the source resistor R_s in parallel to the input current source.

Since v_+=0 V and $v_- \approx v_+$, then $v_-\approx$0 V and the current through R_s is 0 A: the output signal will not depend on R_s, as it happens, actually.

Let's consider the case the input is a voltage generator with its source series resistor R_s.



Now we have that:

$$i_s = \frac{v_s}{R_s}$$

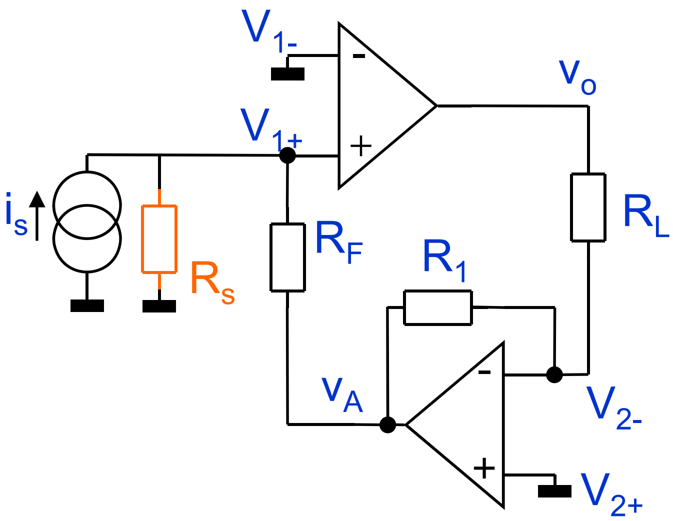
So that:

$$v_o = -\frac{R_1R_2 + R_3(R_1 + R_2)}{R_2} \frac{v_s}{R_s}$$

The output becomes dependent on the source impedance in this case: the amplification is about current.

The negative feedback concept (15)

Let's consider the last of our examples.



We have found that the gain is:

$$I_o = \frac{R_F}{R_1} I_s$$

The input follows the same consideration of the previous case: v_{1+} is at ground and the current in R_s is nulled.

We know that V_{2-} is about 0 V so that:

$$v_o = R_L I_o$$

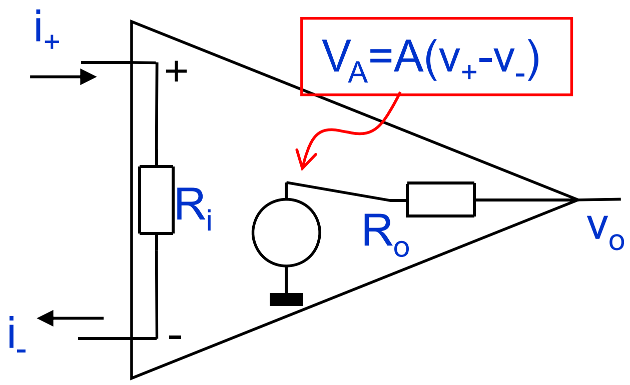
and:

$$v_o = R_L I_o = R_L \frac{R_F}{R_1} I_s$$

The output voltage depends on the load impedance R_L, and the output signal is, therefore, a current.

The feedback of real OAs (1)

We are now in the condition of considering an OA model closer to the actual:



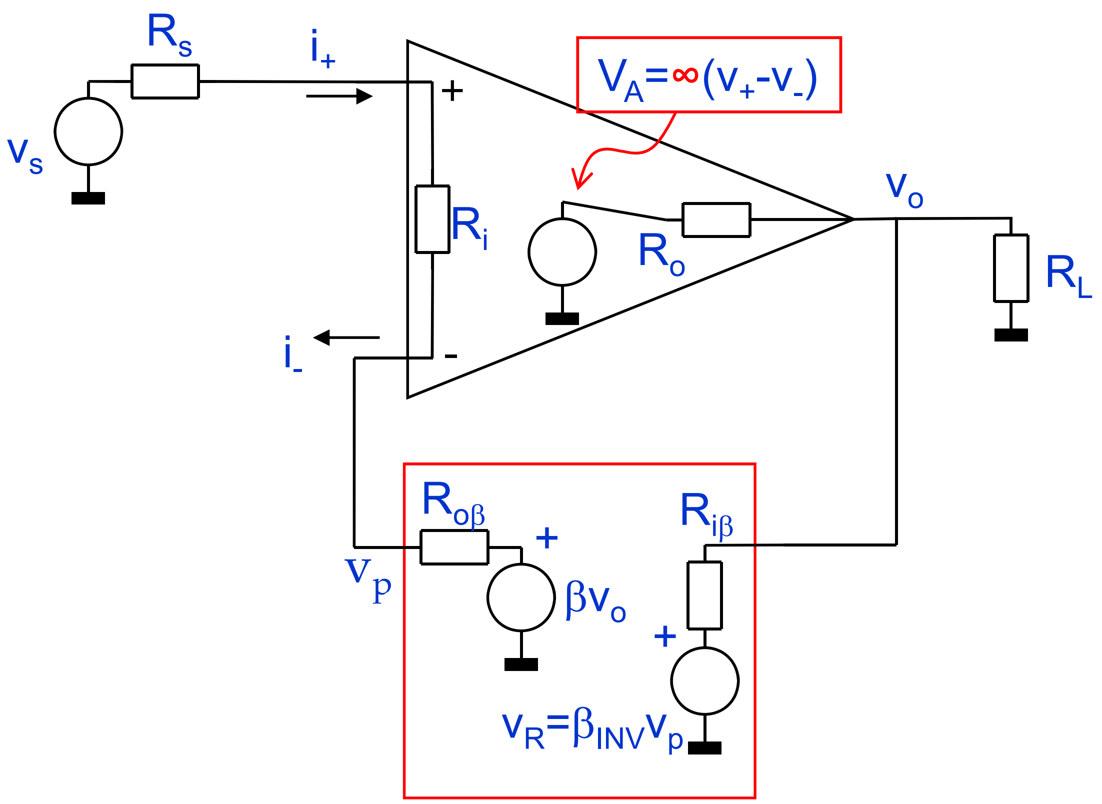
The level of detail of the model can be more sophisticated. For the moment we limit ourselves to consider a finite values of the input impedance, **R_i**, and output impedance, **R_o**, and of the voltage gain, **A**. These, and other parameter values, are listed in the datasheet of the OA we selected.

As it can be seen the gain of the amplifier is modelled with a voltage (or current) dependent source, namely a source whose voltage value depends on the voltage (or current) of another node.

We would develop a procedure for solving the network that allows for a solution having a degree of details depending on the requirements on precision.

The feedback of real OAs (2)

Now we add to our real OA a modelled real feedback, a real source and a real load:



First of all we check if our first approximation is still valid with a real OA and, for a while, let's consider the gain A having a very very large value. We know that:

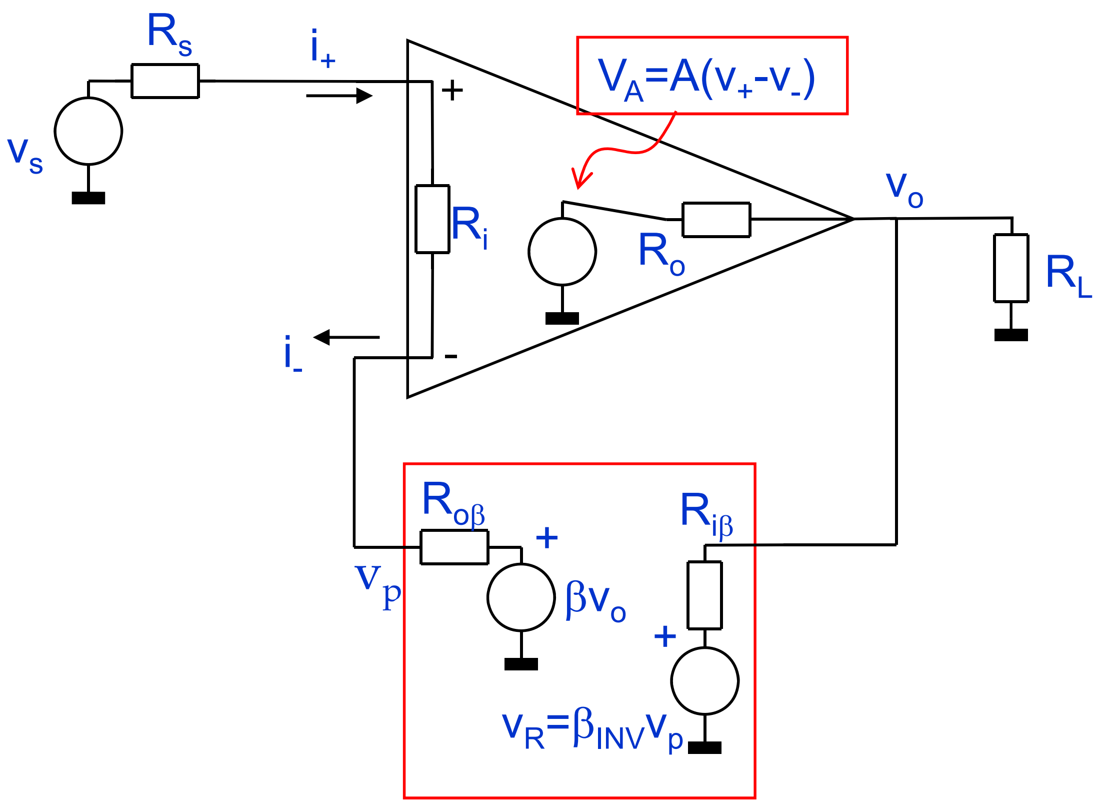
$$(1)\ A = \infty \ \Rightarrow\ (2)\ V_O = \frac{1}{\beta} V_S,$$

$$(3)\ V_+ \approx V_-, \qquad (4)\ I_+ = I_- = 0$$

Applying (4) we have that, at the input mesh: $\mathbf{v_s = v_+ = v_- = \beta v_o}$.

Or: $\mathbf{v_s = \beta v_o} \ \Rightarrow\ \mathbf{v_o = v_s/\beta}$, independent of all the impedances around and our approximation remains still valid and unaffected.

Now let's jump into details.



There are 3 dependent voltage sources and, as expected, the output is zero if the input source is zero: remember that we are now dealing with the signal and we have to think at it as a first order expansion of the static or DC working points, we will come back on this later.

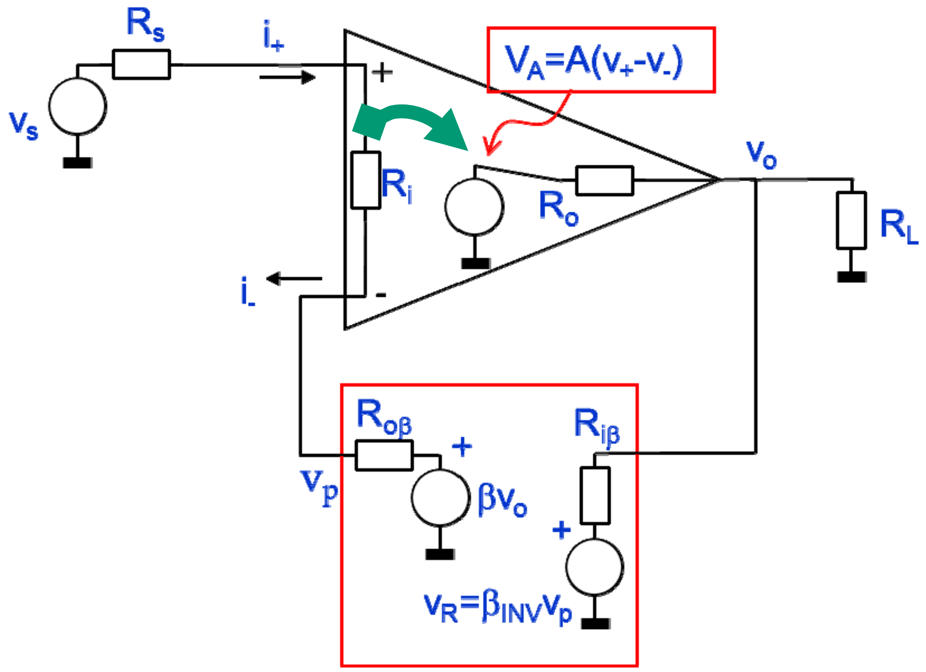
Voltages (or currents) dependant source are mathematically expressed by a law:

$$v_A = A(v_+ - v_-)$$

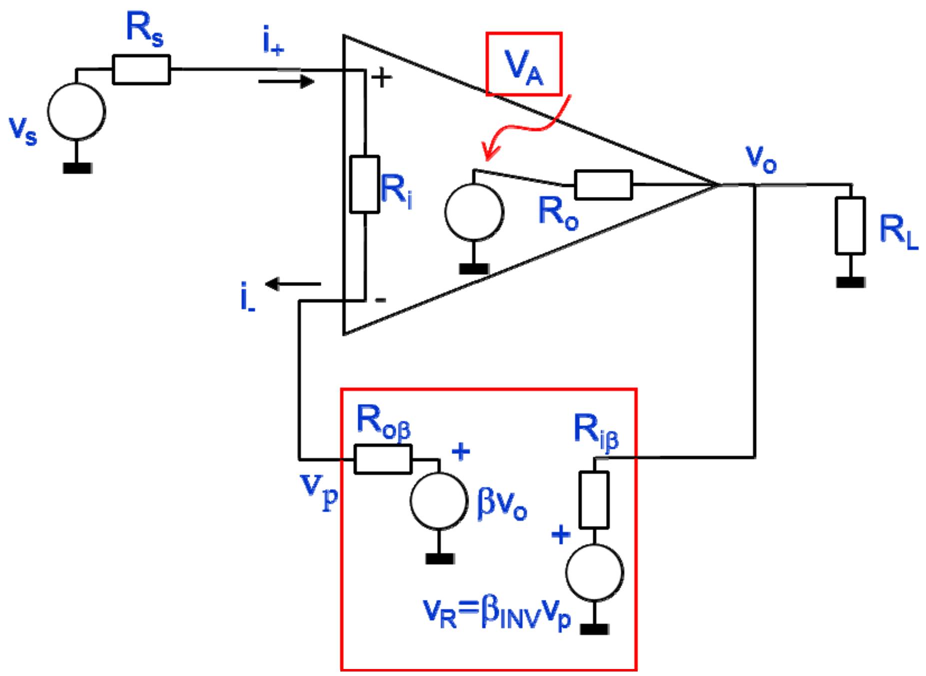
the above expression says that a variable on the left, v_A, can be assigned a function (just as y=f(x)) on the right, $A(v_+ - v_-)$.
We can omit the assignment to v_A when we manage it, for convenience, and considering v_A a "*generic variable* or unknown" and we apply the assignment only at the end of our manipulation.

The feedback of real OAs (4)

The interpretation just given is trivial from the mathematical point of view, but has impact from the physical point of view, and we can exploit this for our purpose.



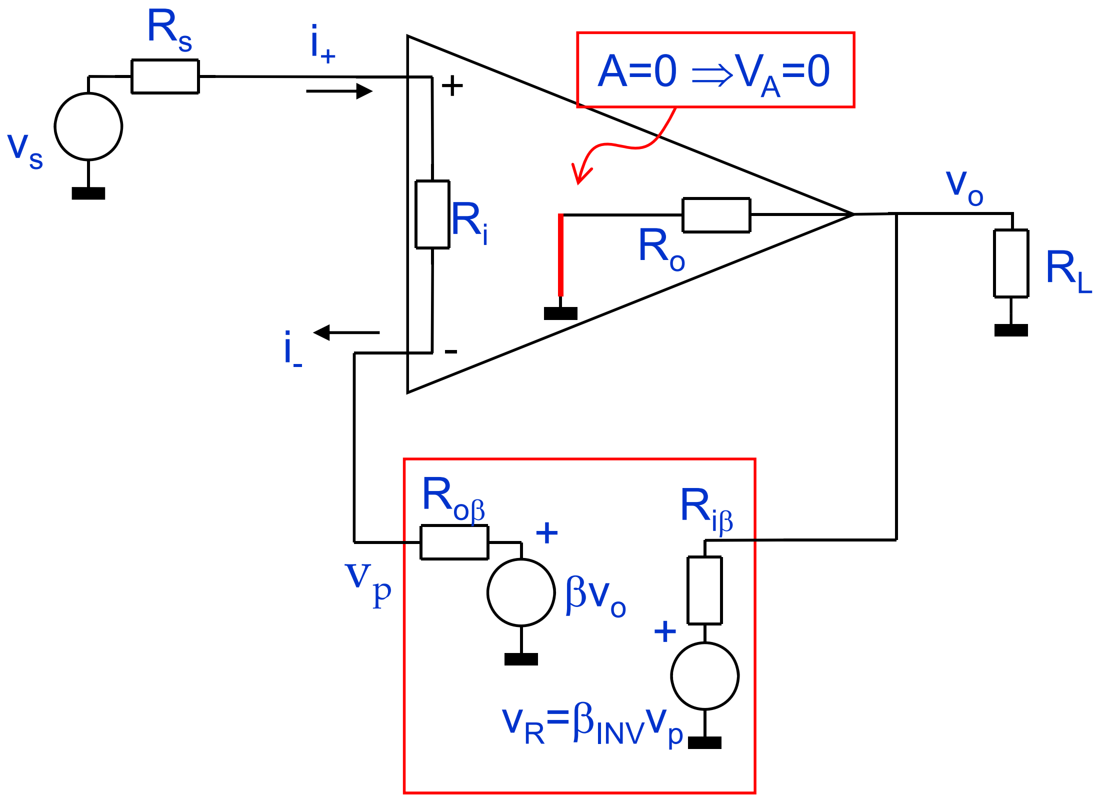
If we let v_A to equal (v_+-v_-) we close the feedback loop, and the OA model works.



If we let v_A to be *"a generic variable"* the OA remains open loop as its input and output are independent: the model is not perfect till we do not apply the actual assignment to v_A, but this un-assignment is useful to study the system as it lets the network unperturbed.

The feedback of real OAs (5)

We set **A=0**, this is equivalent to replace v_A with a short circuit:



Output mesh:

$$\begin{cases} v_{1o} = \dfrac{R_o R_L}{R_o + R_L} \dfrac{1}{\dfrac{R_L R_o}{R_L + R_o} + R_{i\beta}} v_R \\ v_p = \dfrac{R_{o\beta}}{R_{o\beta} + R_i + R_s}(v_s - \beta v_{1o}) + \beta v_{1o} \end{cases}$$

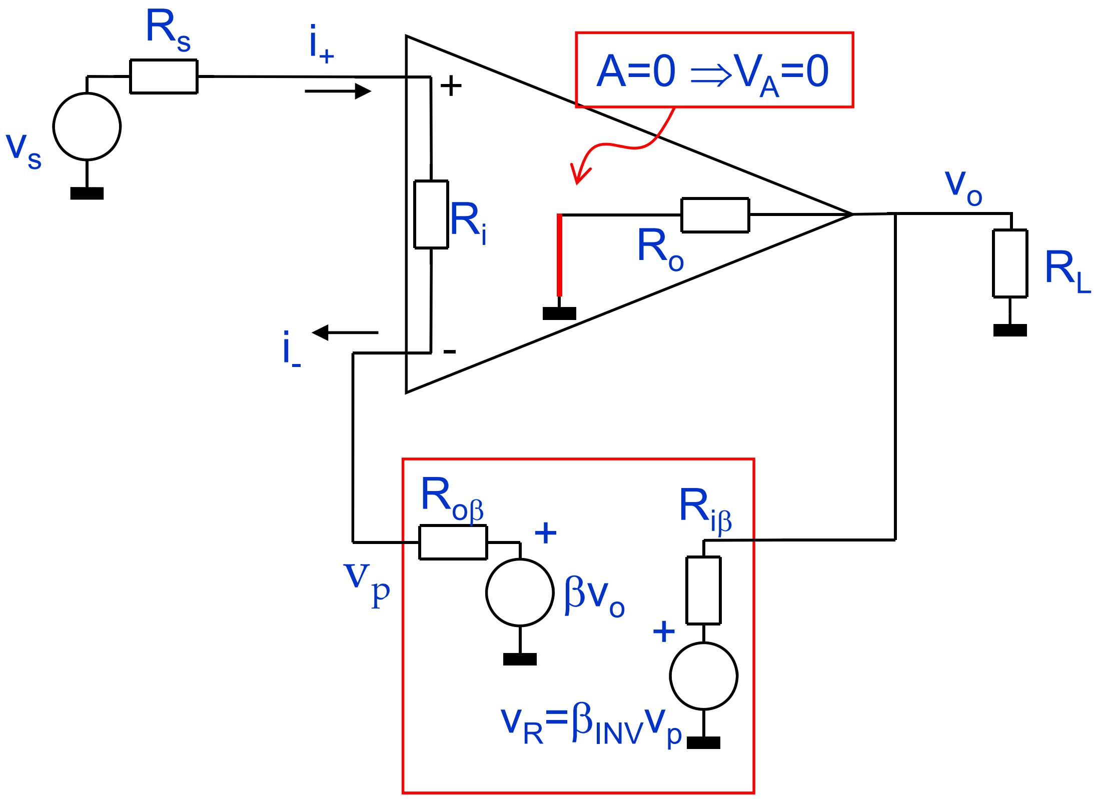
(here we see the application of the concept of "generic variable": v_R is left generic till the end of the calculation, for convenience this time)

$$v_p = \frac{R_{o\beta}}{R_{o\beta} + R_i + R_s} v_s + \frac{R_i + R_s}{R_{o\beta} + R_i + R_s} \beta v_{1o}$$

$$v_p = \frac{R_{o\beta}}{R_{o\beta} + R_i + R_s} v_s + \frac{R_i + R_s}{R_{o\beta} + R_i + R_s} \beta \frac{R_o R_L}{R_o + R_L} \frac{1}{\dfrac{R_L R_o}{R_L + R_o} + R_{i\beta}} v_R$$

The feedback of real OAs (6)

We set **A=0**, this is equivalent to replace v_A with a short circuit:



$$v_p = \frac{R_{o\beta}}{R_{o\beta}+R_i+R_s}v_s + \frac{R_i+R_s}{R_{o\beta}+R_i+R_s}\beta\frac{R_oR_L}{R_o+R_L}\frac{1}{\frac{R_LR_o}{R_L+R_o}+R_{i\beta}}v_R$$

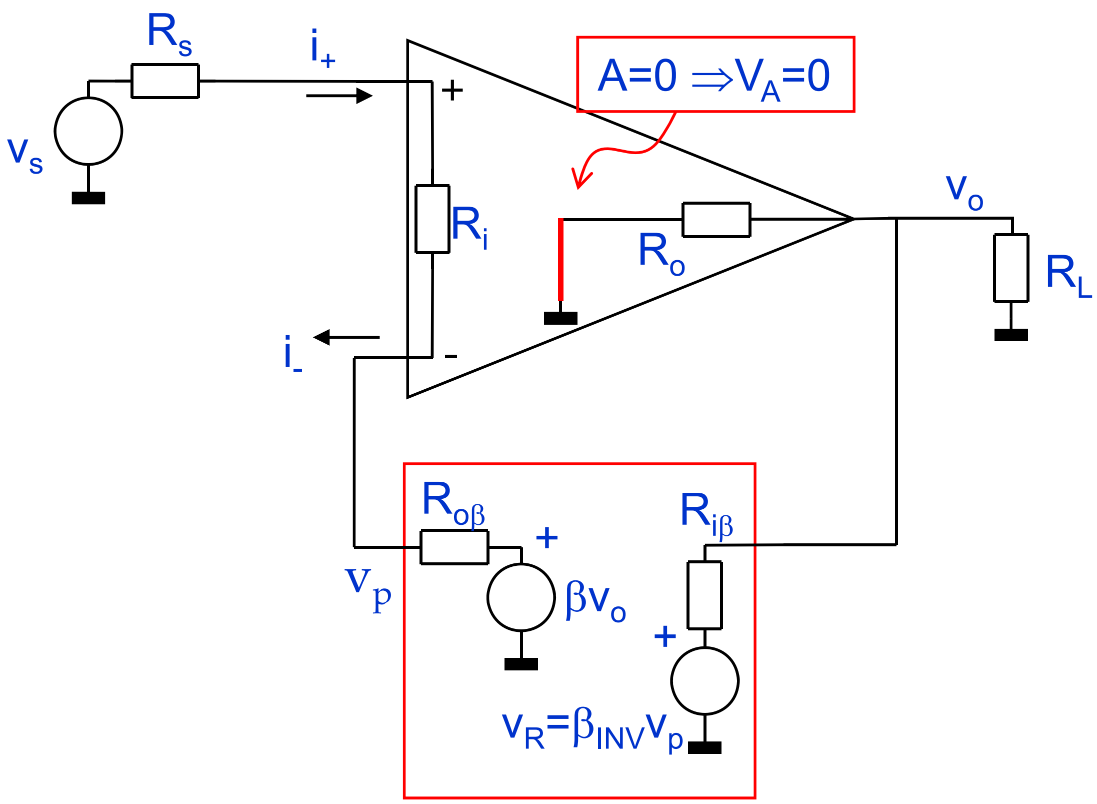
$$v_p = \frac{R_{o\beta}}{R_{o\beta}+R_i+R_s}v_s + \frac{R_i+R_s}{R_{o\beta}+R_i+R_s}\beta\frac{R_oR_L}{R_o+R_L}\frac{1}{\frac{R_LR_o}{R_L+R_o}+R_{i\beta}}\beta_{INV}v_p$$

$$v_p = \frac{R_{o\beta}}{R_{o\beta}+R_i+R_s}\frac{1}{1-\frac{R_i+R_s}{R_{o\beta}+R_i+R_s}\frac{R_oR_L}{R_o+R_L}\frac{\beta\beta_{INV}}{\frac{R_LR_o}{R_L+R_o}+R_{i\beta}}}v_s$$

$$v_p = \frac{R_{o\beta}}{R_{o\beta}+\left(1-\frac{R_oR_L}{R_o+R_L}\frac{\beta\beta_{INV}}{\frac{R_LR_o}{R_L+R_o}+R_{i\beta}}\right)(R_i+R_s)}v_s$$

The feedback of real OAs (7)

We set **A=0**, this is equivalent to replace v_A with a short circuit:



$$v_p = \frac{R_{o\beta}}{R_{o\beta} + \left(1 - \frac{R_o R_L}{R_o + R_L} \frac{\beta\beta_{INV}}{\frac{R_L R_o}{R_L + R_o} + R_{i\beta}}\right)(R_i + R_s)} v_s$$

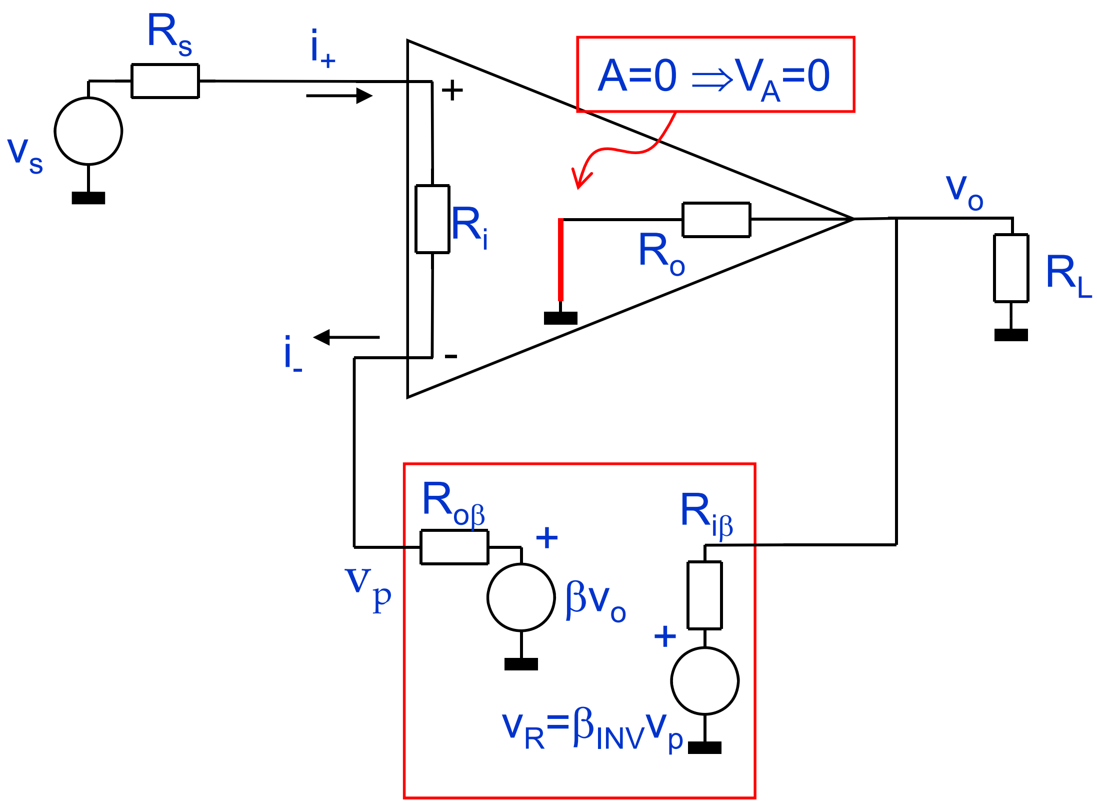
$$v_p = \frac{R_{o\beta} \Big/ \left(1 - \frac{R_o R_L}{R_o + R_L} \frac{\beta\beta_{INV}}{\frac{R_L R_o}{R_L + R_o} + R_{i\beta}}\right)}{R_{o\beta} \Big/ \left(1 - \frac{R_o R_L}{R_o + R_L} \frac{\beta\beta_{INV}}{\frac{R_L R_o}{R_L + R_o} + R_{i\beta}}\right) + (R_i + R_s)} v_s$$

Therefore:

$$v_p = \frac{R_{o\beta A}}{R_{o\beta A} + R_i + R_s} v_s \qquad \left(R_{o\beta A} = \frac{R_{o\beta}}{1 - \frac{R_o R_L}{R_o + R_L} \frac{\beta\beta_{INV}}{\frac{R_L R_o}{R_L + R_o} + R_{i\beta}}} \right)$$

The feedback of real OAs (8)

We set **A=0**, this is equivalent to replace v_A with a short circuit:



$$v_p = \frac{R_{o\beta A}}{R_{o\beta A} + R_i + R_s} v_s$$

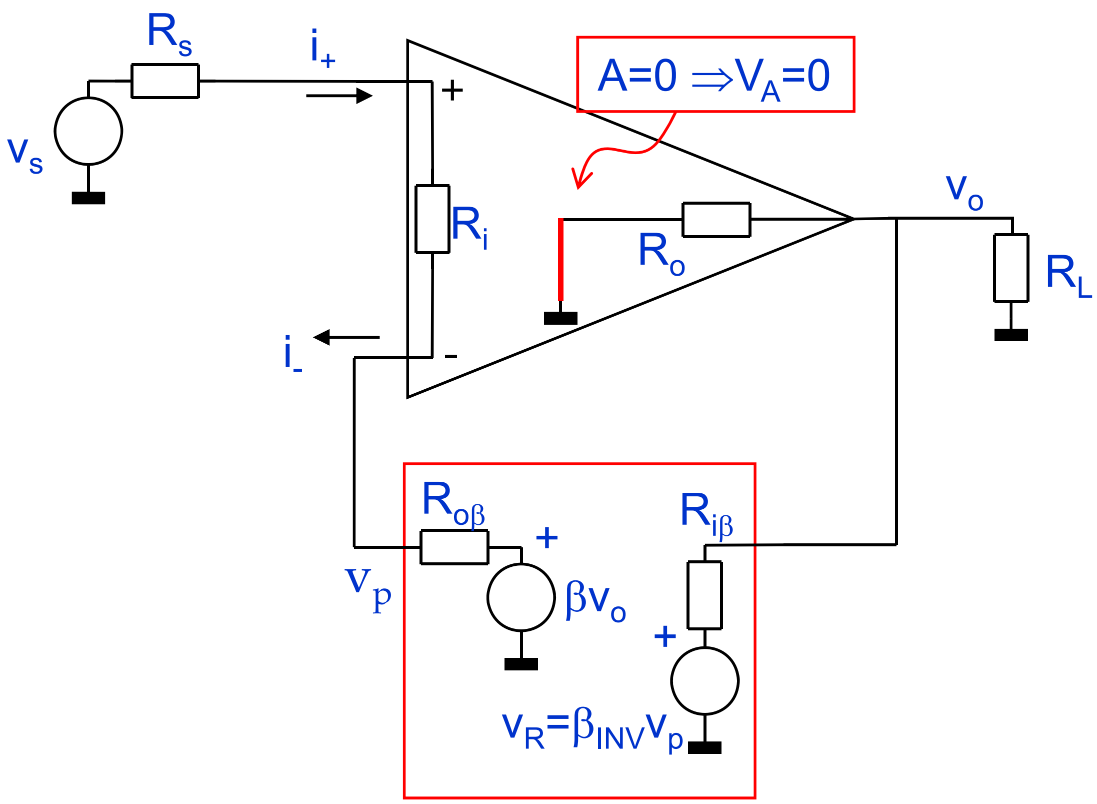
Output mesh:

$$v_{1o} = \frac{R_o R_L}{R_o + R_L} \frac{1}{\frac{R_L R_o}{R_L + R_o} + R_{i\beta}} v_R$$

But $v_R = \beta_{INV} v_p$:

$$v_{1o} = \frac{R_o R_L}{R_o + R_L} \frac{1}{\frac{R_L R_o}{R_L + R_o} + R_{i\beta}} \frac{R_{o\beta A}}{R_{o\beta A} + R_i + R_s} \beta_{INV} v_s$$

We set **A=0**, this is equivalent to replace v_A with a short circuit:



Finally:

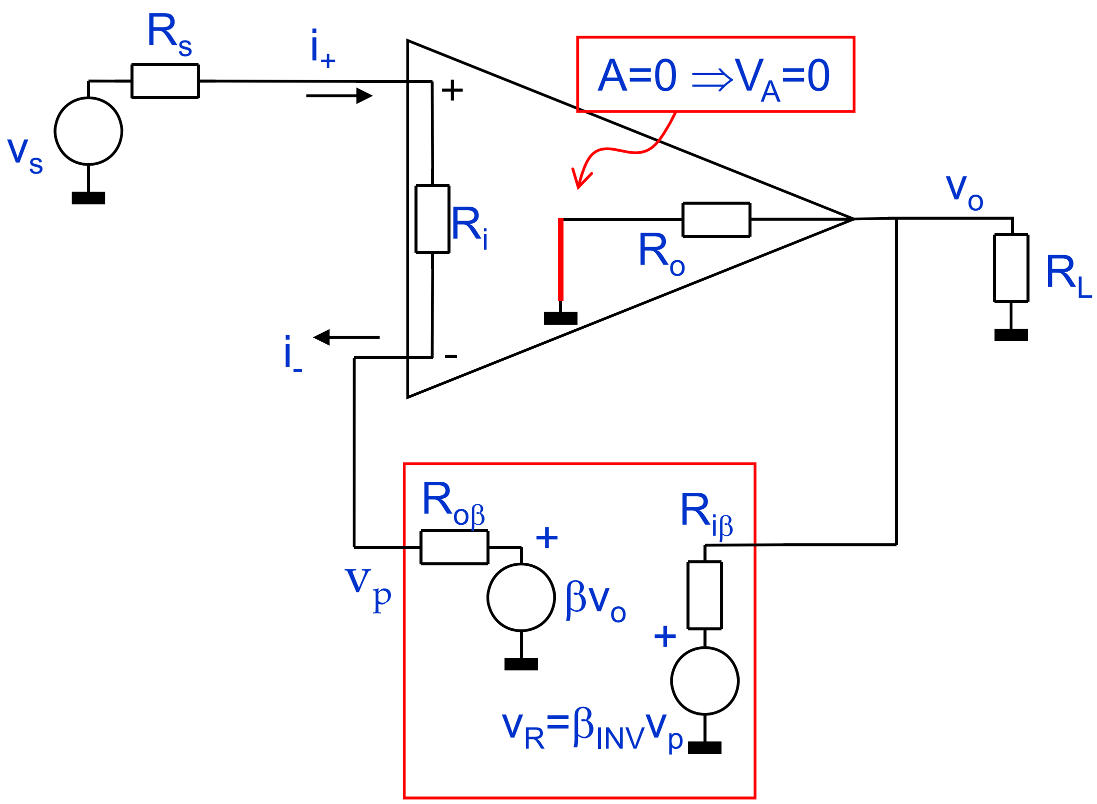
$$v_{1o}=\frac{R_oR_L}{R_o+R_L}\frac{1}{\frac{R_LR_o}{R_L+R_o}+R_{i\beta}}\frac{R_{o\beta A}}{R_{o\beta A}+R_i+R_s}\beta_{INV}v_s$$

Let's define:

$$\beta_{INVA}=\frac{R_{o\beta A}}{R_{o\beta A}+R_i+R_s}\beta_{INV}$$

The feedback of real OAs (10)

We set **A=0**, this is equivalent to replace v_A with a short circuit:



So we can write::

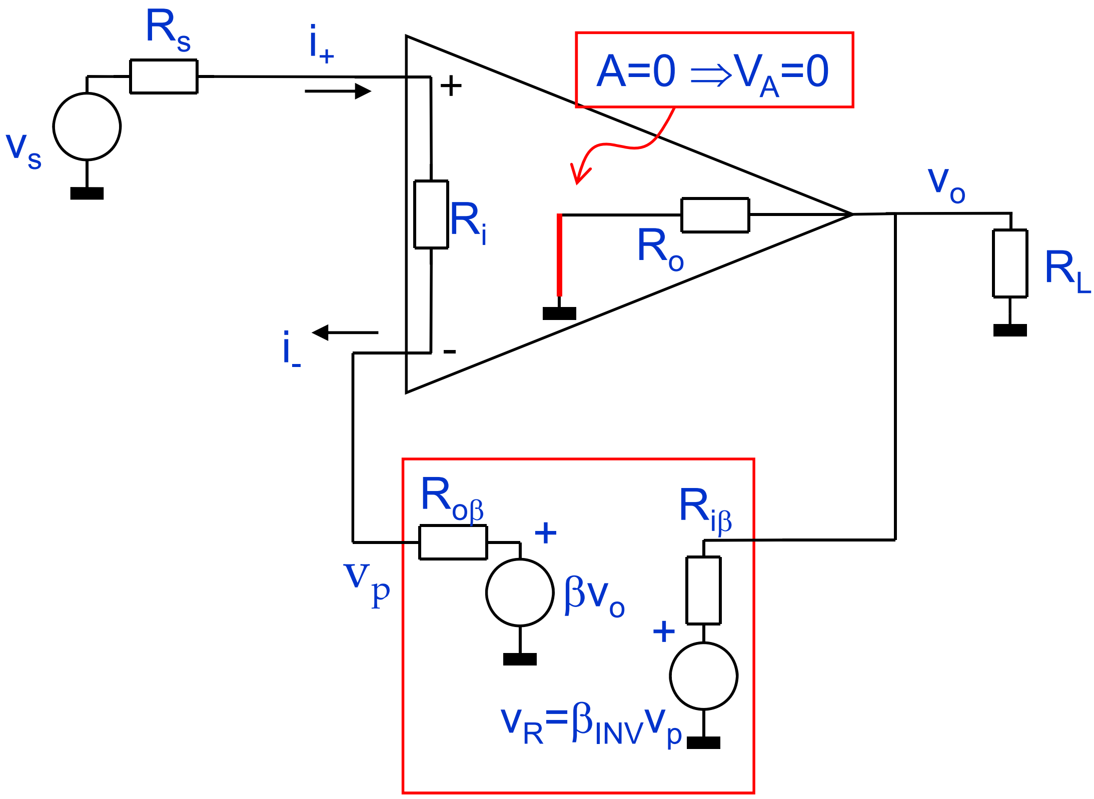
$$v_{1o}=\frac{R_oR_L}{R_o+R_L}\frac{1}{\frac{R_LR_o}{R_L+R_o}+R_{i\beta}}\beta_{INVA}v_s$$

$$v_{1o}=A_Rv_s$$

Rule #1: if we set A=0 the output voltage we measure is the direct transmission, namely the signal we have at the output which comes from the fact that the feedback is not ideal, unidirectional from the output to the input of the OA. This result is quite general and works also with the actual feedback, since we did not make special assumptions.

The feedback of real OAs (11)

We set **A=0**, this is equivalent to replace v_A with a short circuit:



Input mesh:

$$v_{1+}-v_{1-}=\frac{v_S-\beta v_o}{R_i+R_S+R_{o\beta}}R_i$$

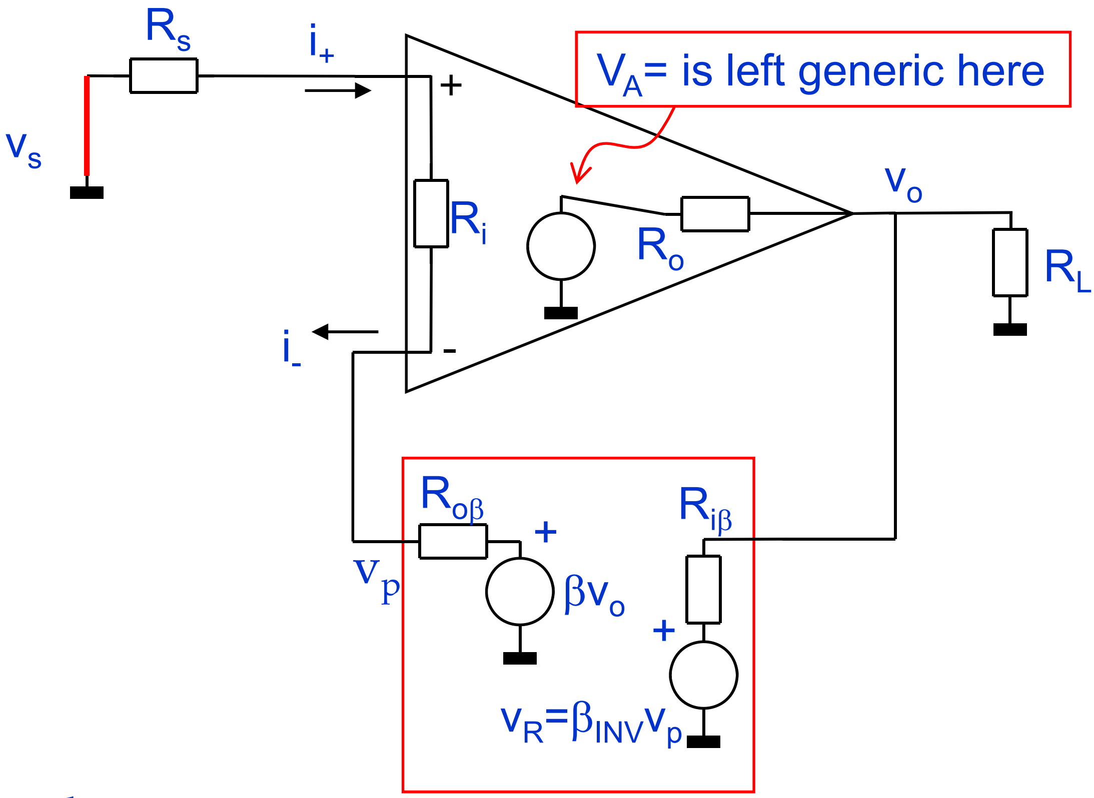
$$v_{1+}-v_{1-}=\frac{R_i}{R_i+R_S+R_{o\beta}}v_S-\frac{R_i}{R_i+R_S+R_{o\beta}}\beta v_{1o}$$

$$v_{1+}-v_{1-}=\frac{R_i}{R_i+R_S+R_{o\beta}}v_S-\frac{\beta R_i}{R_i+R_S+R_{o\beta}}A_R v_S$$

This step is needed now, but will be not necessary in the final procedure. We need it now since v_A will be dependent on this difference at the end.

The feedback of real OAs (12)

Now we concentrate on the source v_A, that we make independent for a moment, while we null the input V_s:



Output mesh:

$$\begin{cases} \dfrac{v_A - v_{2o}}{R_o} = \dfrac{v_{2o}}{R_L} + \dfrac{v_{2o} - \beta_{INV} v_p}{R_{i\beta}} \\[2ex] v_p = \dfrac{R_i + R_S}{R_i + R_S + R_{o\beta}} \beta v_{2o} \end{cases}$$

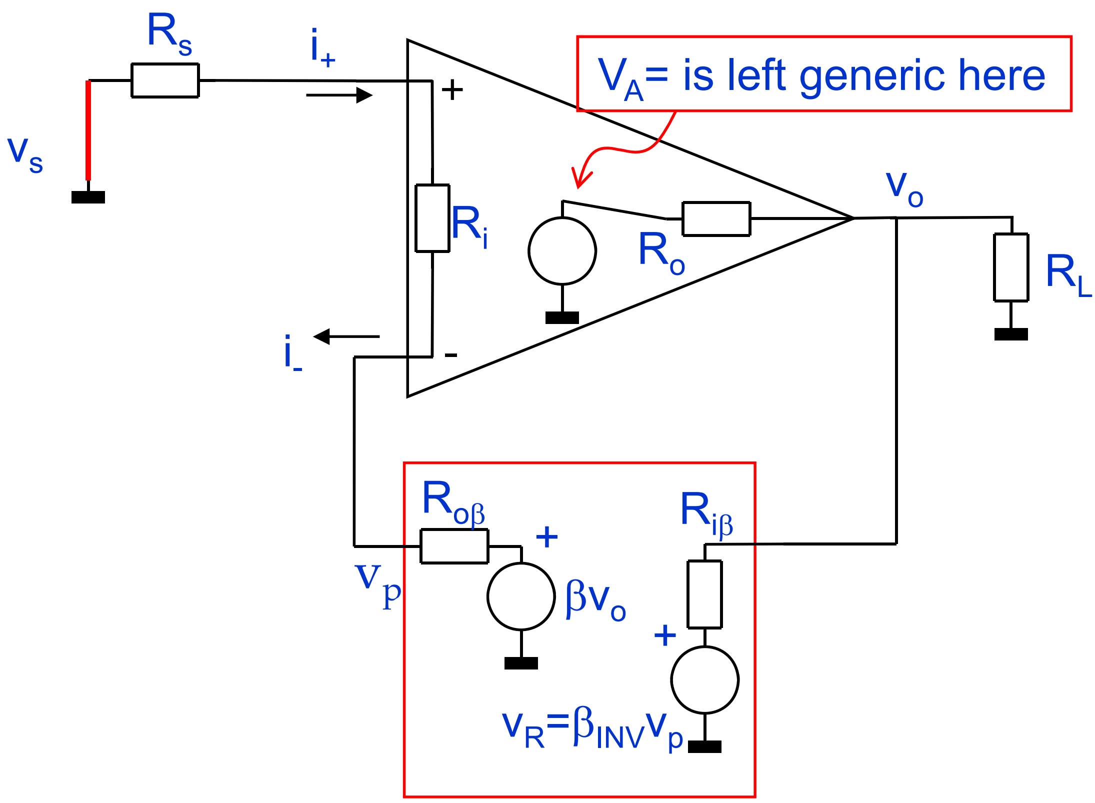
(here we see again the application of the concept of "generic variable": v_A is left generic for letting open the loop and to allow the study of it)

$$\frac{v_A}{R_o} = \frac{v_{2o}}{R_o} + \frac{v_{2o}}{R_L} + \frac{v_{2o}}{R_{i\beta}} \left(1 - \frac{R_i + R_S}{R_i + R_S + R_{o\beta}} \beta_{INV} \beta \right)$$

$$\frac{v_A}{R_o} = \frac{v_{2o}}{R_o} + \frac{v_{2o}}{R_L} + \frac{v_{2o}}{R_{i\beta A}} \qquad \left(R_{i\beta A} = R_{i\beta} \Big/ \left(1 - \frac{R_i + R_S}{R_i + R_S + R_{o\beta}} \beta_{INV} \beta \right)\right)$$

$$v_{2o} = \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o} v_A$$

The feedback of real OAs (13)



Output mesh:

$$v_{2o}=\frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A}+R_o} v_A$$

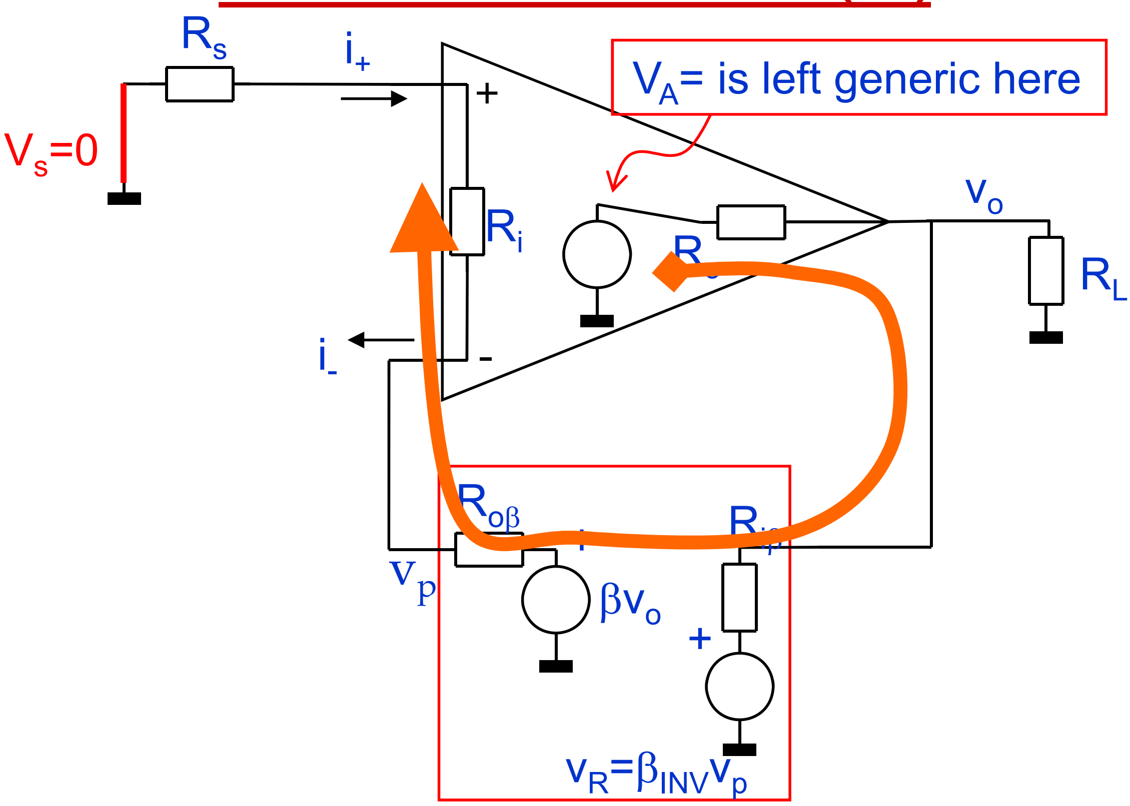
(here we see again the application of the concept of "generic variable": v_A is left generic for letting open the loop and to allow the study of it)

Input mesh:

$$v_{2+}-v_{2-}=-\frac{R_i}{R_i+R_s+R_{o\beta}}\beta v_{2o}$$

$$v_{2+}-v_{2-}=-\frac{\beta R_i}{R_i+R_s+R_{o\beta}}\frac{R_L R_{i\beta A}}{R_L+R_{i\beta A}}\frac{1}{\frac{R_L R_{i\beta A}}{R_L+R_{i\beta A}}+R_o} v_A$$

The feedback of real OAs (14)



We can see the analogy of the result obtained with the former calculation we did with feedback:

$$V_i = V_s - \beta V_o \quad \Rightarrow \quad \text{now} \quad V_+ - V_- = V_s - \beta f(\beta) V_o$$

$$V_o = A(V_s - \beta V_o) \quad \Rightarrow \quad \text{now } V_o = A(V_s - \beta f(\beta) V_o)$$

$$V_o = AV_s - A\beta V_o \quad \Rightarrow \quad \text{now } V_o = AV_s - A\beta f(\beta) V_o$$

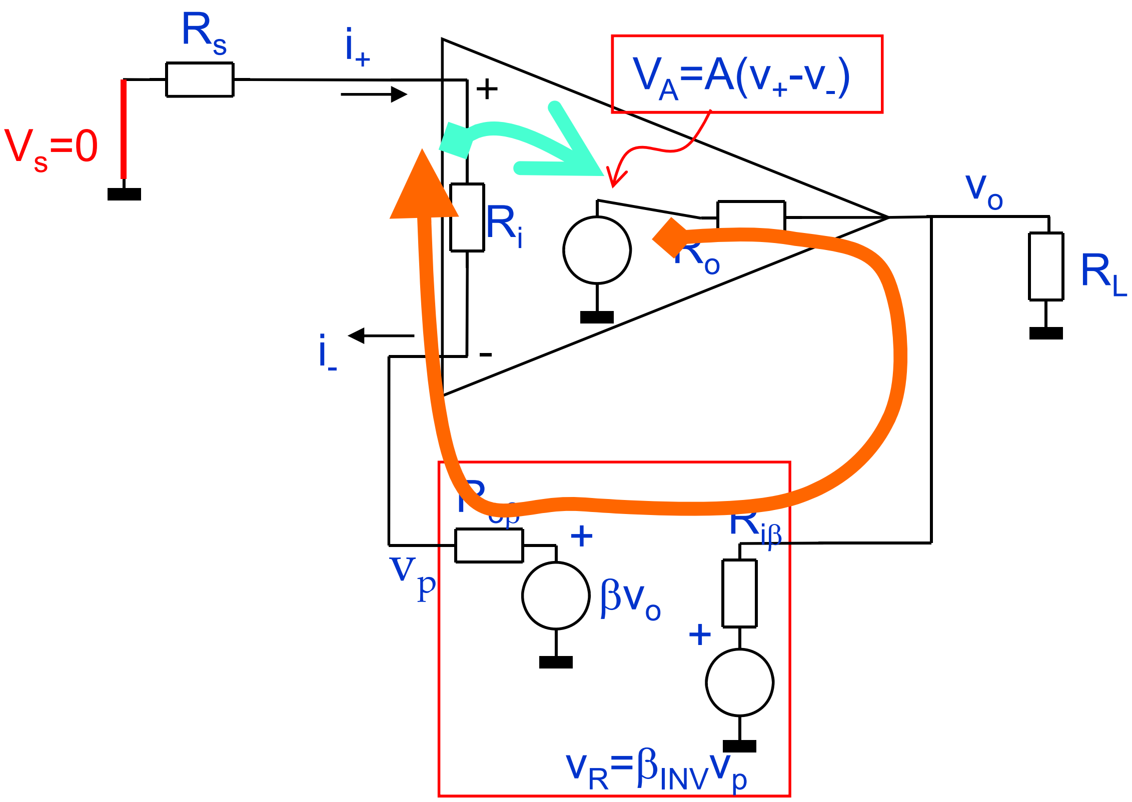
$$V_o = AV_s + TV_o$$

The difference between the present case and that case is only because v_s has been nulled and there are some impedances around. From the first equation we have, with V_s=0:

$$V_+ - V_- = -\beta f(\beta) V_o$$

The eqs which follow the first in the above system are active only when V_A is made dependent on the input nodes, and in this case the loop closes and T is simply the product of the above eq by –A…

The feedback of real OAs (15)



$$V_i = -\beta f(\beta) V_o \quad \Rightarrow \quad T = (-A) \times -\beta f(\beta)$$

Therefore, multiply by A/A:

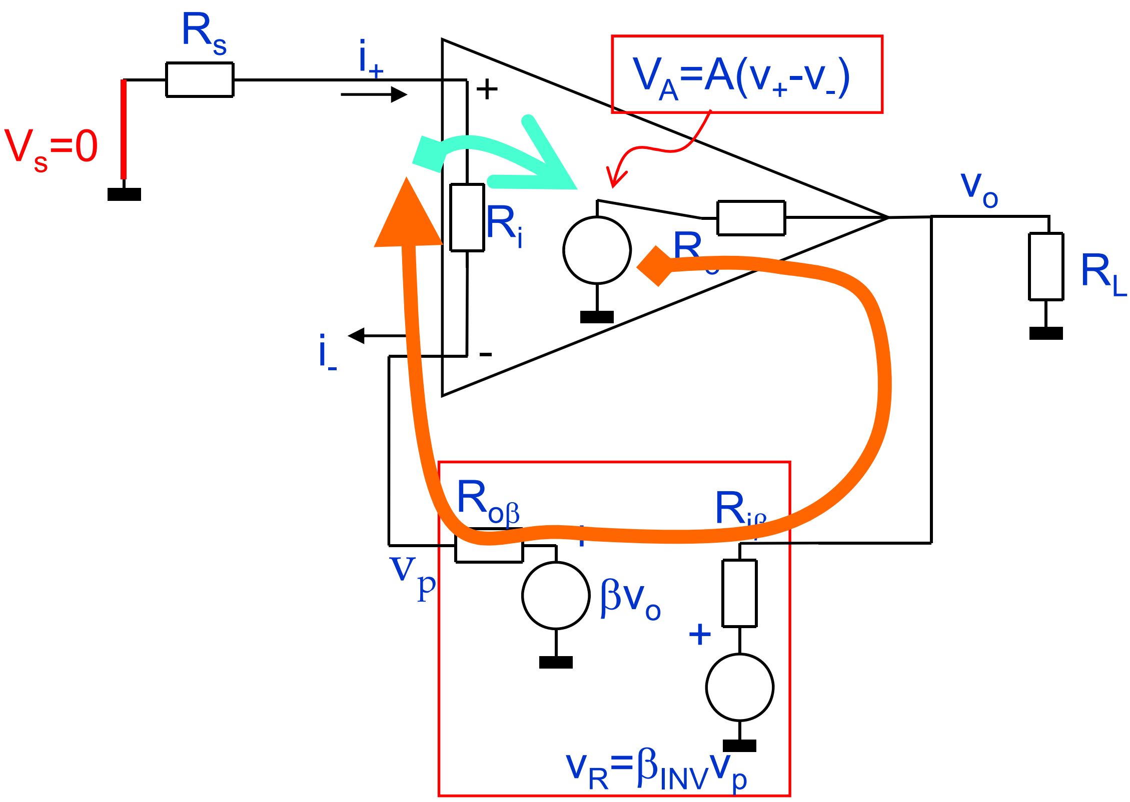
$$v_{2+} - v_{2-} = -\frac{A\beta R_i}{R_i + R_s + R_{o\beta}} \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o} \frac{v_A}{A}$$

$$v_{2+} - v_{2-} = T\frac{v_A}{A}$$

Or: $$T = \frac{A(v_{2+} - v_{2-})}{v_A}$$

(note that we refer now to the term T, and not $A\beta f(\beta)$, that has lost its meaning since the components around complicate its expression.)

The feedback of real OAs (16)



To say in words: in this last calculation we have followed the path marked by the orange line from the OA output to its input. This orange line is the feedback path, which is closed by the green line, i.e. when to v_A is returned its original function: $v_A=A(v_+-v_-)$.

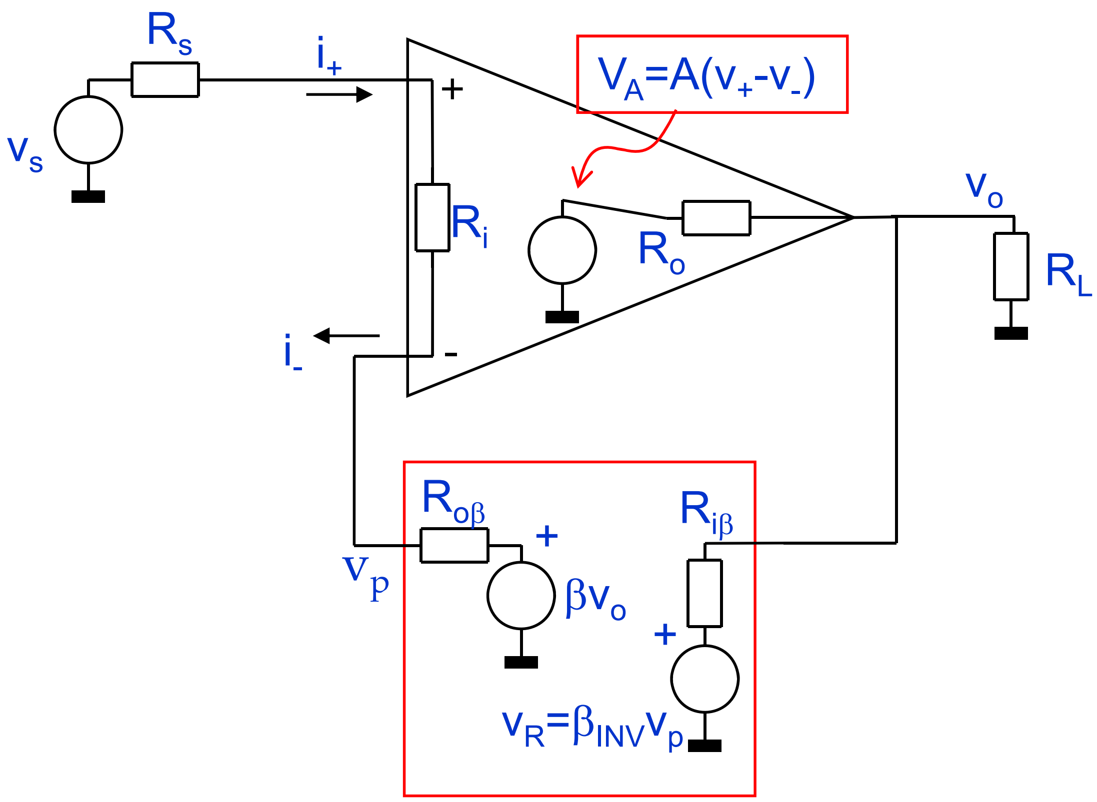
$$v_{2+}-v_{2-}=T\frac{v_A}{A}$$

(note that the loop gain T is the parameter we are interest in. Remember: T must be dimensionless and negative.)

$$T=\frac{A(v_{2+}-v_{2-})}{v_A}$$

Rule #2: nulling the input source, the ratio between the input signal and the amplifier source (the dependent generator made independent for a while) equals the ratio between the loop gain and the amplifier gain.

The feedback of real OAs (17)



Now let's put together the 2 results obtained:

$$v_+ - v_- = v_{1+} - v_{1-} + v_{2+} - v_{2-}$$

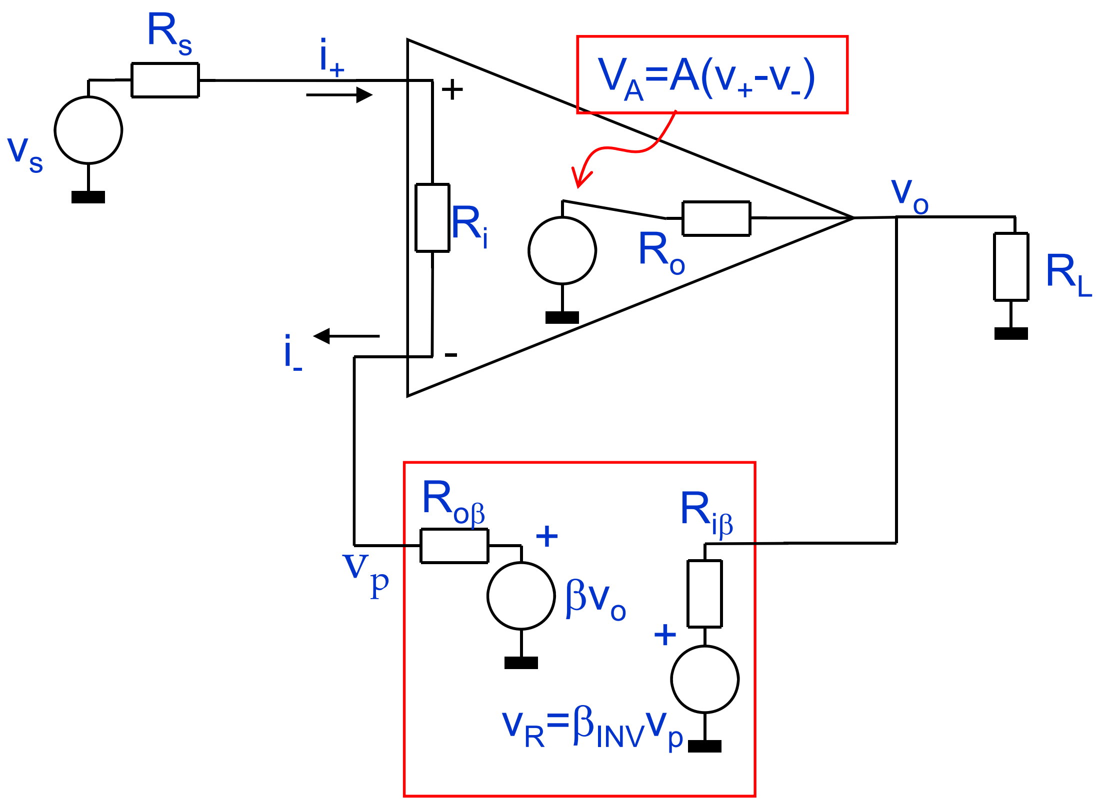
$$v_+ - v_- = \frac{R_i}{R_i + R_s + R_{o\beta}} v_s - \frac{\beta R_i}{R_i + R_s + R_{o\beta}} A_R v_s + T\frac{v_A}{A}$$

Where:

$$T = -\frac{A\beta R_i}{R_i + R_s + R_{o\beta}} \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o}$$

From which we can write:

$$v_+ - v_- = -\frac{T}{\beta A} \frac{R_L \| R_{i\beta A} + R_o}{R_L \| R_{i\beta A}} v_s + \frac{T}{A} \frac{R_L \| R_{i\beta A} + R_o}{R_L \| R_{i\beta A}} A_R v_s + T\frac{v_A}{A}$$



$$v_+ - v_- = -\frac{T}{\beta A}\frac{R_L \| R_{i\beta A} + R_o}{R_L \| R_{i\beta A}} v_S + \frac{T}{A}\frac{R_L \| R_{i\beta A} + R_o}{R_L \| R_{i\beta A}} A_R v_S + T\frac{v_A}{A}$$

Now let's remember that v_A is a dependent source, namely we close the feedback:

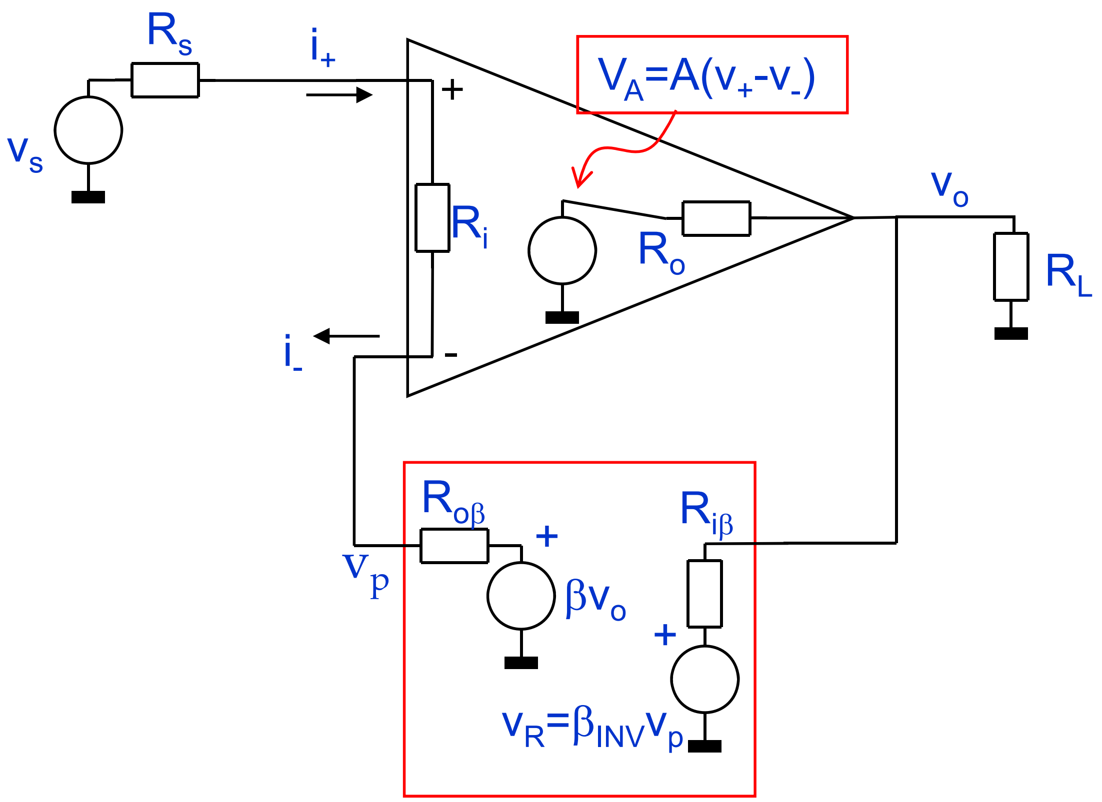
$$v_A = A(v_+ - v_-)$$

(now that we have finished our calculation we stop to consider v_A a "generic variable" and we assigned to it its expected dependence on the input nodes, closing the loop, now)

…and rearrange (multiply by A to express in terms of V_A):

$$v_A(1-T) = \frac{-T}{\beta}\frac{R_L \| R_{i\beta A} + R_o}{R_L \| R_{i\beta A}} v_S + T\frac{R_L \| R_{i\beta A} + R_o}{R_L \| R_{i\beta A}} A_R v_S$$

The feedback of real OAs (19)



A further step ahead:

$$v_A=\frac{-T}{\beta(1-T)}\frac{R_L\|R_{i\beta A}+R_o}{R_L\|R_{i\beta A}}v_s+\frac{T}{(1-T)}\frac{R_L\|R_{i\beta A}+R_o}{R_L\|R_{i\beta A}}A_R v_s$$

We prefer to know what is v_o, rather than v_A. And v_o was found proportional to v_A a few pages ago:

$$v_o=v_{1o}+v_{2o}=A_R v_s+\frac{R_L\|R_{i\beta A}}{R_L\|R_{i\beta A}+R_o}v_A$$

The feedback of real OAs (20)

$$v_O = A_R v_S + \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o} v_A$$

Remember v_A:

$$v_A = \frac{-T}{\beta(1-T)} \frac{R_L \| R_{i\beta A} + R_o}{R_L \| R_{i\beta A}} v_S + \frac{T}{(1-T)} \frac{R_L \| R_{i\beta A} + R_o}{R_L \| R_{i\beta A}} A_R v_S$$

We obtain:

$$v_O = A_R v_S + \frac{-T}{\beta(1-T)} v_S + \frac{T}{(1-T)} A_R v_S$$

…. and finally:

$$v_O = \frac{1}{\beta} \frac{-T}{(1-T)} v_S + \frac{A_R}{(1-T)} v_S$$

The output signal needs 3 parameters to be calculated and these 3 parameters can be obtained independently. Moreover, depending of the level of precision or approximation, we would like to obtain, we can limit the calculation to less than the 3 parameters.
Remember: the result so far obtained is not approximated.

The feedback of real OAs (21)

$$v_o = \frac{1}{\beta}\frac{-T}{(1-T)}v_s + \frac{A_R}{(1-T)}v_s$$

We see that if T is very very large or, as we know, A is very very large, the first term reduces to 1/β and the second term disappears (also from the fact that A_R is already small, in general).

We can then consider 3 levels of accuracy:

1. T is very very large: v_o reduces to 1/β;
2. T is large, but we would be a bit more accurate: v_o is 1/β [–T/(1-T)];
3. T is large, or even not too large. We can be very accurate and consider both terms in the expression above for v_o.

Whatever is the level of approximation considered, we can improve it simply adding terms that can be easily evaluated. At the maximum level of accuracy the result will be not approximated any more.

Feedback procedure (1)

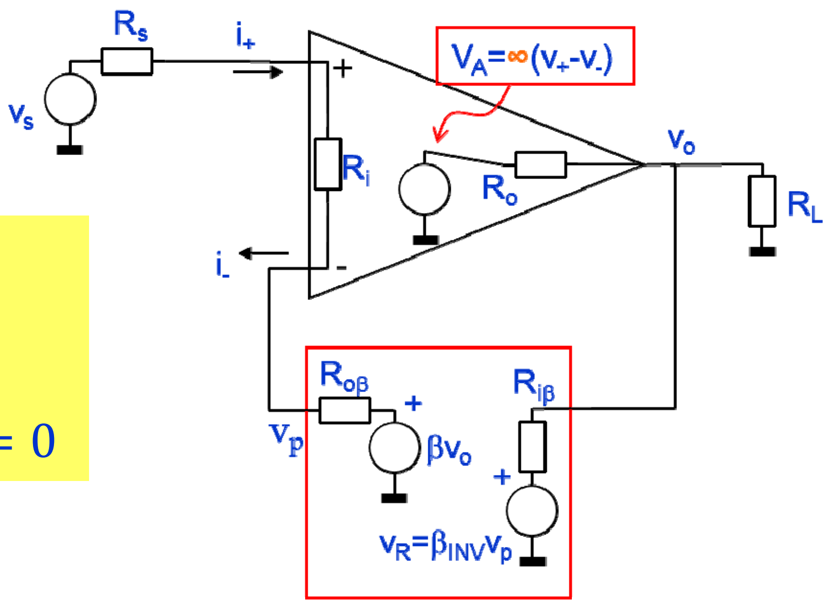
Step #0:

$$v_O = \frac{1}{\beta}\frac{-T}{(1-T)}v_S + \frac{A_R}{(1-T)}v_S$$

The only way to obtain the gain 1/β is to consider A=∞, even if it is not: if A=∞ in the expression for v_o all terms disappear except 1/β.

$$(1)\ A = \infty \ \Rightarrow\ (2)\ V_O = \frac{1}{\beta}V_S,$$

$$(3)\ V_+ \approx V_-, \qquad (4)\ I_+ = I_- = 0$$



If A=∞ the 4 rules above apply and we can easily arrive at the result.

Once 1/β has been found we can decide if we need or not to proceed to find the other terms.

The following step is to calculate T and see if it is worth to consider –T/(1-T) or not.

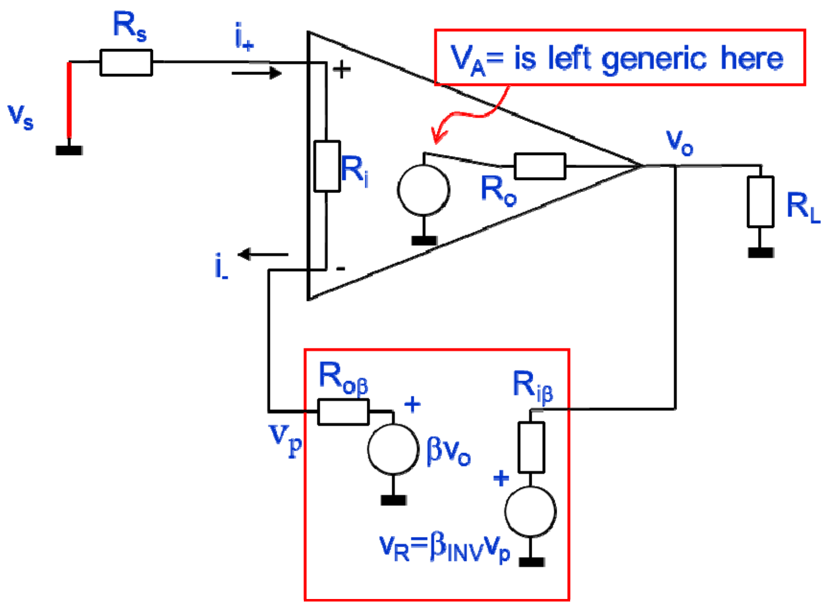
Feedback procedure (2)

Step #1:

$$v_O = \frac{1}{\beta}\frac{-T}{(1-T)}v_S + \frac{A_R}{(1-T)}v_S$$

To evaluate the loop gain T we null the input source and let v_A to be "*generic*".

$$v_+ - v_- = T\frac{v_A}{A}$$



Once that we know T we can decide if –T/(1-T) can be considered well approximated with 1 or not.

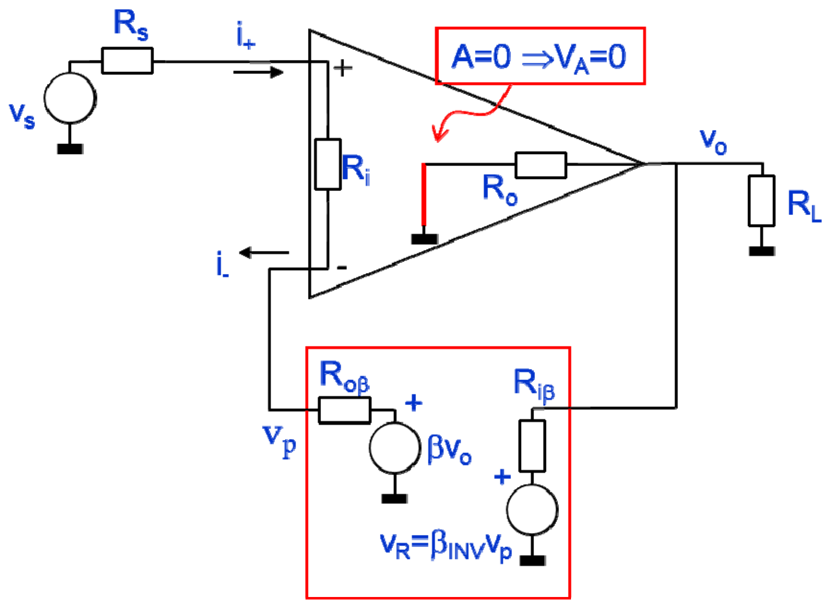
Feedback procedure (3)

Step #2:

$$v_O = \frac{1}{\beta}\frac{-T}{(1-T)}v_S + \frac{A_R}{(1-T)}v_S$$

Finally, once we know β and T we can calculate A_R by nulling the amplifier gain A.

$v_O = A_R v_S$

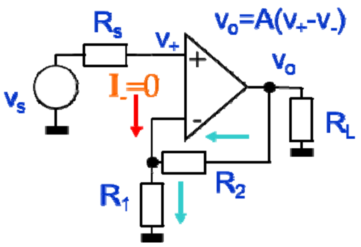


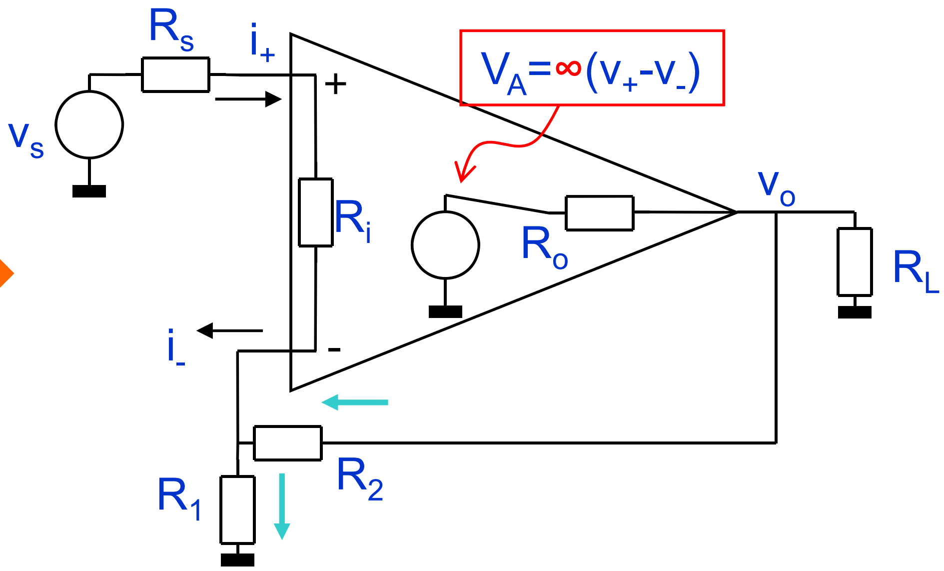
Combining all together the non-approximated result is so far obtained.

The application of the 3 steps to the practical case is straightforward and we do not need to model the feedback with a 4-terminals block.

Examples of complete feedback evaluation (1)

Let's consider some examples.





Step #0:

$$(1)\ A = \infty \Rightarrow (2)\ V_O = \frac{1}{\beta} V_S,$$

$$(3)\ V_+ \approx V_-, \qquad (4)\ I_+ = I_- = 0$$

If (1) is valid then (3) is valid and $v_- \approx v_+ = v_s$ and:

$$\frac{v_S}{R_1} = \frac{v_O - v_S}{R_2}$$

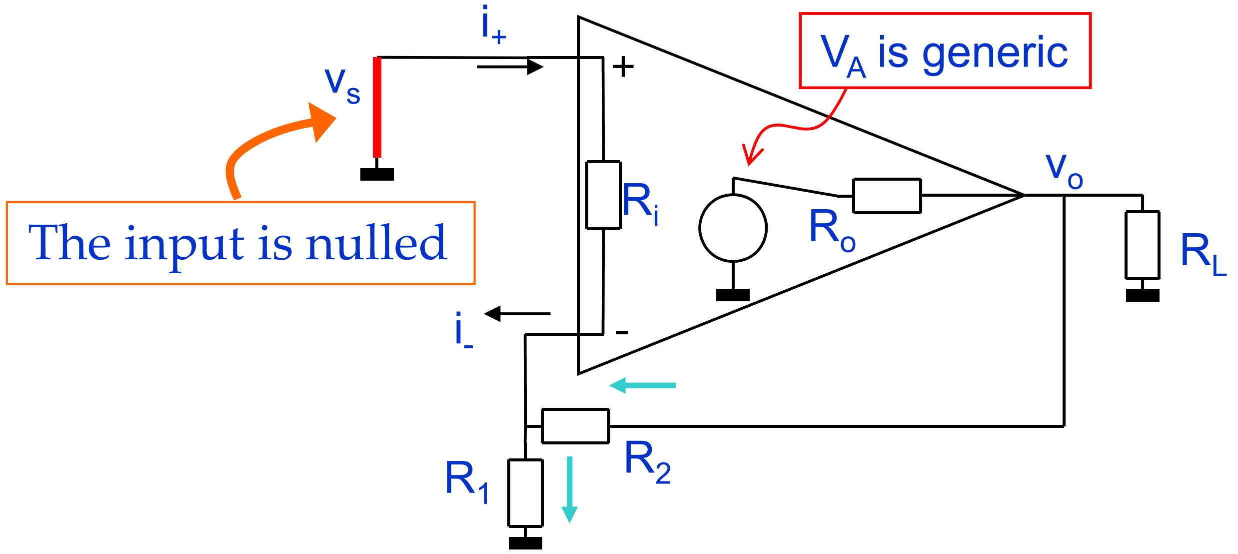
From which we obtain the same result of the ideal case:

$$v_O = \frac{R_1 + R_2}{R_1} v_S$$

…or:

$$\frac{1}{\beta} = \frac{R_1 + R_2}{R_1}$$

Examples of complete feedback evaluation (2)



Step #1:

$$v_+ - v_- = T\frac{v_A}{A}$$

Let's solve the network as the feedback is not present for finding the loop gain T:

$$\begin{cases} \dfrac{v_A - v_o}{R_o} = \dfrac{v_o}{R_L} + \dfrac{v_o - v_-}{R_2} \\ \dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1} + \dfrac{v_-}{R_i} \\ v_+ - v_- = -v_- \end{cases}$$

Let's define: $\dfrac{1}{R_{1'}} = \dfrac{1}{R_1} + \dfrac{1}{R_i}$, namely: $R_{1'} = R_1 \| R_i$

From the second eq:

$$\frac{v_o - v_-}{R_2} = \frac{v_-}{R_{1'}} \Rightarrow \frac{v_o}{R_2} = \left(\frac{1}{R_{1'}} + \frac{1}{R_2}\right) v_- \Rightarrow v_o = \frac{R_{1'} + R_2}{R_{1'}} v_-$$

$$\Rightarrow v_- = \frac{R_{1'}}{R_{1'} + R_2} v_o$$

Examples of complete feedback evaluation (3)

$$\begin{cases} \frac{v_A - v_o}{R_o} = \frac{v_o}{R_L} + \frac{v_o - v_-}{R_2} \\ \frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} + \frac{v_-}{R_i} \\ v_+ - v_- = -v_- \end{cases}$$

$$v_- = \frac{R_{1'}}{R_{1'} + R_2} v_o$$

Now the first eq:

$$\frac{v_A}{R_o} = \frac{v_o}{R_o} + \frac{v_o}{R_L} + \frac{v_o}{R_2} - \frac{1}{R_2}\frac{R_{1'}}{R_{1'} + R_2} v_o$$

$$\frac{v_A}{R_o} = \frac{v_o}{R_o} + \frac{v_o}{R_L} + \frac{v_o}{R_2}\left(1 - \frac{R_{1'}}{R_{1'} + R_2}\right)$$

$$\frac{v_A}{R_o} = \frac{v_o}{R_o} + \frac{v_o}{R_L} + \frac{1}{R_{1'} + R_2} v_o$$

With the position:

$$\frac{1}{R_{L'}} = \frac{1}{R_L} + \frac{1}{R_{1'} + R_2},$$

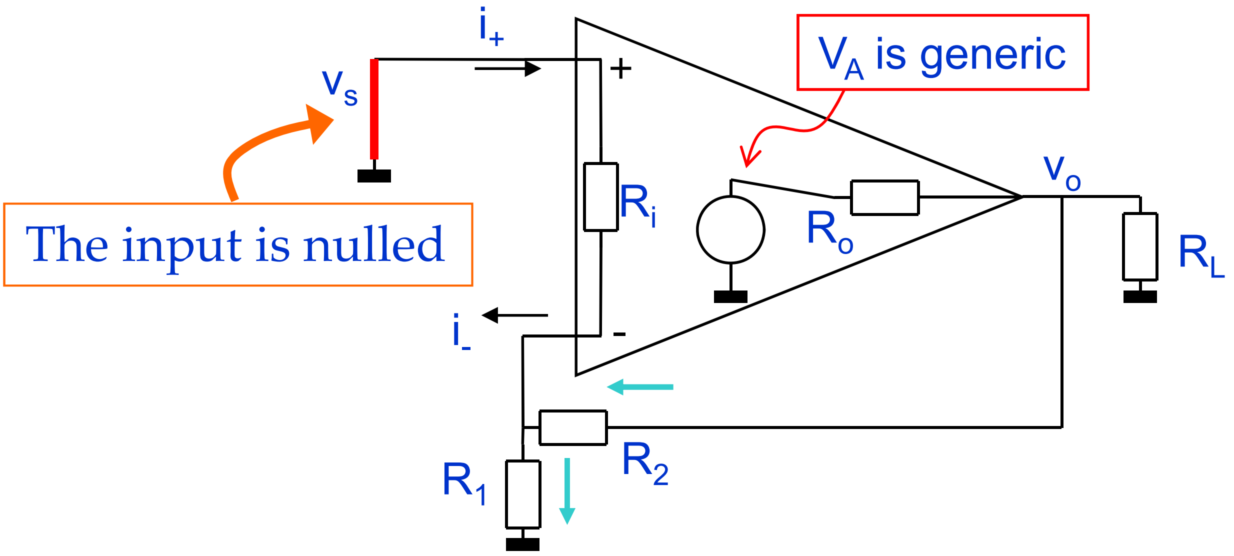
namely: $R_{L'} = R_L \| (R_{1'} + R_2)$

$$\frac{v_A}{R_o} = \frac{v_o}{R_o} + \frac{v_o}{R_{L'}} \Rightarrow v_A = \frac{R_o + R_{L'}}{R_{L'}} v_o \Rightarrow v_o = \frac{R_{L'}}{R_o + R_{L'}} v_A$$

And:

$$v_- = \frac{R_{1'}}{R_{1'} + R_2} \frac{R_{L'}}{R_o + R_{L'}} v_A$$

Examples of complete feedback evaluation (4)



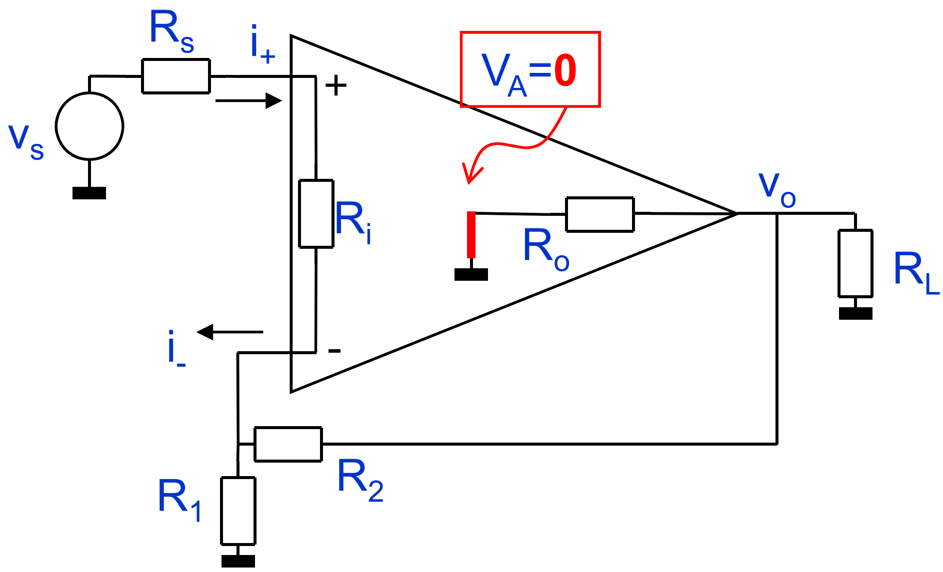
Remembering that:

$$T\frac{v_A}{A} = v_+ - v_- = -v_- = -\frac{R_{1'}}{R_{1'} + R_2}\frac{R_{L'}}{R_o + R_{L'}}v_A$$

We, finally, obtain:

$$T = -\frac{AR_{1'}}{R_{1'} + R_2}\frac{R_{L'}}{R_o + R_{L'}}$$

Examples of complete feedback evaluation (5)



Step #2: $v_o = A_R v_s$

We null the gain source:

$$\begin{cases} \dfrac{v_s - v_-}{R_s + R_i} = \dfrac{v_-}{R_1} + \dfrac{v_- - v_o}{R_2} \\[2ex] \dfrac{v_- - v_o}{R_2} = \dfrac{v_o}{R_0} + \dfrac{v_o}{R_L} \end{cases}$$

We put: $$\frac{1}{R_{o'}} = \frac{1}{R_0} + \frac{1}{R_L}$$

From the second:

$$\frac{v_-}{R_2} = \frac{v_o}{R_2} + \frac{v_o}{R_{0'}} \Rightarrow v_- = \frac{R_2 + R_{0'}}{R_{0'}} v_o \Rightarrow v_o = \frac{R_{0'}}{R_2 + R_{0'}} v_-$$

From the first:

$$\frac{v_s}{R_s + R_i} = \frac{v_-}{R_s + R_i} + \frac{v_-}{R_1} + \frac{v_-}{R_2} - \frac{1}{R_2} \frac{R_{0'}}{R_2 + R_{0'}} v_-$$

$$\frac{v_s}{R_s + R_i} = \frac{v_-}{R_s + R_i} + \frac{v_-}{R_1} + \frac{v_-}{R_2}\left(1 - \frac{R_{0'}}{R_2 + R_{0'}}\right)$$

Examples of complete feedback evaluation (6)

$$\begin{cases} \dfrac{v_s - v_-}{R_s + R_i} = \dfrac{v_-}{R_1} + \dfrac{v_- - v_o}{R_2} \\ \dfrac{v_- - v_o}{R_2} = \dfrac{v_o}{R_0} + \dfrac{v_o}{R_L} \end{cases}$$

$$\frac{v_s}{R_s + R_i} = \frac{v_-}{R_s + R_i} + \frac{v_-}{R_1} + \frac{v_-}{R_2}\left(1 - \frac{R_{0'}}{R_2 + R_{0'}}\right)$$

Again, for convenience, we can put:

$$\frac{1}{R_{1'}} = \frac{1}{R_1} + \frac{1}{R_2 + R_{0'}}$$

$$\frac{v_s}{R_s + R_i} = \frac{v_-}{R_s + R_i} + \frac{v_-}{R_{1'}} \quad \Rightarrow \quad v_s = \frac{R_s + R_i + R_{1'}}{R_{1'}} v_-$$

$$v_- = \frac{R_{1'}}{R_s + R_i + R_{1'}} v_s$$

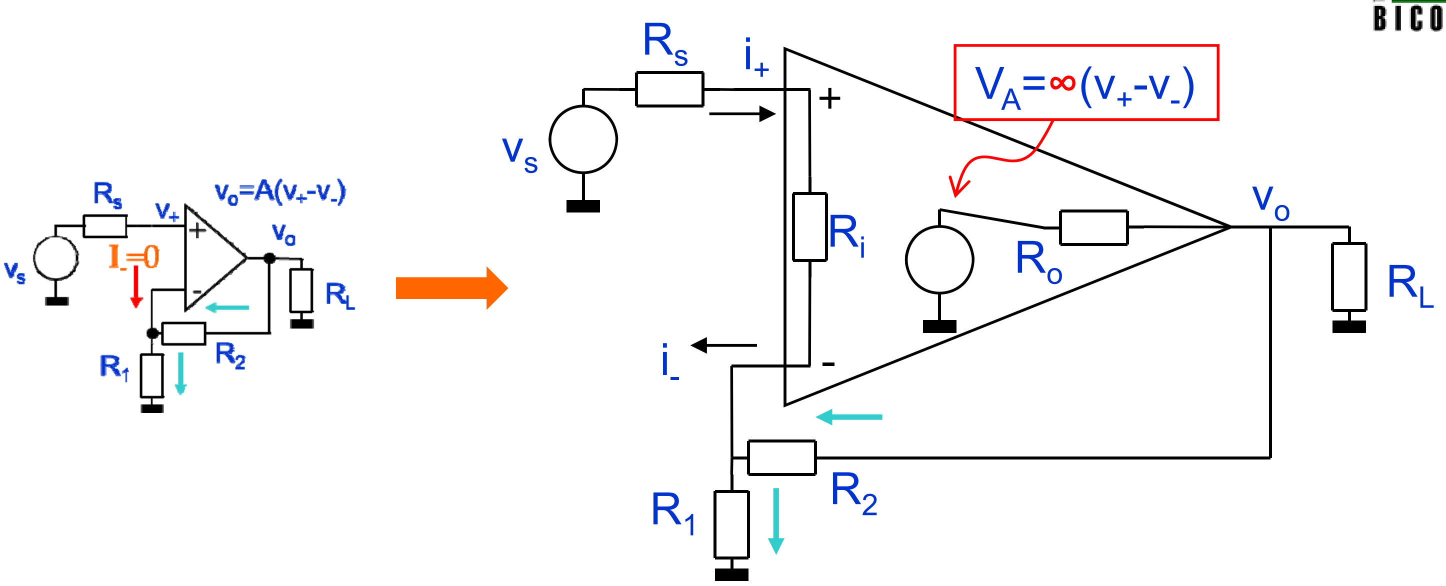
Finally:

$$v_o = \frac{R_{0'}}{R_2 + R_{0'}} v_- = \frac{R_{0'}}{R_2 + R_{0'}} \frac{R_{1'}}{R_s + R_i + R_{1'}} v_s$$

Or:

$$A_R = \frac{v_o}{v_s} = \frac{R_{0'}}{R_2 + R_{0'}} \frac{R_{1'}}{R_s + R_i + R_{1'}}$$

Examples of complete feedback evaluation (7)



Putting all together:

$$v_o = \frac{R_1 + R_2}{R_1}\frac{-T}{1-T} v_s + \frac{R_{0'}}{R_2 + R_{0'}}\frac{R_{1'}}{R_s + R_i + R_{1'}}\frac{1}{1-T} v_s$$

And:

$$T = -\frac{AR_{1'}}{R_{1'} + R_2}\frac{R_{L'}}{R_o + R_{L'}}$$

One consideration

So far we calculated the loop gain T, which is the product of few parts. Although not needed in the practical cases, one can try to disentangle T in its constituents.
We know β and T, so:

$$A_{and\_other} = -\frac{T}{\beta} = Af(\beta)$$

Dividing T by the feedback β what remains is the product of the OA gain and a term that depends on all the components of the network: the elements that compose the OA itself and the elements connected around (source and load impedances and the feedback network, that is not ideal).

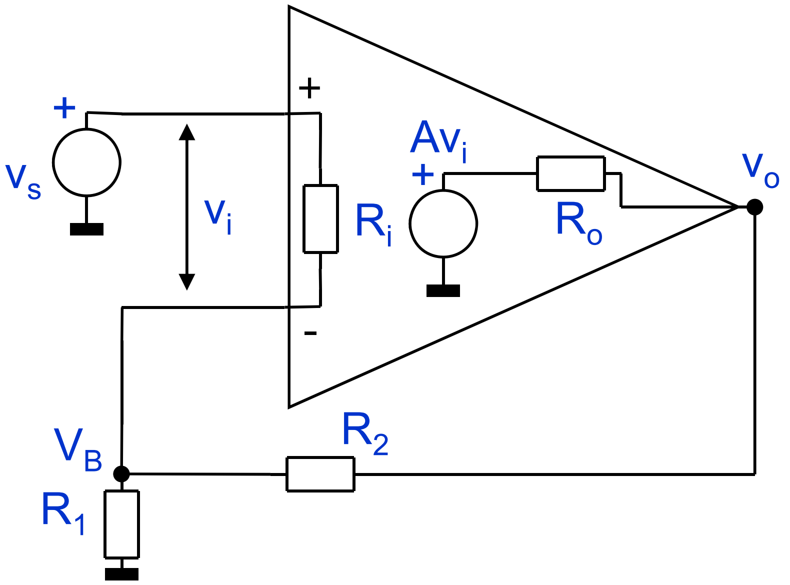
For instance, in the previous example:

$$A_{and\_other} = -\frac{T}{\beta} = \frac{R_1 + R_2}{R_1} \frac{AR_{1\prime}}{R_{1\prime} + R_2} \frac{R_{L\prime}}{R_o + R_{L\prime}}$$

or:

$$f(all\_around) = \frac{A_{and\_other}}{A} = \frac{R_1 + R_2}{R_1} \frac{R_{1\prime}}{R_{1\prime} + R_2} \frac{R_{L\prime}}{R_o + R_{L\prime}}$$

…and another consideration



At first it might seem a complicated procedure the following of the 3 steps. The alternative way is to solve the network as one were not be aware of the feedback, as it follows:

This is the system of equations to be written to solve the network as if we did not know the feedback

$$\begin{cases} \dfrac{v_s - v_B}{R_i} + \dfrac{v_o - v_B}{R_2} = \dfrac{v_B}{R_1} \\[2ex] \dfrac{A(v_s - v_B) - v_o}{R_o} = \dfrac{v_o - v_B}{R_2} \end{cases} \qquad (v_i = v_s - v_B)$$

$$\begin{cases} \dfrac{v_s}{R_i} = v_B\left(\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}\right) - \dfrac{v_o}{R_2} \\[2ex] v_s\dfrac{A}{R_o} = v_B\left(\dfrac{A}{R_o} - \dfrac{1}{R_2}\right) + v_o\left(\dfrac{1}{R_2} + \dfrac{1}{R_o}\right) \end{cases}$$

$$\begin{cases} \dots \\ \dots \end{cases}$$

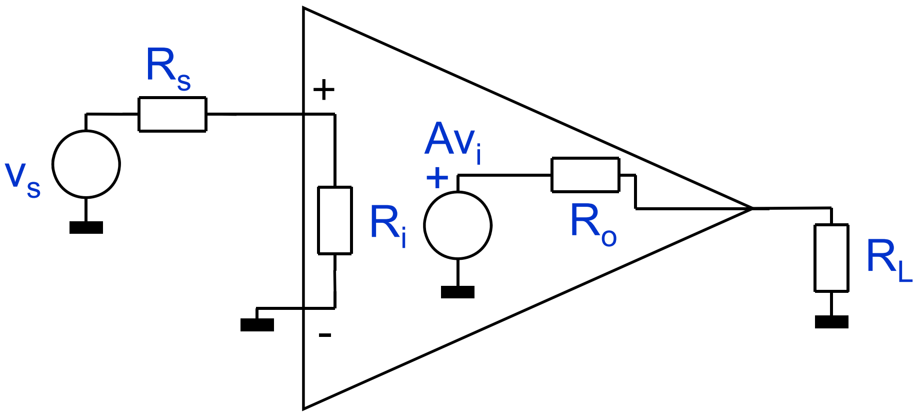
To come to the final solution several intermediate steps are needed.

$$v_o = \frac{AR_2R_i(R_1 + R_2) + R_1R_2R_o}{(R_1 + R_i)(R_2 + R_o)R_2 + (A+1)R_1R_2R_i} v_s$$

$$\approx \frac{AR_2R_i(R_1 + R_2)}{(A+1)R_1R_2R_i} v_s \approx \frac{(R_1 + R_2)}{R_1} v_s \quad \text{cvd!}$$

At the end the final result does not distinguish the different contributions.

Input and output impedance of a fed backed OA



The source and load impedances introduce signal partitions, in a real OA, at the input and output across which part of the signal is lost.

Nature of feedback is kindly in making the input and output of the fed backed OA close to an ideal condition.

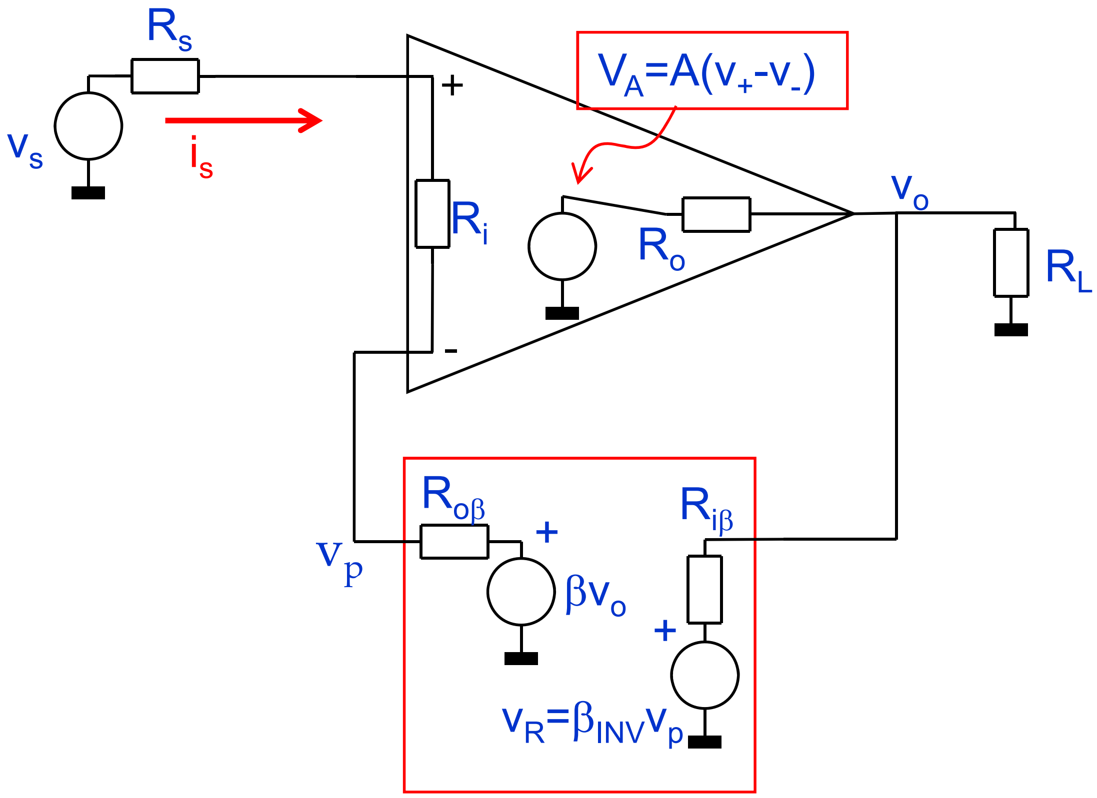
This was implicit in the results we have obtained in our calculation as we have taken into account the presence of the source and load impedance, obtaining little effects.

We can try to estimate the input and output impedances of the fed backed OA in view of a final modeling, as we will see later.

Again, we will try to exploit the parameters we are learned to estimate.

Input impedance of a v-to-v fed backed OA 1

v-to-v= voltage input and voltage output amplifier



The input impedance of the network is that seen at the input source node, and is given by:

$$R_{if} = \frac{v_s}{i_s}$$

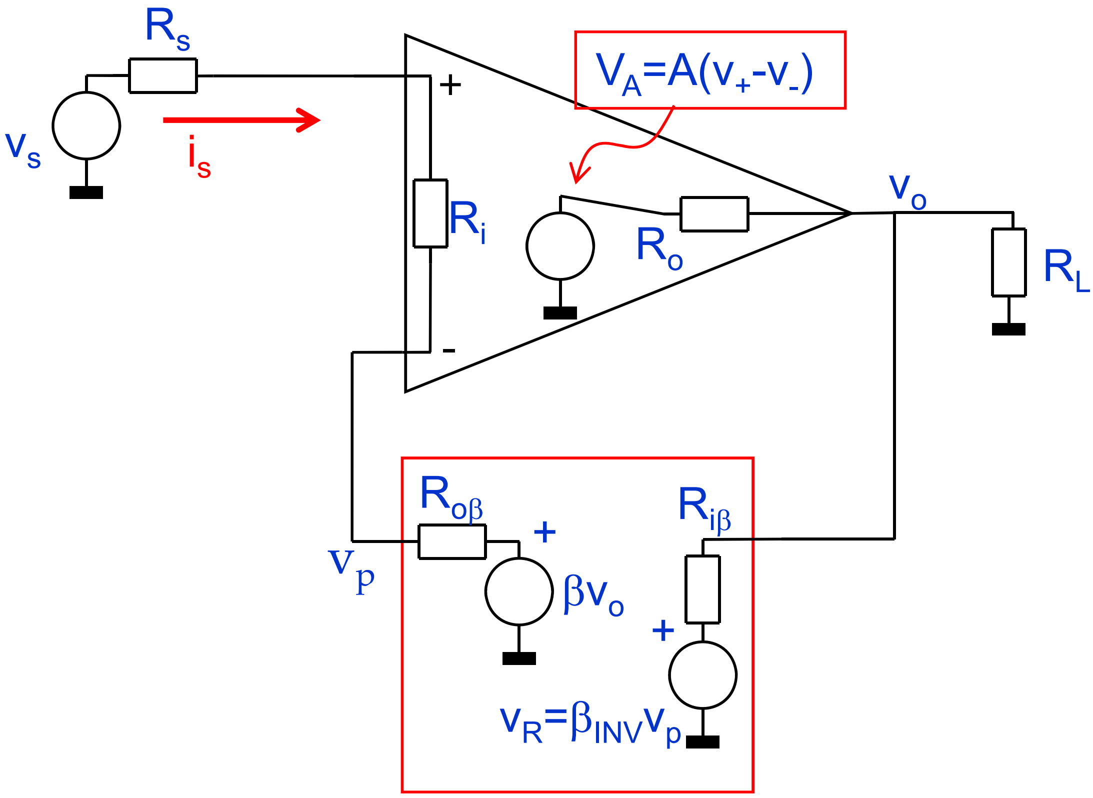
The current i_s is flowing through R_s and also thought R_i and through this latter satisfies:

$$i_s = \frac{v_+ - v_-}{R_i}$$

or:

$$i_s = \frac{v_+ - v_-}{R_i} = \frac{v_A}{AR_i}$$

Input impedance of a v-to-v fed backed OA 2



Let's try to exploit what we have learned. Node v_A has not special properties with respect node v_o or any other node of the network; therefore we can write:

$$v_A = \frac{1}{\beta'}\frac{-T}{1-T}v_s + \frac{A_{R'}}{1-T}v_s$$

Here β' and $A_{R'}$ refer the fact that node v_A and node v_o are different, but T is the same, as expected all around the loop.

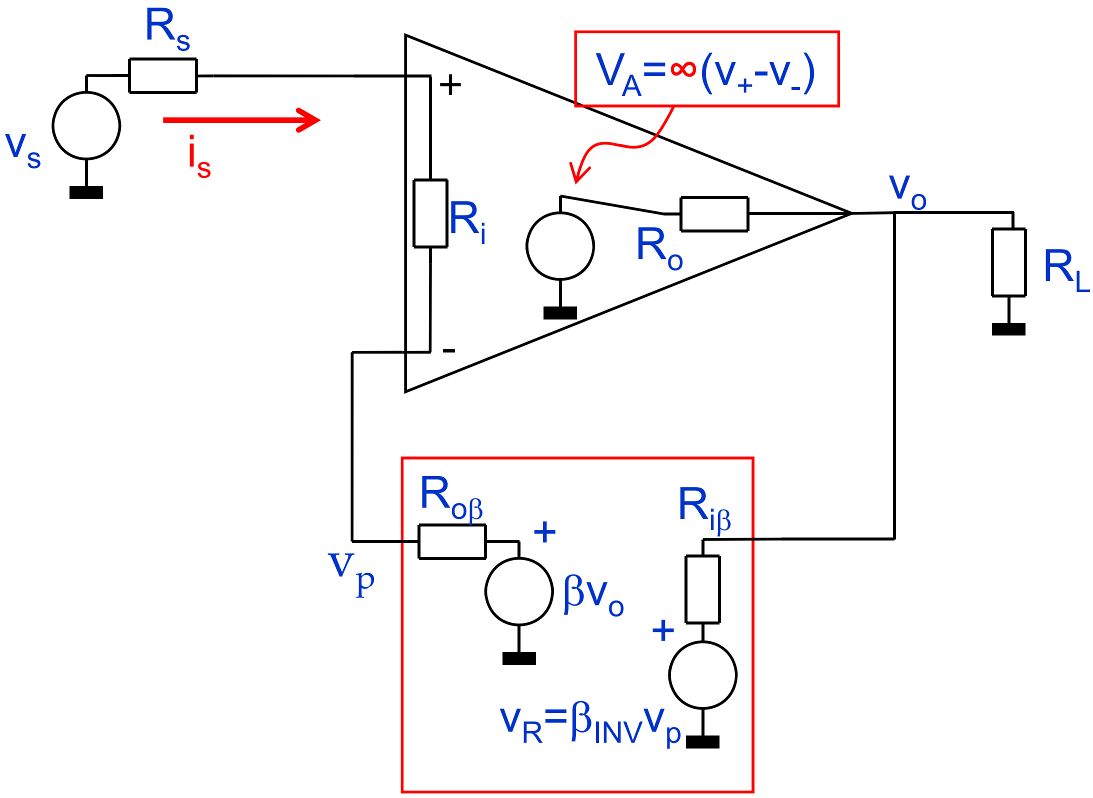
To calculate β' we start from considering T=∞:

$$\beta v_o = v_s \quad \rightarrow \quad v_o = \frac{v_s}{\beta}$$

and:

$$\frac{v_A - v_o}{R_o} = \frac{v_o}{R_L} + \frac{v_o - \beta_{INV} v_p}{R_{i\beta}}$$

Input impedance of a v-to-v fed backed OA 3



$$\beta v_o = v_s \quad \rightarrow \quad v_o = \frac{v_s}{\beta}$$

With the position above:

$$\frac{v_A - v_o}{R_o} = \frac{v_o}{R_L} + \frac{v_o - \beta_{INV} v_p}{R_{i\beta}} \quad \overset{v_p = v_s}{\Longrightarrow} \quad \frac{v_A}{R_o} = \left(\frac{1}{R_o} + \frac{1}{R_L} + \frac{1 - \beta\beta_{INV}}{R_{i\beta}}\right)\frac{v_s}{\beta}$$

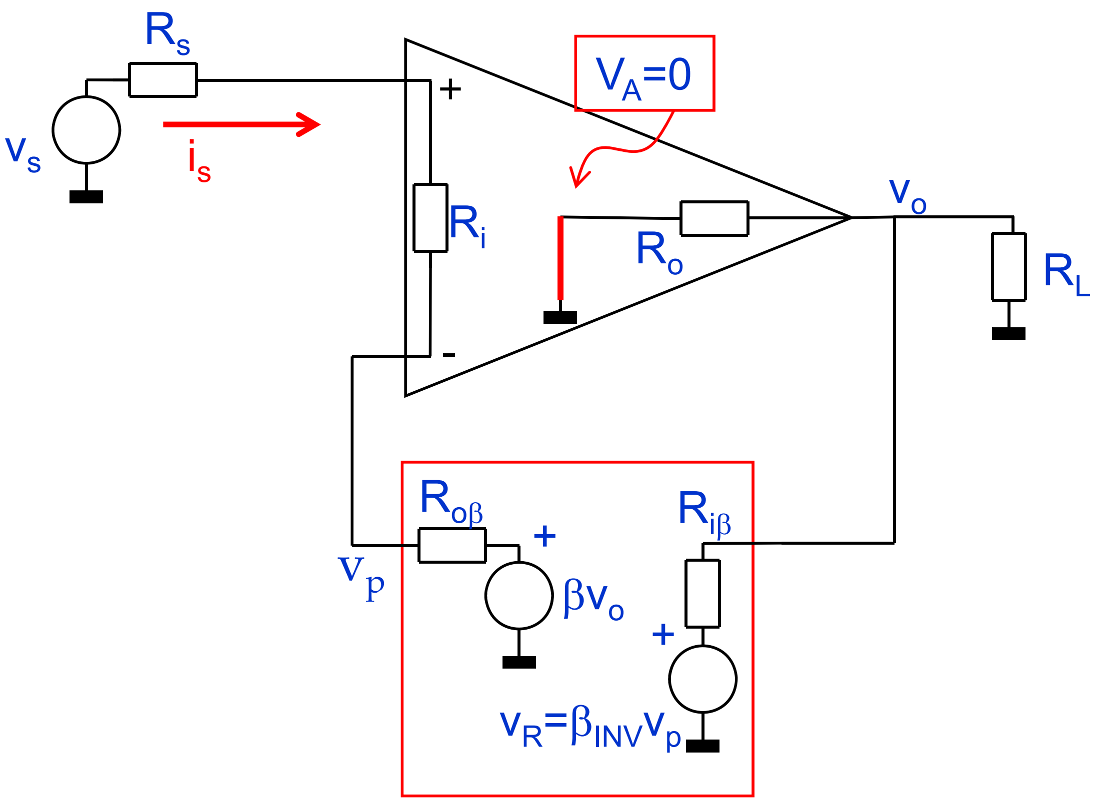
or:

$$v_A = \left(1 + \frac{R_o}{R_L} + \frac{(1 - \beta\beta_{INV})R_o}{R_{i\beta}}\right)\frac{v_s}{\beta}$$

namely:

$$\left(\frac{v_A}{v_S}\right)^{-1} = \beta' = \beta\left(1 + \frac{R_o}{R_L} + \frac{(1 - \beta\beta_{INV})R_o}{R_{i\beta}}\right)^{-1}$$

Input impedance of a v-to-v fed backed OA 4



Now, if the gain is reduced to 0 we see that node v_A cannot change for direct transmission and $A_{R'}$ results to be 0:

$$v_A = \frac{1}{\beta'}\frac{-T}{1-T}v_s$$

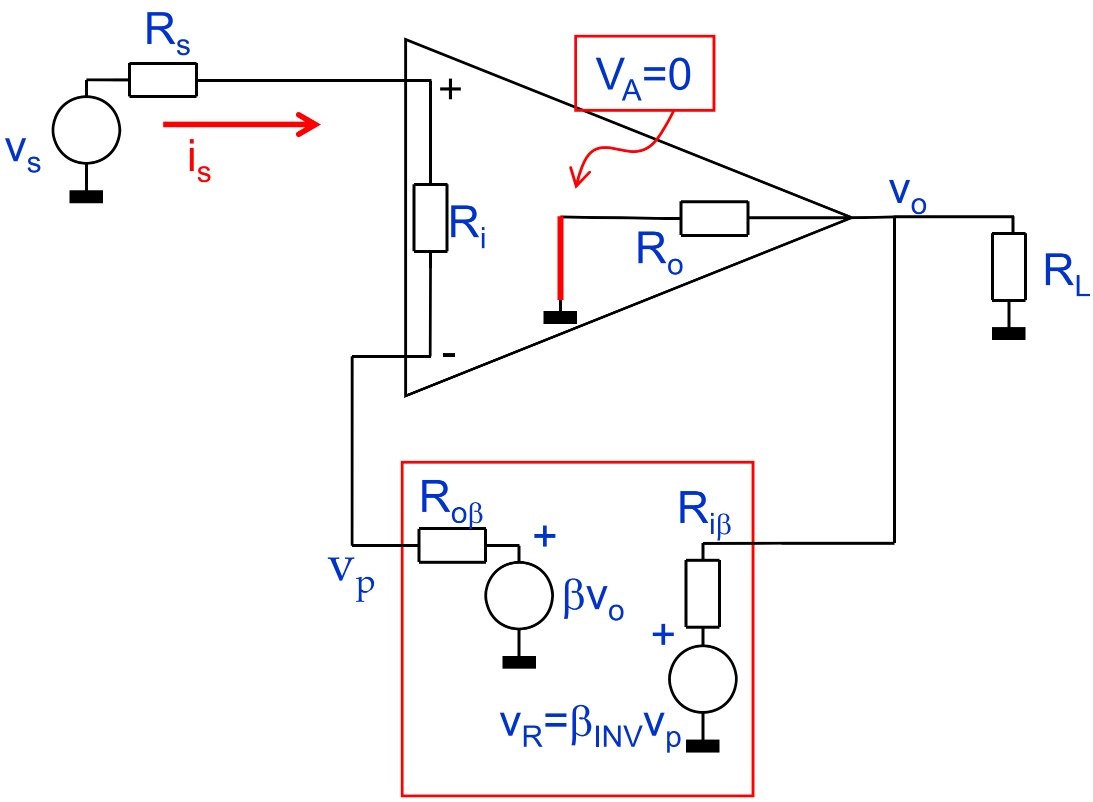
Now: $\qquad (T = -\beta' A(s) f(\beta', s))$

$$v_+ - v_- = \frac{v_A}{A} = \frac{1}{A\beta'}\frac{-T}{1-T}v_s = \frac{1}{A\beta'}\frac{\beta' A(s) f(\beta', s)}{1-T}v_s$$

$$v_+ - v_- = \frac{f(\beta')}{1-T}v_s$$

$$i_s = \frac{v_+ - v_-}{R_i} = \frac{f(\beta')}{(1-T)R_i}v_s$$

Input impedance of a v-to-v fed backed OA 5



$$i_s = \frac{v_+ - v_-}{R_i} = \frac{f(\beta')}{(1-T)R_i} v_s$$

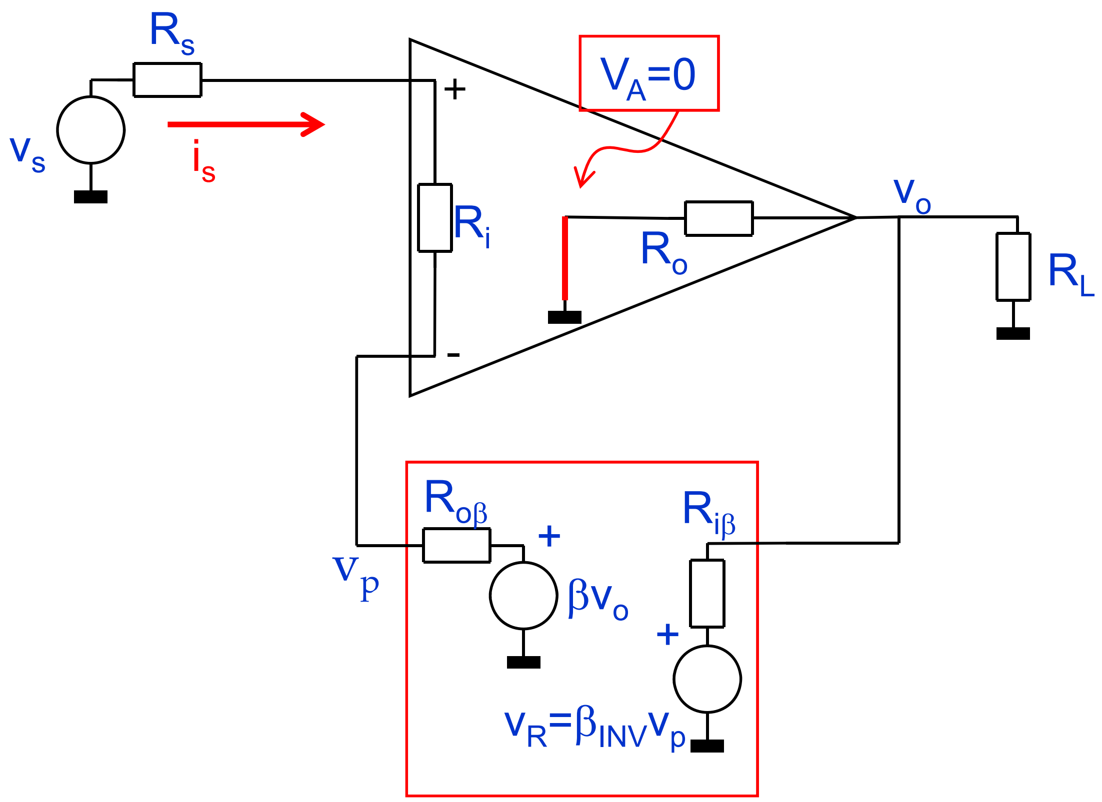
Finally:

$$R_{if} = \frac{v_s}{i_s} = \frac{R_i}{f(\beta')}(1-T)$$

The feedback multiplies the parameter $R_i/f(\beta')$ by 1-T, with the aim of increasing the input impedance, obtaining a condition closer to the ideal case.

The result obtained has a physical interpretation. If we set T to 0 by nulling the OA gain, as it is the case above, the input impedance reduces to that can be measured in open loop condition and can be easily calculated…

Input impedance of a v-to-v fed backed OA 6



We can write that:

$$R_{if} = \frac{v_s}{i_s} = R_{iol}(1 - T)$$

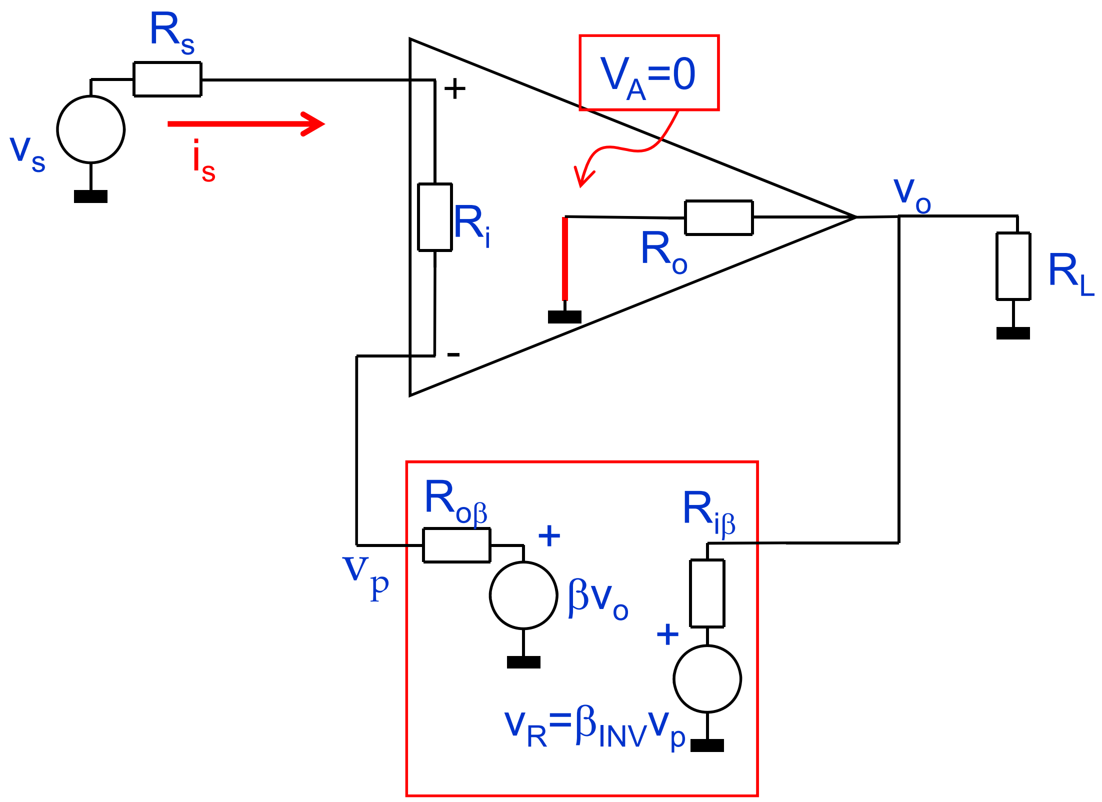
where R_{iol} (where iol is for input, open loop) is the impedance that is measured when the OA gain is set to 0, in order to have T=0.

This measurement is easily done with the network above.
In the next page the assumption above is proved.

Let's try now to determine R_{if} first from the open loop condition, as indicated above, namely from R_{iol}, than from the original $R_{if}/f(\beta')$ found. This to show that the 2 approaches are similar.

A practical example is given in Appendix A.

Input impedance of a v-to-v fed backed OA 7



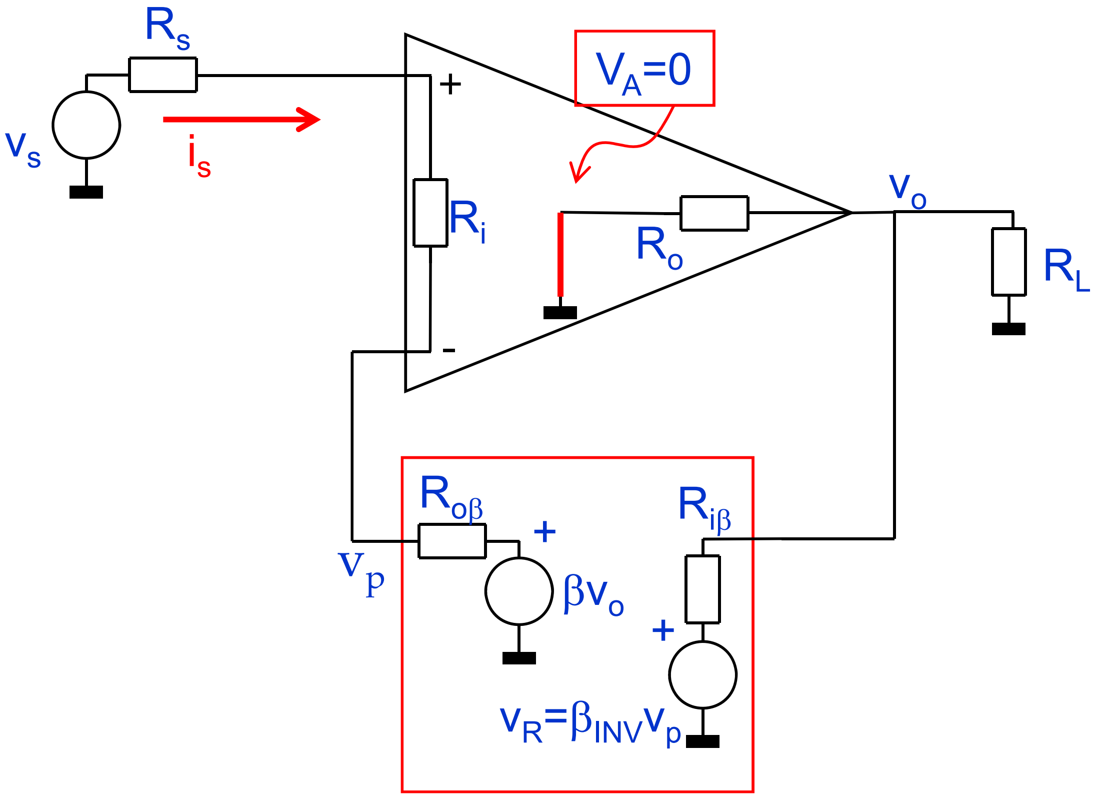
First method: evaluation of R_{if} from R_{iol}.

$$\begin{cases} i_s = \dfrac{v_s - v_p}{R_s + R_i} \\ v_p = \dfrac{R_{o\beta}}{R_s + R_i + R_{o\beta}}(v_s - \beta v_o) + \beta v_o \\ v_o = \dfrac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta_{INV} v_p \end{cases}$$

From which we already obtained:

$$v_p = \frac{R_{o\beta A}}{R_{o\beta A} + R_i + R_s} v_s \qquad \left(R_{o\beta A} = \frac{R_{o\beta}}{1 - \dfrac{R_o R_L}{R_o + R_L} \dfrac{\beta \beta_{INV}}{\dfrac{R_L R_o}{R_L + R_o} + R_{i\beta}}} \right)$$

Input impedance of a v-to-v fed backed OA 8



$$v_p = \frac{R_{o\beta A}}{R_{o\beta A} + R_s + R_i} v_s$$

Now:

$$i_s = \frac{v_s - v_p}{R_s + R_i} = \frac{v_s}{R_s + R_i}\left(1 - \frac{R_{o\beta A}}{R_{o\beta A} + R_s + R_i}\right)$$

$$i_s = \frac{v_s - v_p}{R_s + R_i} = \frac{v_s}{R_{o\beta A} + R_s + R_i}$$

And:

$$\mathbf{R_{iol\_practical\_method}} = \frac{v_s}{i_s} = \mathbf{R_{o\beta A} + R_s + R_i}$$

Input impedance of a v-to-v fed backed OA 9

Second method: evaluation of R_{if} from $R_{if}/f(\beta')$

We have shown that: $$R_{if} = \frac{v_s}{i_s} = \frac{R_i}{f(\beta')}(1-T)$$

So we need to elaborate: $$f(\beta') = \frac{-T}{A\beta'}$$

T was already evaluated at the beginning to be:

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta}} \frac{R_L R_{i\beta A}}{R_L + R_{i\beta A}} \frac{1}{\frac{R_L R_{i\beta A}}{R_L + R_{i\beta A}} + R_o}$$

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta}} \frac{R_L R_{i\beta A}}{R_L R_{i\beta A} + R_L R_o + R_{i\beta A} R_o}$$

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta}} \frac{R_L R_{i\beta A}}{R_L + R_o} \frac{1}{R_{i\beta A} + R_o \| R_L}$$

$$\beta' = \beta\left(1 + \frac{R_o}{R_L} + \frac{(1-\beta\beta_{INV})R_o}{R_{i\beta}}\right)^{-1}$$

$$\beta' = \beta\left(\frac{R_L R_{i\beta} + R_o R_{i\beta} + (1-\beta\beta_{INV})R_L R_o}{R_L R_{i\beta}}\right)^{-1}$$

$$\beta' = \beta\left(\frac{\left(R_{i\beta} + (1-\beta\beta_{INV})R_o \| R_L\right)(R_L + R_o)}{R_L R_{i\beta}}\right)^{-1}$$

Input impedance of a v-to-v fed backed OA 10

$$T=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{R_L R_{i\beta}}{R_L+R_O}\frac{1}{R_{i\beta}+R_o\|R_L}$$

$$\beta'=\beta\left(\frac{\left(R_{i\beta}+(1-\beta\beta_{INV})R_o\|R_L\right)\left(R_L+R_O\right)}{R_L R_{i\beta}}\right)^{-1}$$

Then:

$$f(\beta')=\frac{-T}{A\beta'}=\frac{1}{A}\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{R_L R_{i\beta A}}{R_L+R_O}\frac{1}{R_{i\beta A}+R_o\|R_L}\times$$

$$\times\frac{1}{\beta}\frac{\left(R_{i\beta}+(1-\beta\beta_{INV})R_o\|R_L\right)\left(R_L+R_O\right)}{R_L R_{i\beta}}$$

$$f(\beta')=\frac{R_i}{R_i+R_S+R_{o\beta}}\frac{R_{i\beta}+(1-\beta\beta_{INV})R_o\|R_L}{R_{i\beta A}+R_o\|R_L}\frac{R_{i\beta A}}{R_{i\beta}}$$

Now:

$$R_{iol\_evaluation\_method}=\frac{R_i}{f(\beta')}$$

$$=\left(R_i+R_S+R_{o\beta}\right)\frac{R_{i\beta A}+R_o\|R_L}{R_{i\beta}+(1-\beta\beta_{INV})R_{o,}}\frac{R_{i\beta}}{R_{i\beta A}}$$

$$=\left(R_i+R_S+R_{o\beta}\right)\frac{R_{i\beta}+R_o\|R_L\left(1-\frac{R_i+R_S}{R_i+R_S+R_{o\beta}}\beta_{INV}\beta\right)}{R_{i\beta}+(1-\beta\beta_{INV})R_o\|R_L}$$

Input impedance of a v-to-v fed backed OA 11

$$R_{iol\_evaluation\_method} =$$

$$= \left(R_i + R_S + R_{o\beta}\right) \frac{R_{i\beta} + R_o \| R_L \left(1 - \frac{R_i + R_S}{R_i + R_S + R_{o\beta}} \beta_{INV}\beta\right)}{R_{i\beta} + (1 - \beta\beta_{INV}) R_o \| R_L}$$

$$R_{iol\_evaluation\_method} =$$

$$= \left(R_i + R_S + R_{o\beta}\right) \frac{R_{i\beta} + R_o \| R_L - \frac{R_i + R_S}{R_i + R_S + R_{o\beta}} \beta_{INV}\beta R_o \| R_L}{R_{i\beta} + R_o \| R_L - \beta\beta_{INV} R_o \| R_L}$$

$$R_{iol\_evaluation\_method} =$$

$$= \left(R_i + R_S + R_{o\beta}\right) \frac{1 - \frac{R_o \| R_L}{R_{i\beta} + R_o \| R_L} \frac{R_i + R_S}{R_i + R_S + R_{o\beta}} \beta_{INV}\beta}{1 - \frac{R_o \| R_L}{R_{i\beta} + R_o \| R_L} \beta\beta_{INV}}$$

$$R_{iol\_evaluation\_method} =$$

$$= \frac{R_{o\beta} + R_i + R_S - \frac{R_o \| R_L}{R_{i\beta} + R_o \| R_L} (R_i + R_S) \beta_{INV}\beta}{1 - \frac{R_o \| R_L}{R_{i\beta} + R_o \| R_L} \beta\beta_{INV}}$$

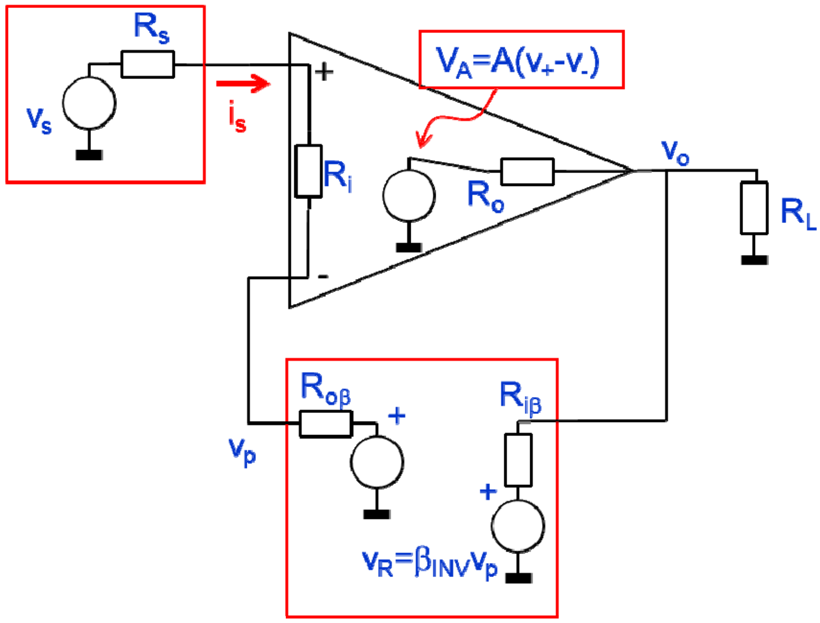
$$\mathbf{R_{iol\_evaluation\_method}} =$$

$$= \frac{R_{o\beta} + \left(1 - \frac{R_o \| R_L}{R_{i\beta} + R_o \| R_L} \beta\beta_{INV}\right)(R_i + R_S)}{1 - \frac{R_o \| R_L}{R_{i\beta} + R_o \| R_L} \beta\beta_{INV}} = R_{o\beta A} + R_i + R_S$$

cvd!

Input impedance of a v-to-v fed backed OA 12

A consideration (a):



In determining the input impedance the source of test signal was a voltage. That was a good choice as in this way we were sure that the driving current was that flowing through R_{if}.

Namely, in this way the feedback is not perturbed when we apply the A=∞ rule: when A=∞ the current thorough R_i drops to a negligible value and is managed from the feedback.

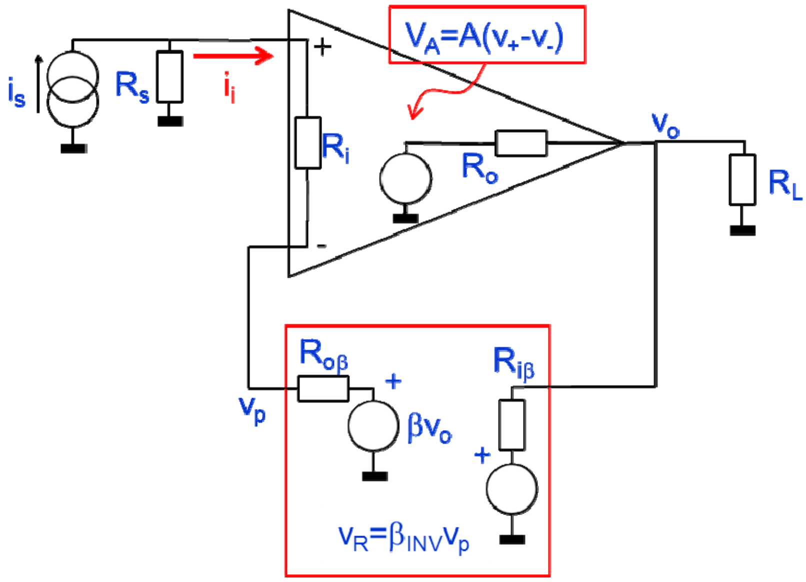
Moreover, if the testing voltage source is not ideal, namely R_s is not completely negligible, the measured input impedance value is not contributed, or is marginally contributed, by R_s itself.

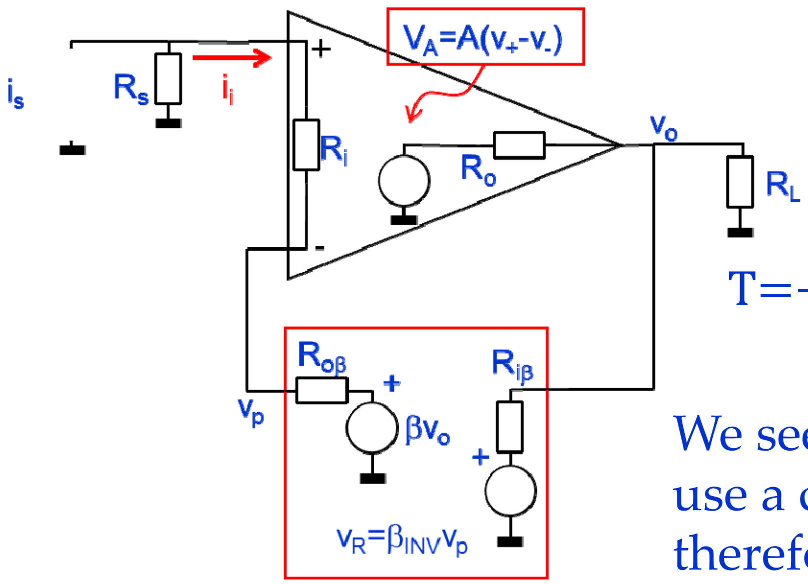
Input impedance of a v-to-v fed backed OA 13

A consideration (b):

If a current source were used for the characterization we would perturb the feedback, acting on the actual value of T.

We see the way this perturbs T, or validity of the feedback:





$$T=-\frac{A\beta R_i}{R_i+R_s+R_{o\beta}}\frac{R_L R_{i\beta A}}{R_L+R_o}\frac{1}{R_{i\beta A}+R_{o'}}$$

We see that if $R_s \rightarrow \infty$ because we use a current source, than $T \rightarrow 0$, therefore, that means feedback broken. Omitting the direct gain for brevity:

$$v_o=\frac{1}{\beta}\frac{-T}{1-T}v_s=\frac{1}{\beta}\frac{-T}{1-T}R_s i_s$$

$$v_o=\frac{AR_i}{R_i+R_s+R_{o\beta}}\frac{R_L R_{i\beta A}}{R_L+R_o}\frac{1}{R_{i\beta A}+R_{o'}}\frac{1}{1-T}R_s i_s$$

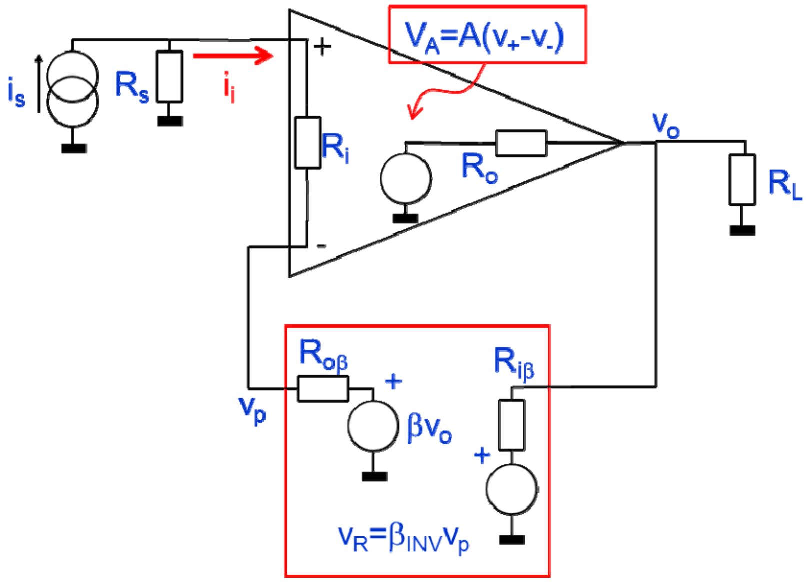
$$v_o \xrightarrow[R_s\rightarrow\infty]{} AR_i\frac{R_L R_{i\beta A}}{R_L+R_o}\frac{1}{R_{i\beta A}+R_{o'}}i_s$$

Namely, the gain depends on A and no more on $1/\beta$, so the amplifier works open loop.

Input impedance of a v-to-v fed backed OA 14

A consideration (c):

In case a current source were used anyway, it would help to consider a source impedance, R_s, in parallel to it.
Remembering the Thevenin theorem, this would be equivalent to consider a voltage source with a series resistance in series to it.



GENERAL RULE:
In characterizing the input impedance use a voltage source to measure large impedance and a current source to measure small impedances; namely, if the feedback reads a voltage, then use a voltage source; if the feedback read a current use a current source.
Said in another way: a voltage amplifier tries to show a large input impedance, then use a source with a small impedance for its characterization; a current amplifier tries to show a small input impedance, then use a source with a large impedance for its characterization.

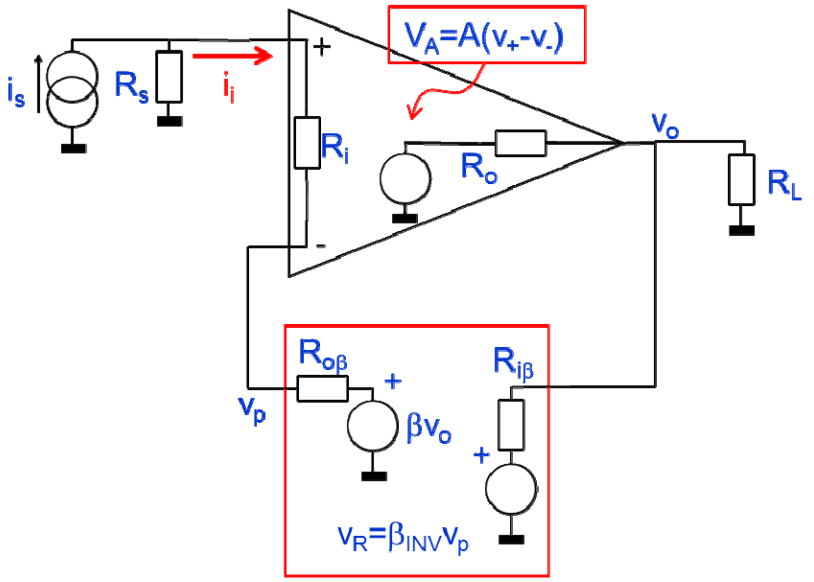
The golden suggestion: look at the found loop gain and see what happens if the source impedance is very big or very small: only in one case T will be 0; than, take a current source if $T \to 0$ when $R_s \to 0$, or take a voltage source if $T \to 0$ when $R_s \to$

$$T=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{R_L R_{i\beta A}}{R_L+R_o}\frac{1}{R_{i\beta A}+R_{o'}}$$

Here we saw that $T \to 0$ if $R_s \to \infty$, than, the voltage source is the choice.

A consideration (d):

Using the configuration on the figure is anyway possible, provided that some considerations are done.



It has to be considered that:

$$i_s = \frac{v_+}{R_s} + \frac{v_+ - v_-}{R_i}$$

Where, as we have already shown:

$$v_A = \frac{1}{\beta'}\frac{-T}{1-T}v_s = \frac{1}{\beta'}\frac{-T}{1-T}R_s i_s$$

$$i_s = \frac{v_+}{R_s} + \frac{f(\beta')}{(1-T)R_i}R_s i_s$$

$$i_s\left(1 - \frac{f(\beta')}{(1-T)R_i}R_s\right) = \frac{v_+}{R_s}$$

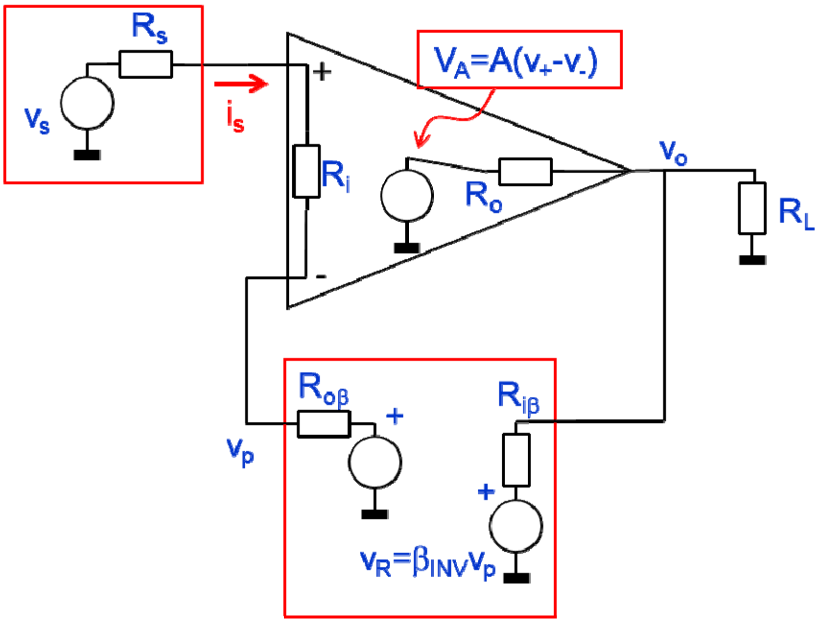
And:

$$R_{ifp} = \frac{v_+}{i_s} = R_s\left(1 - \frac{f(\beta')}{(1-T)R_i}R_s\right) = R_s^2\left(\frac{1}{R_s} - \frac{1}{R_{if}}\right)$$

$$= R_s^2\left(\frac{1}{R_s} - \frac{1}{R_s + R_{ifo}}\right) = R_s\left(\frac{R_s + R_{ifo} - R_s}{R_s + R_{ifo}}\right) = \frac{R_s R_{ifo}}{R_s + R_{ifo}}$$

And R_{if0} is the feedback input impedance measured when R_s is set to 0 in the network, see the next consideration for detail.

A consideration (e):



An useful utility is to disentangle the source impedance, R_s, from the amplifier input impedance. In practice the impedance so far evaluated, R_{if}, we would like to be express by:

$$R_{if} = R_s + R_{if0}$$

Starting from the zero gain, or open loop, input impedance:

$$R_{iol} = R_s + R_{o\beta A} + R_i$$

We can write, obviously:

$$R_{iol} = R_s + R_{iol0}$$

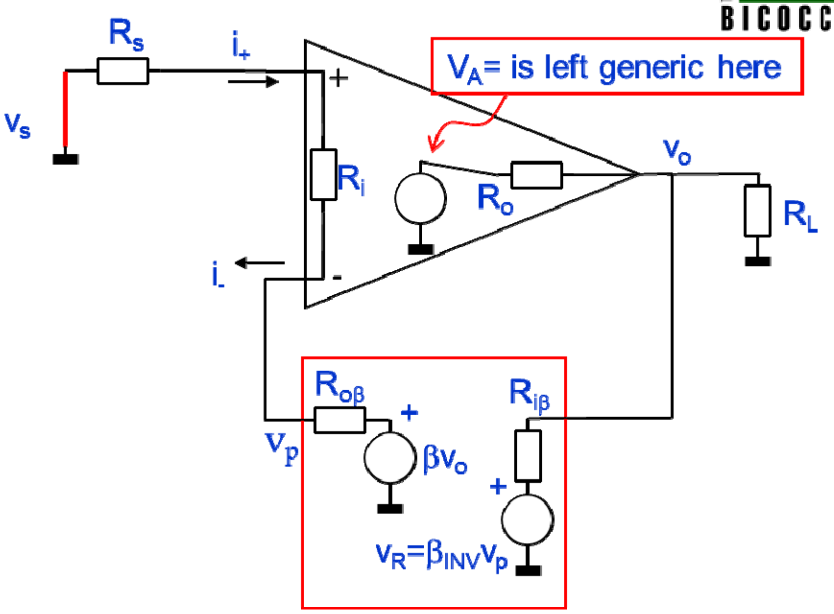
We would be happy to arrive at an expression of this kind:

$$R_{if} = R_s + R_{iol0}(1 - T_0)$$

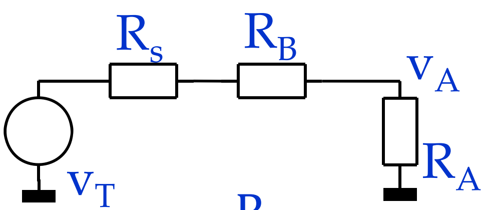
Where R_{iol0} and T_0 refer to R_{iol} and T evaluated when R_s=0 Ω.

Input impedance of a v-to-v fed backed OA 17

The loop gain T is the combination of several voltage attenuators. Indeed the node v_o is the first attenuation of v_A at the output mesh. After the link of the 2 dependent voltage sources another attenuation is applied at the input mesh.

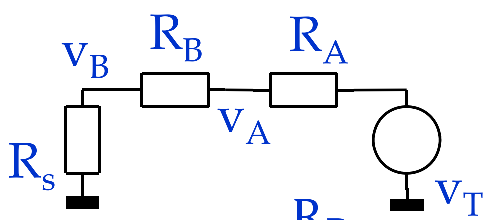


Now, let's study the voltage divider below:



$$R_T = R_A + R_B + R_s$$

$$v_A = \frac{R_A}{R_T} v_T$$



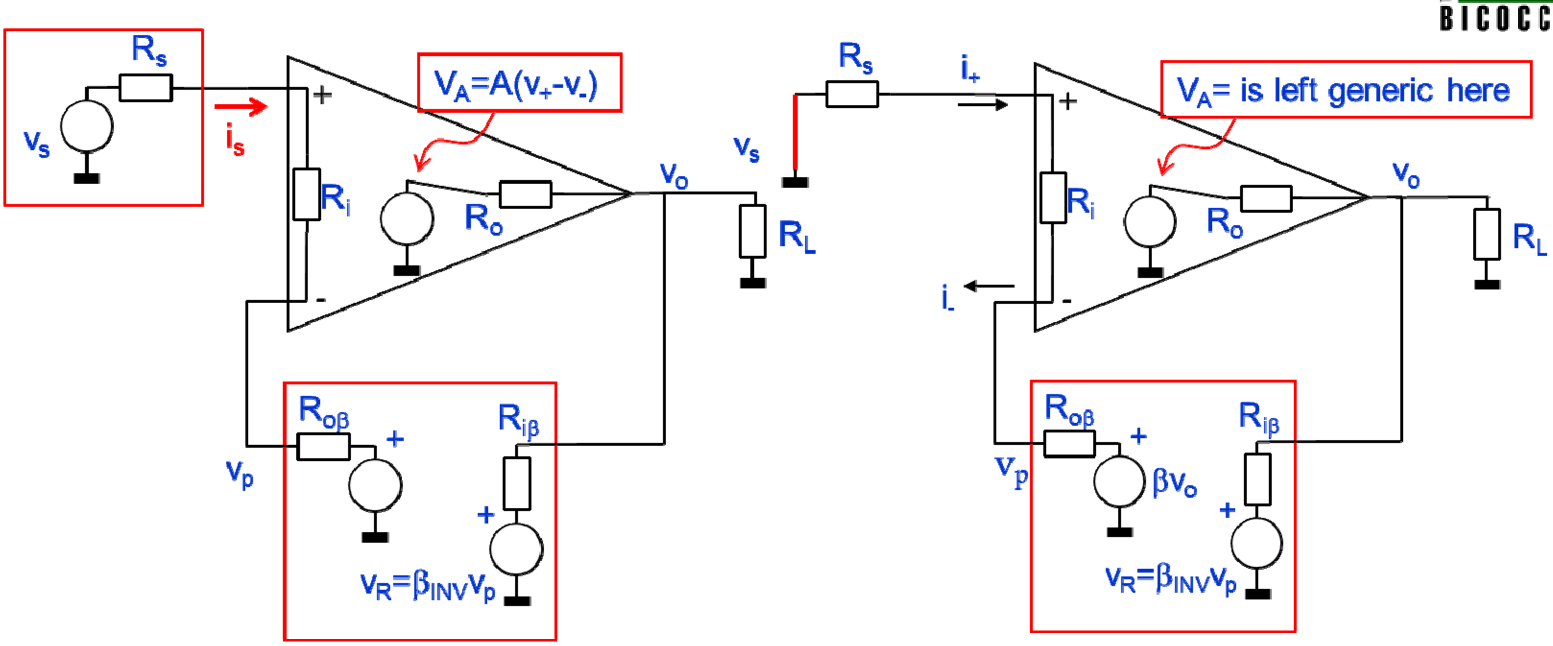
$$v_A - v_B = \frac{R_B}{R_T} v_T$$

We see that (a), irrespectively of where we apply the excitation, the node voltage to which we are interested is inversely proportional to the total resistor, R_T.

(b) The numerator of the voltage drop of interest is proportional only to the resistor to which the drop is considered.
This proves that:

$R_T v_x$ = depend only on the resistor for which v_x is the dropout, and $R_{T0}v_{x0}=R_T v_x$, if R_{T0} and v_{x0} are R_T and v_x when $R_s=0$, and x=A or B.

Input impedance of a v-to-v fed backed OA 18



Considering the meaning of R_{iol} , R_{iol0} and T, namely, R_{iol} is the total resistor at the input mesh, T is a voltage partition of v_A and that T is proportional to v_+ - v_- , which is not the drop across R_s , we expect that :

$$R_{iol}(v_+ - v_-) = R_{iol}\frac{T}{A}v_A = f(R_i)$$

$$R_{iol0}(v_+ - v_-) = R_{iol0}\frac{T_o}{A}v_A = f(R_i)$$

$$\Rightarrow \quad R_{iol}T = R_{iol0}T_o$$

$$T = \frac{R_{iol0}}{R_{iol}}T_0$$

So that:

$$R_{if} = R_{iol}(1 - T)$$

$$=R_{iol}-R_{iol}T$$

$$=R_{iol} - R_{iol0}T_0$$

$$=R_s+R_{iol0} - R_{iol0}T_0$$

$$=R_s+R_{iol0}(1 - T_0)$$

Input impedance of a v-to-v fed backed OA 19

We can verify our result remembering that:

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta}} \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o}$$

Now, since:

$$R_{iol} = R_s + R_{o\beta A} + R_i$$

We need to elaborate $R_{i\beta A}$ for having T as a function of $R_{o\beta A}$ instead of $R_{o\beta}$ or vice versa:

$$R_{i\beta A} = \frac{R_{i\beta}}{1 - \frac{R_i + R_S}{R_i + R_S + R_{o\beta}} \beta_{INV} \beta}$$

$$R_{o\beta A} = \frac{R_{o\beta}}{1 - \frac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta \beta_{INV}}$$

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta}} \frac{R_o \| R_L}{R_o \| R_L + R_{i\beta A}} \frac{R_{i\beta A}}{R_o}$$

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta}} \frac{R_o \| R_L}{R_o \| R_L \left(1 - \frac{R_i + R_S}{R_i + R_S + R_{o\beta}} \beta_{INV} \beta\right) + R_{i\beta}} \frac{R_{i\beta}}{R_o}$$

Continuing:

$$T=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{R_o\|R_L}{R_o\|R_L\left(1-\frac{R_i+R_S}{R_i+R_S+R_{o\beta}}\beta_{INV}\beta\right)+R_{i\beta}}\frac{R_{i\beta}}{R_o}$$

$$T=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{R_o\|R_L}{R_o\|R_L+R_{i\beta}+\frac{R_i+R_S}{R_i+R_S+R_{o\beta}}\beta_{INV}\beta R_o\|R_L}\frac{R_{i\beta}}{R_o}$$

$$T=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{R_o\|R_L}{1-\frac{R_o\|R_L}{R_o\|R_L+R_{i\beta}}\frac{R_i+R_S}{R_i+R_S+R_{o\beta}}\beta_{INV}\beta}\frac{R_{i\beta}}{R_o\left(R_o\|R_L+R_{i\beta}\right)}$$

$$T=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\times\frac{\left(R_i+R_S+R_{o\beta}\right)R_o\|R_L}{R_{o\beta}+\left(1-\frac{R_o\|R_L}{R_o\|R_L+R_{i\beta}}\beta_{INV}\beta\right)\left(R_i+R_S\right)}\frac{R_{i\beta}}{R_o\left(R_o\|R_L+R_{i\beta}\right)}$$

Continuing:

$$R_{o\beta A} = \frac{R_{o\beta}}{1 - \frac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta\beta_{INV}}$$

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta A}} \frac{R_o \| R_L}{1 - \frac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta_{INV}\beta} \frac{R_{i\beta}}{R_O\left(R_o \| R_L + R_{i\beta}\right)}$$

Slightly more precise:

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta A}} \frac{R_o \| R_L}{R_{i\beta} + (1 - \beta_{INV}\beta) R_o \| R_L} \frac{R_{i\beta}}{R_O}$$

$$T = -\frac{A\beta R_i}{R_i + R_S + R_{o\beta A}} \frac{R_{i\beta B} \| R_L}{R_{i\beta B} \| R_L + R_O} \qquad R_{i\beta B} = \frac{R_{i\beta B}}{1 - \beta\beta_{INV}}$$

By remembering

$$R_{iol} = R_s + R_{o\beta A} + R_i$$

We have proven that:

$$R_{iol} T = -\frac{(A\beta R_i) R_{i\beta B} \| R_L}{R_{i\beta B} \| R_L + R_O}$$

is independent on both R_{iol} and R_s, namely R_{iol0}.

cvd!

Input impedance of a v-to-v fed backed OA 22

In our particular case from:

$$R_{iol} = R_s + R_{o\beta A} + R_i$$

We have:

$$R_{if} = R_s + R_{iol0}(1 - T_0)$$

$$R_{if} = R_s + R_{o\beta A} + R_i + \frac{(A\beta R_i)R_{i\beta B} \| R_L}{R_{i\beta B} \| R_L + R_o}$$

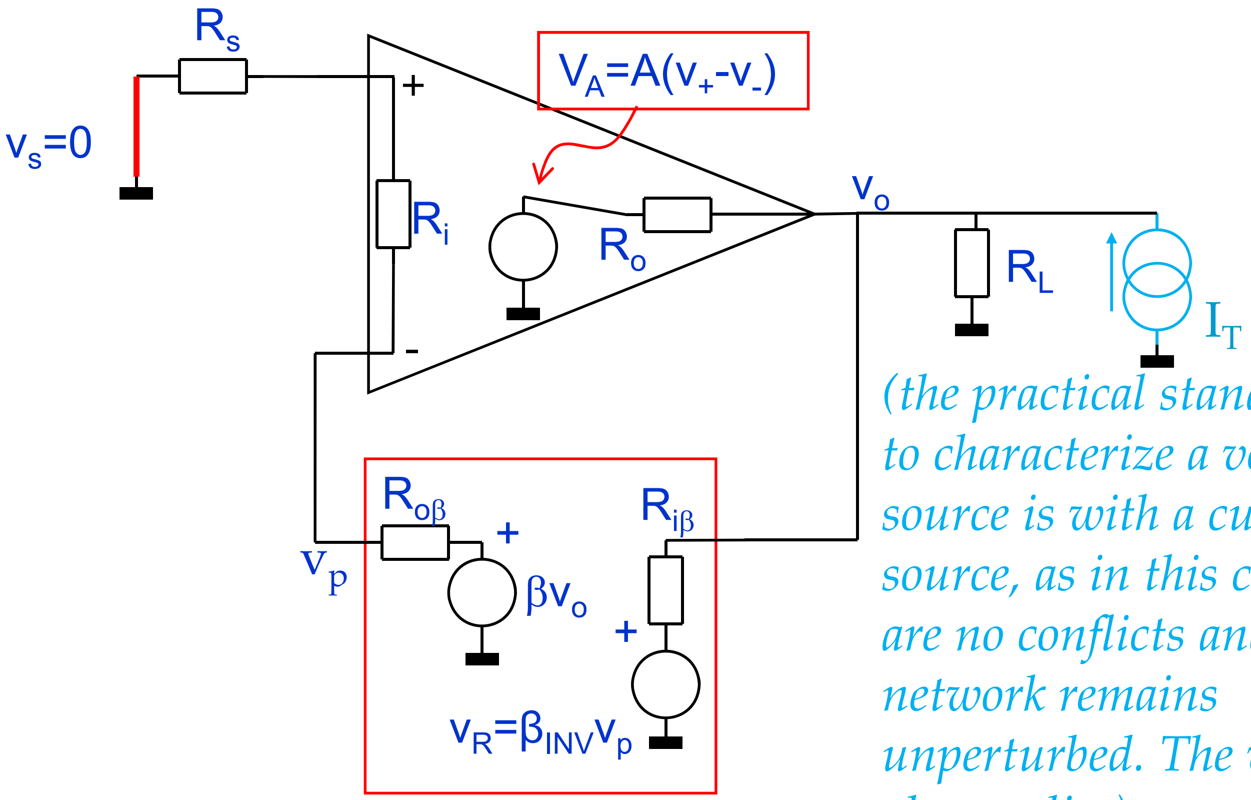
$$R_{if} = R_s + R_{o\beta A} + R_i\left(1 + \frac{(A\beta)R_{i\beta B} \| R_L}{R_{i\beta B} \| R_L + R_o}\right)$$

$$R_{if} = R_{if0} + R_s$$

where R_{if0} is R_{if} obtained when R_s is set to 0.

Output impedance of a v-to-v fed backed OA 1

The output impedance is the impedance seen from the load and is characterized as shown below:



(the practical standard way to characterize a voltage source is with a current source, as in this case there are no conflicts and the network remains unperturbed. The vice versa also applies)

The source of the signal is nulled and the test signal is applied at the node where the impedance has to be measured. Note that this mathematical procedure reflects what it needs to be done in the lab.

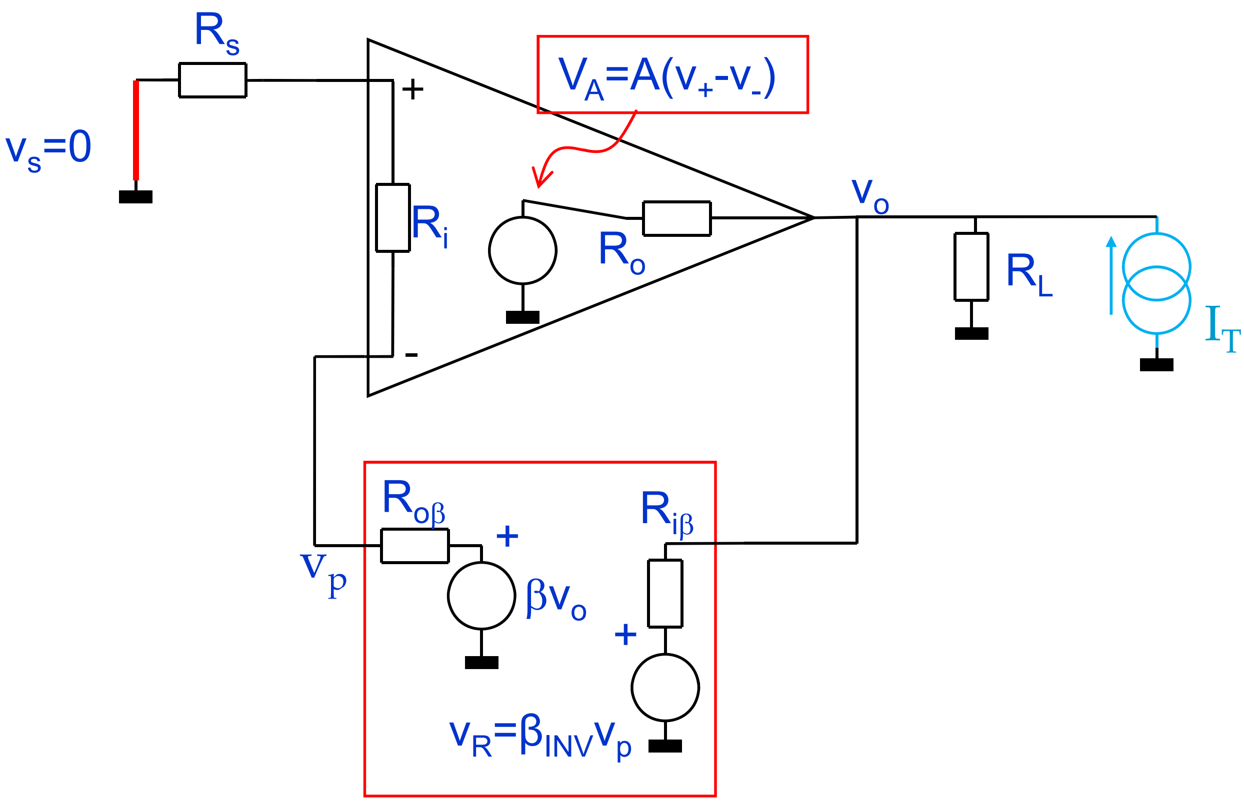
Also for this case we would try to apply our known procedure as the node where the signal is applied has not a particular property, provided that the source does not perturb the network, hence the loop gain T.

Again, we expect that:

$$v_o = \frac{1}{\beta''}\frac{-T}{1-T}i_T + \frac{A_{R''}}{1-T}i_T$$

Parameters β'' and $A_{R''}$ are only to indicate that v_o is excited by a source other than v_s, and are different from the respective β and A_R.

Output impedance of a v-to-v fed backed OA 2



Let's begin with considering T=∞:

$$0 = v_+ = v_- = \beta v_o$$

Therefore v_o results 0, that means: β''=∞:

$$v_o = \frac{A_{R''}}{1 - T} i_T$$

So that:

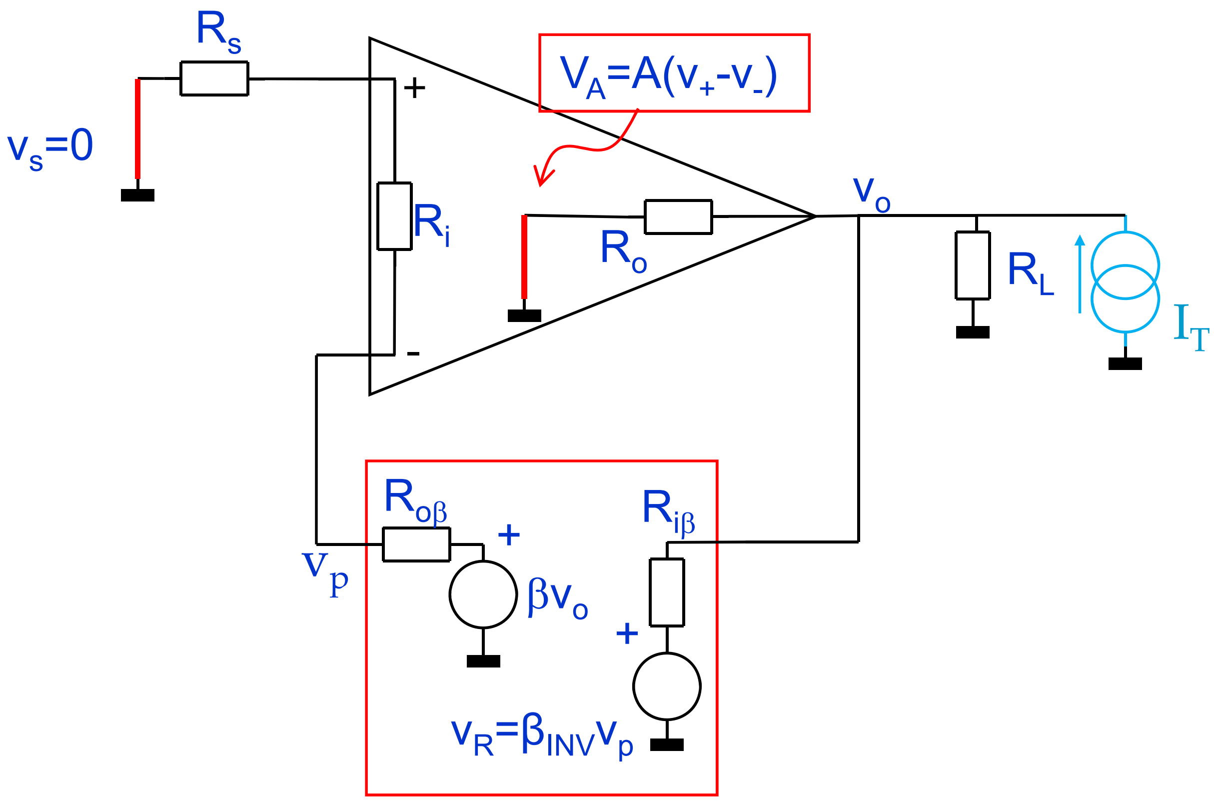
$$R_{ofp} = \frac{v_o}{i_T} = \frac{A_{R''}}{1 - T}$$

With T=0 R_{ofp} reduces to R_{ool}, hence:

$$R_{ofp}\Big|_{T=0} = R_{ool} = A_{R''}$$

Now the parameter $A_{R''}$, or R_{ool},…

Output impedance of a v-to-v fed backed OA 3

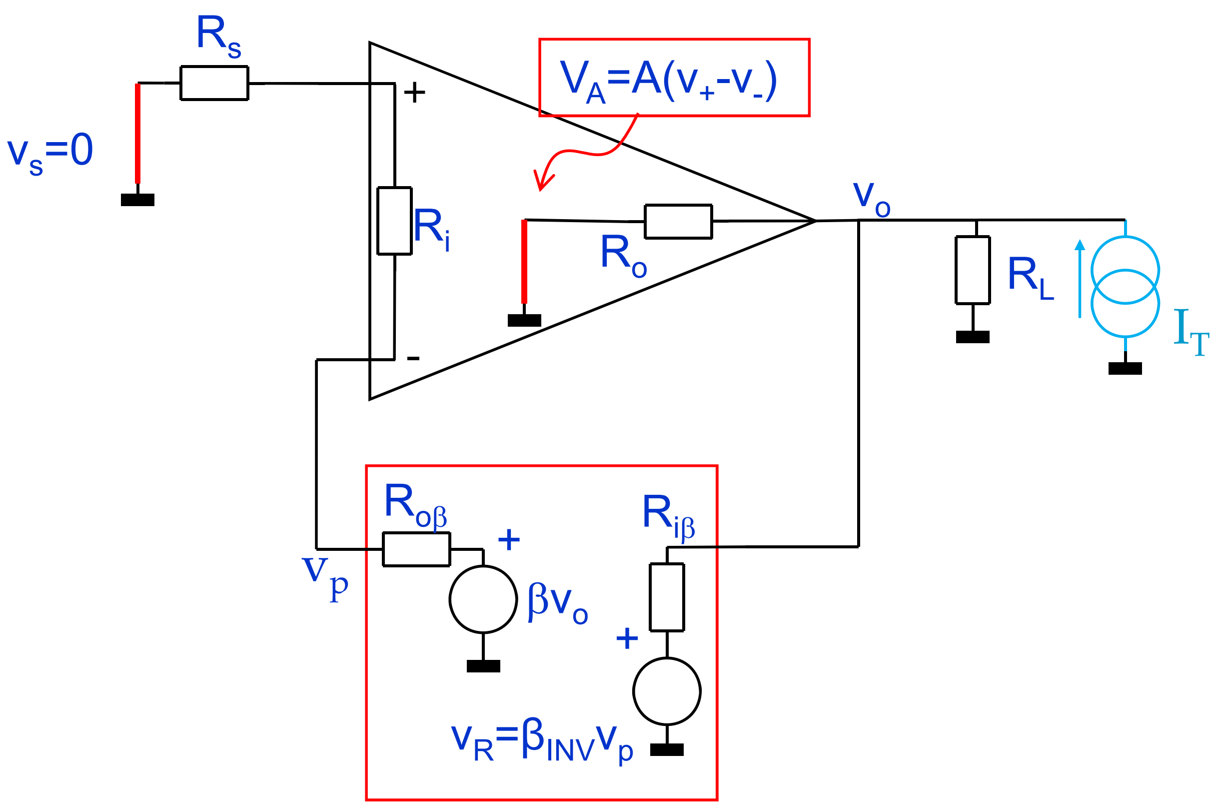


Now we have that:

$$\begin{cases} i_T = \dfrac{v_o}{R_o} + \dfrac{v_o}{R_L} + \dfrac{v_o - \beta_{INV} v_p}{R_{i\beta}} \\ v_p = \beta \dfrac{R_s + R_i}{R_s + R_i + R_{o\beta}} v_o \end{cases}$$

$$i_T = \frac{v_o}{R_o} + \frac{v_o}{R_L} + \frac{v_o}{R_{i\beta}}\left(1 - \frac{R_s + R_i}{R_s + R_i + R_{o\beta}} \beta\beta_{INV}\right)$$
$$= \frac{v_o}{R_o} + \frac{v_o}{R_L} + \frac{v_o}{R_{i\beta A}}$$

Output impedance of a v-to-v fed backed OA 4



Now we have that:

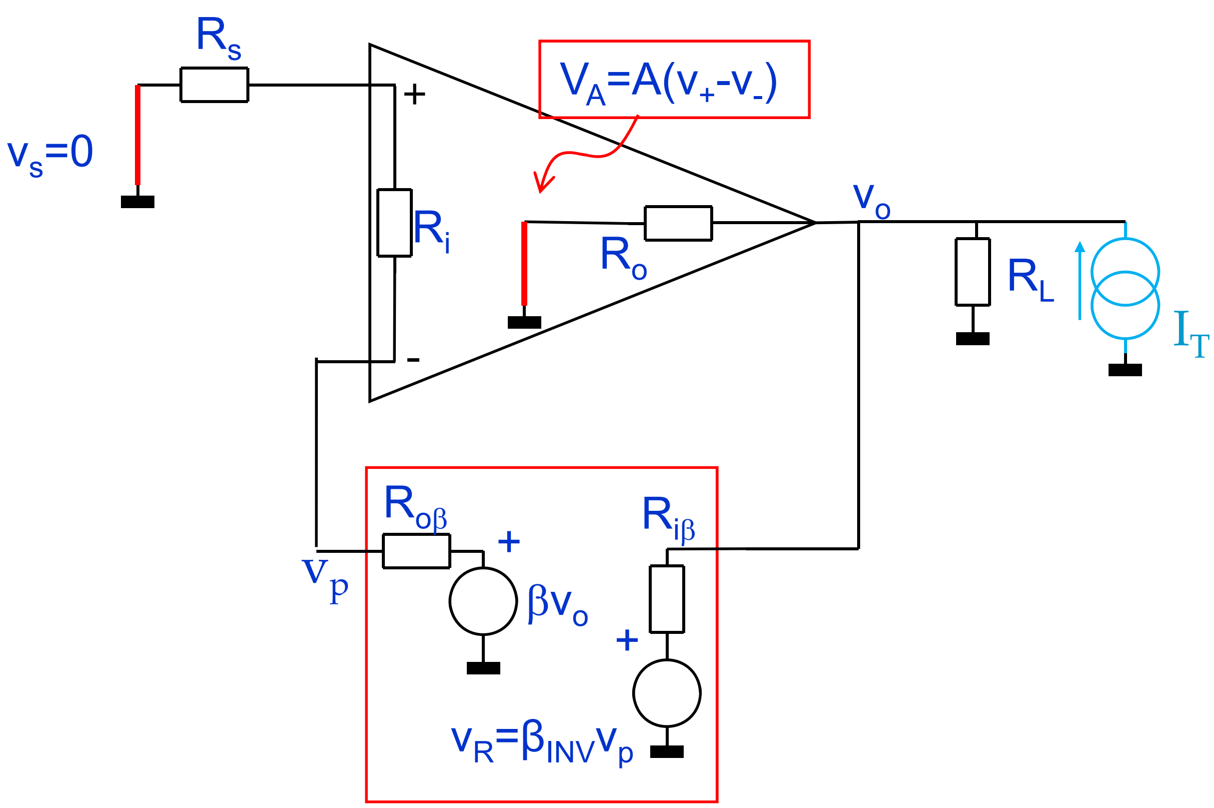
$$i_T = \frac{v_o}{R_o} + \frac{v_o}{R_L} + \frac{v_o}{R_{i\beta A}} \Rightarrow i_T = \frac{v_o}{R_{o''}}, \qquad R_{o''} = R_L \| R_o \| R_{i\beta A}$$

Therefore: $$A_{R''} = \frac{v_o}{i_T} = R_{o''}$$

Then: $$v_o = \frac{A_{R''}}{1-T} = \frac{R_{o''}}{1-T} i_T$$

And finally: $$\mathbf{R_{ofp}} = \frac{v_o}{i_T} = \frac{R_{o''}}{1-T}$$

Output impedance of a v-to-v fed backed OA 5



$$\mathbf{R_{of}} = \frac{v_o}{i_T} = \frac{R_{o''}}{1-T}$$

Again the final result has an important physical meaning: the output impedance after the feedback is closed is given by the ratio of the output impedance measured when the gain is nulled, i.e. the feedback is open, and 1-T: once again the feedback tries to approach the behavior of the fed backed OA to the ideal condition.

In this case is quite easy to verify that $R_{o''}$ is the impedance that is measured when the gain is set to 0.

The impedance characterized in this way is in the form (see also later for details):

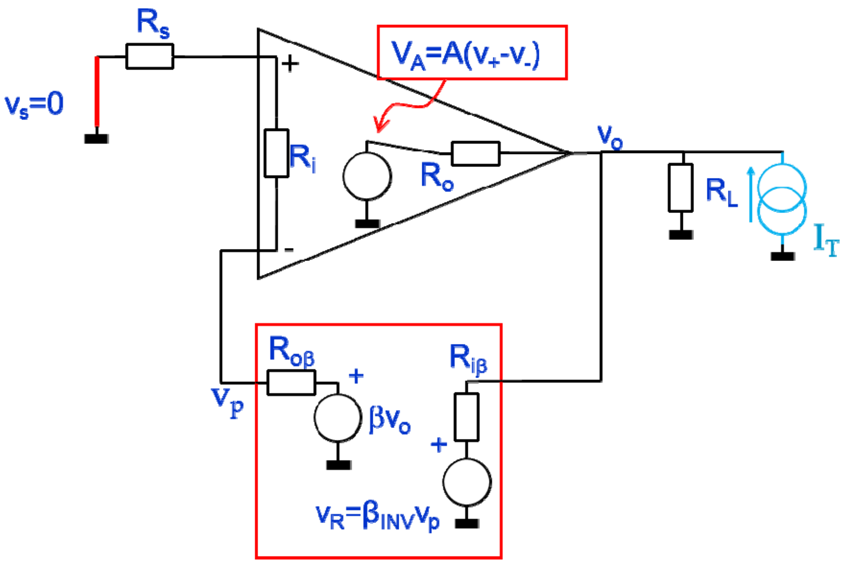
$$\mathbf{R_{ofp}} = R_L \| R_{of\infty}$$

$R_{of\infty}$ being the impedance measured when $R_L=\infty$.

(see appendix A for an example)

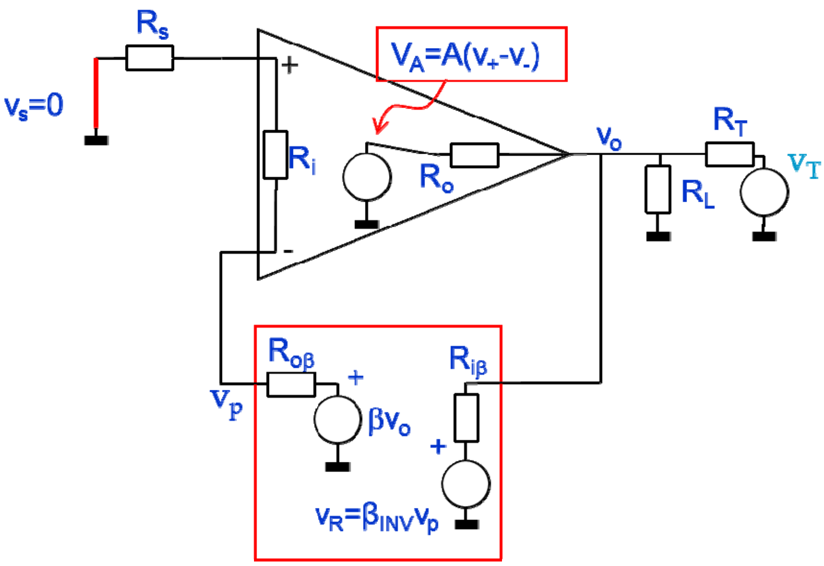
Output impedance of a v-to-v fed backed OA 6

A consideration (a):



We expect to evaluate a very small output impedance, then, for not perturbing the loop, it is recommended to use a current source for the test signal.

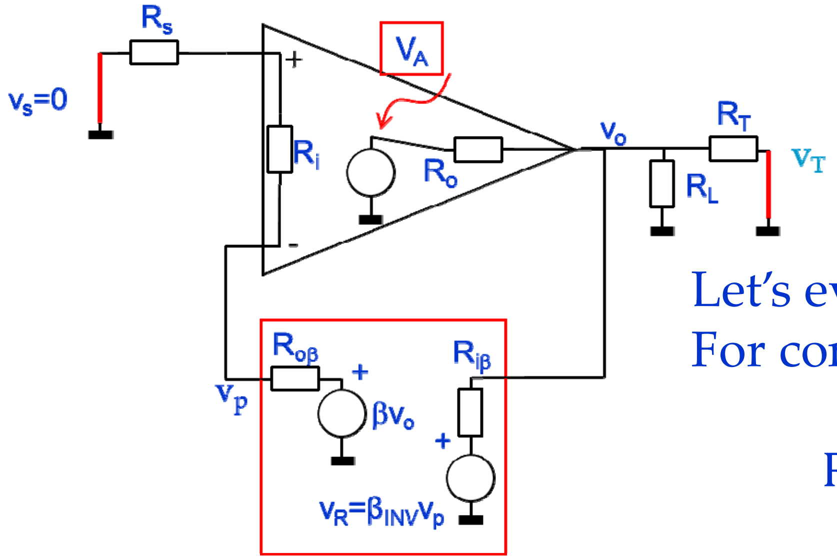
Below, we use a voltage source. See what happens.



It is easy to imagine that if the voltage source is close to ideal, namely R_T close to 0 Ω, the output node is shorted and the loop does not work.
See next slide for details.

<u>The golden suggestion</u>: To measure a small impedance use a source with a large impedance, a current source; to measure a large impedance use a source with a small impedance, a voltage source.

A consideration (b):



Let's evaluate the loop gain.
For convenience we define:

$$R_{L'} = R_L \| R_T$$

Then:

$$T = -\frac{A\beta R_i}{R_i + R_s + R_{o\beta}} \frac{R_{L'} R_{i\beta A}}{R_{L'} + R_o} \frac{1}{R_{i\beta A} + R_{o'}}$$

$$T \xrightarrow[R_T \to 0]{} 0$$

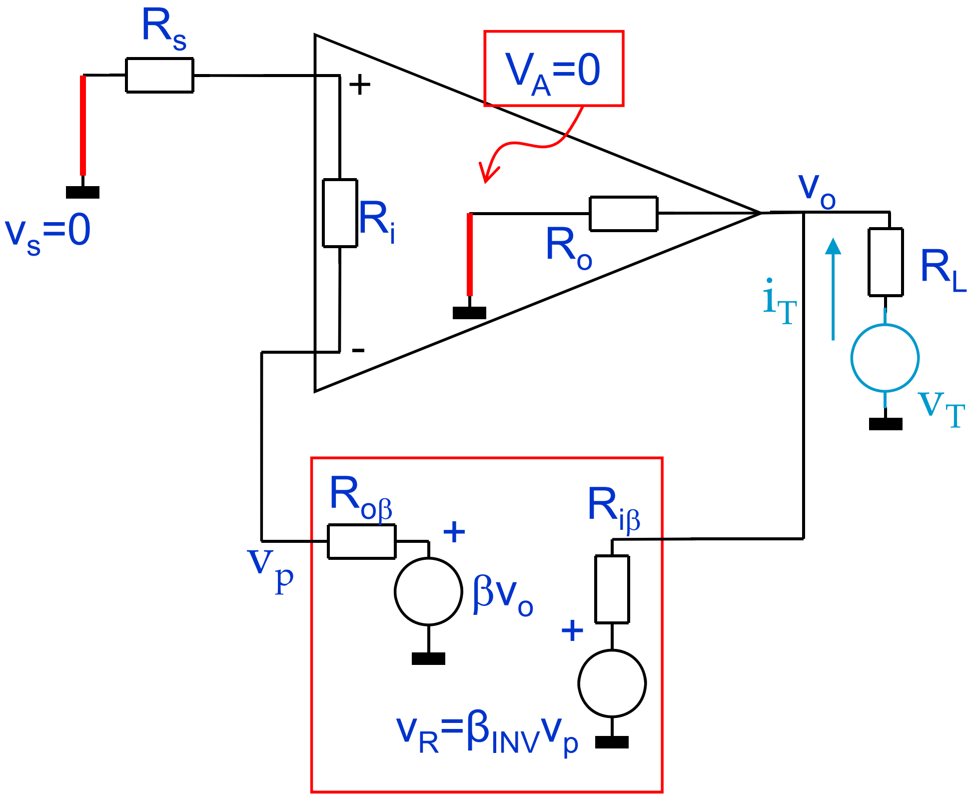
In this case, obviously, again neglect the direct transmission:

$$v_o = \frac{1}{\beta}\frac{-T}{1-T} v_s$$

$$v_o = \frac{AR_i}{R_i + R_s + R_{o\beta}} \frac{R_{L'} R_{i\beta A}}{R_{L'} + R_o} \frac{1}{R_{i\beta A} + R_{o'}} \frac{1}{1-T} v_s \xrightarrow[R_T \to 0]{} 0$$

The golden suggestion: look at the found loop gain and see what happens if the load test impedance is very big or very small: only in one case T will be 0; than, take a current source if $T \to 0$ when $R_s \to 0$, or take a voltage source if $T \to 0$ when $R_s \to \infty$.

A consideration (c): nevertheless…



The test signal in the position shown does not affect the loop and can be used for testing.
Also in this case we have only direct transmission, given by:

$$\begin{cases} \dfrac{v_T - v_o}{R_L} = \dfrac{v_o}{R_o} + \dfrac{v_o - \beta_{INV} v_p}{R_{i\beta}} \\ v_p = \dfrac{R_s + R_i}{R_s + R_i + R_{o\beta}} v_o \end{cases}$$

$$\frac{v_T - v_o}{R_L} = \frac{v_o}{R_o} + \frac{v_o}{R_{i\beta A}}$$

$$\frac{v_T}{R_L} = \frac{v_o}{R_L} + \frac{v_o}{R_o} + \frac{v_o}{R_{i\beta A}}$$

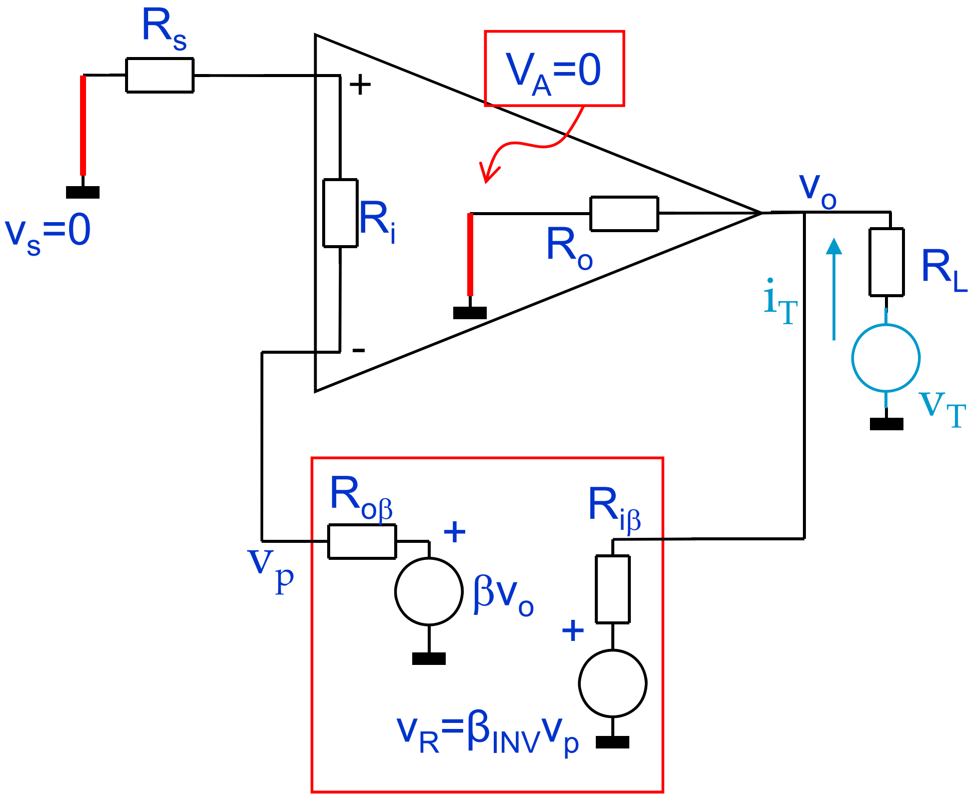
$$v_T = \frac{(R_o + R_{i\beta A})R_L + R_o R_{i\beta A}}{R_o R_{i\beta A}} v_o$$

Therefore, finally:

$$v_o = \frac{R_{i\beta A} R_o}{R_{i\beta A} + R_o} \frac{1}{\dfrac{R_{i\beta A} R_o}{R_{i\beta A} + R_o} + R_L} \frac{v_T}{1 - T} = \frac{A_{R'''}}{1 - T} v_T$$

Output impedance of a v-to-v fed backed OA 9

A consideration (c): nevertheless…



$$v_o = \frac{R_{i\beta A} R_o}{R_{i\beta A} + R_o} \frac{1}{\frac{R_{i\beta A} R_o}{R_{i\beta A} + R_o} + R_L} \frac{v_T}{1-T} = \frac{A_{R''}}{1-T} v_T$$

Therefore:

$$i_T = \frac{v_T - v_o}{R_L} = \left(1 - \frac{A_{R'''}}{1-T}\right) \frac{v_T}{R_L}$$

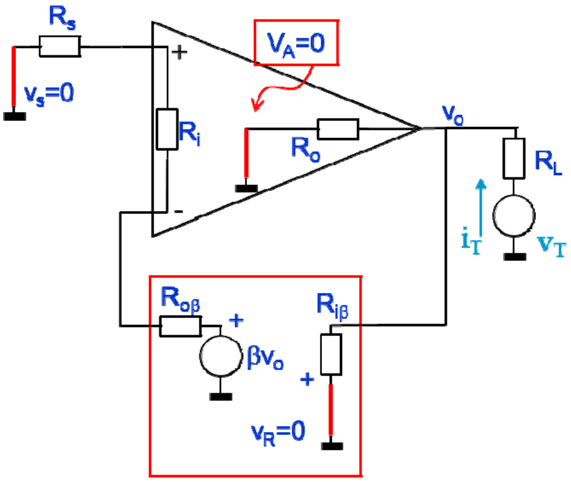
$$R_{ofs} = R_L + R_{of\infty} = \frac{v_T}{i_T} = R_L \frac{1}{1 - \frac{A_{R'''}}{1-T}}$$

It can be shown that the result obtained in this measurement configuration for $R_{of\infty}$ and in the previous configuration with a current source as test signal are the same.

The adoption of the test current source approach is a bit more physical as it links the impedance measured in open loop condition with the impedance measured in closed loop condition.

Output impedance of a v-to-v fed backed OA 10

The output impedances R_{ofp} and R_{ofs}:



Just for verification, we repeat the calculation, starting from R_{ofs};

$$v_o = \frac{R_{i\beta A}R_o}{R_{i\beta A}+R_o}\frac{1}{\frac{R_{i\beta A}R_o}{R_{i\beta A}+R_o}+R_L}\frac{v_T}{1-T}$$

Remembering:

$$T=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{R_L R_{i\beta A}}{R_L+R_{i\beta A}}\frac{1}{\frac{R_L R_{i\beta A}}{R_L+R_{i\beta A}}+R_o}$$

We have:

$$v_o = \frac{R_{i\beta A}R_o}{R_o\left(R_L+R_{i\beta A}\right)+\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)R_L R_{i\beta A}}v_T$$

$$R_{ofs} = R_L\frac{1}{1-\frac{R_{i\beta A}R_o}{R_o\left(R_L+R_{i\beta A}\right)+\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)R_L R_{i\beta A}}}$$

$$R_{ofs} = \frac{R_o\left(R_L+R_{i\beta A}\right)+\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)R_L R_{i\beta A}}{R_o+\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)R_{i\beta A}}$$

Output impedance of a v-to-v fed backed OA 11

Now:

$$R_{ofs} = R_L + R_{of\infty}$$

Then:

$$R_{of\infty} = \frac{R_o\left(R_L + R_{i\beta A}\right) + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_L R_{i\beta A}}{R_o + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_{i\beta A}} - R_L$$

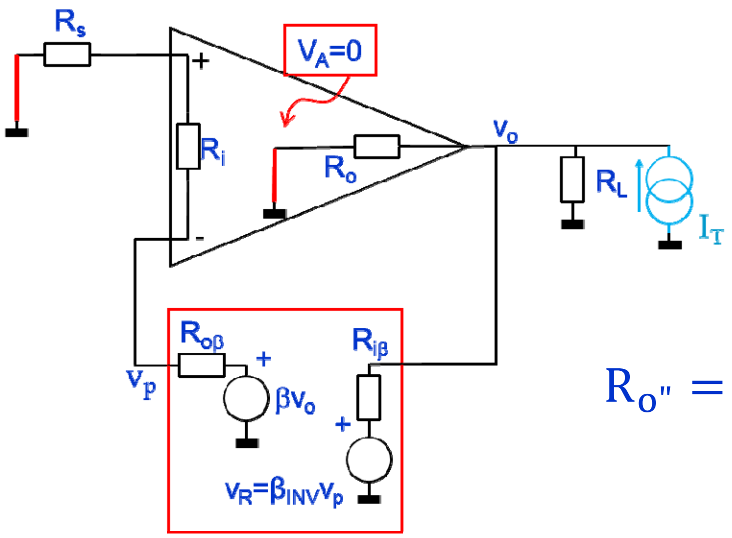
$$R_{of\infty} = \frac{R_L R_o + R_o R_{i\beta A} + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_L R_{i\beta A}}{R_o + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_{i\beta A}} + \frac{-R_L R_o - \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_L R_{i\beta A}}{R_o + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_{i\beta A}}$$

Finally:

$$R_{of\infty} = \frac{R_o R_{i\beta A}}{R_o + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_{i\beta A}}$$

$$R_{of\infty} = R_{i\beta A} \,\Big\|\, \frac{R_o}{\left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right)}$$

Output impedance of a v-to-v fed backed OA 12



Now the output impedance with this characterizing configuration:

$$R_{o''} = R_L \| R_o \| R_{i\beta A}$$

$$R_{ofp} = \frac{R_{o''}}{1-T} = \frac{R_L R_{i\beta A}}{R_L + R_{i\beta A}} \frac{R_o}{\frac{R_L R_{i\beta A}}{R_L + R_{i\beta A}} + R_o} \frac{1}{1-T}$$

$$R_{ofp} = \frac{R_L R_{i\beta A}}{R_L + R_{i\beta A}} \frac{R_o}{R_o + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) \frac{R_L R_{i\beta A}}{R_L + R_{i\beta A}}}$$

Now:

$$\mathbf{R_{ofp}} = R_L \| R_{of\infty} \quad \Rightarrow \quad \frac{1}{R_{of\infty}} = \frac{1}{R_{ofp}} - \frac{1}{R_L}$$

$$\frac{1}{R_{of\infty}} = \frac{R_o(R_L + R_{i\beta A}) + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_L R_{i\beta A}}{R_L R_{i\beta A} R_o} - \frac{1}{R_L}$$

$$\frac{1}{R_{of\infty}} = \frac{R_L R_o + \left(1 + \frac{A\beta R_i}{R_i + R_S + R_{o\beta}}\right) R_L R_{i\beta A}}{R_L R_{i\beta A} R_o}$$

Output impedance of a v-to-v fed backed OA 13

$$\frac{1}{R_{of\infty}}=\frac{R_L R_o+\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)R_L R_{i\beta A}}{R_L R_{i\beta} R_o}$$

$$\frac{1}{R_{of\infty}}=\frac{R_o+\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)R_{i\beta A}}{R_{i\beta A} R_o}$$

Hence:

$$R_{of\infty}=\frac{R_{i\beta A} R_o}{R_o+\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)R_{i\beta A}}$$

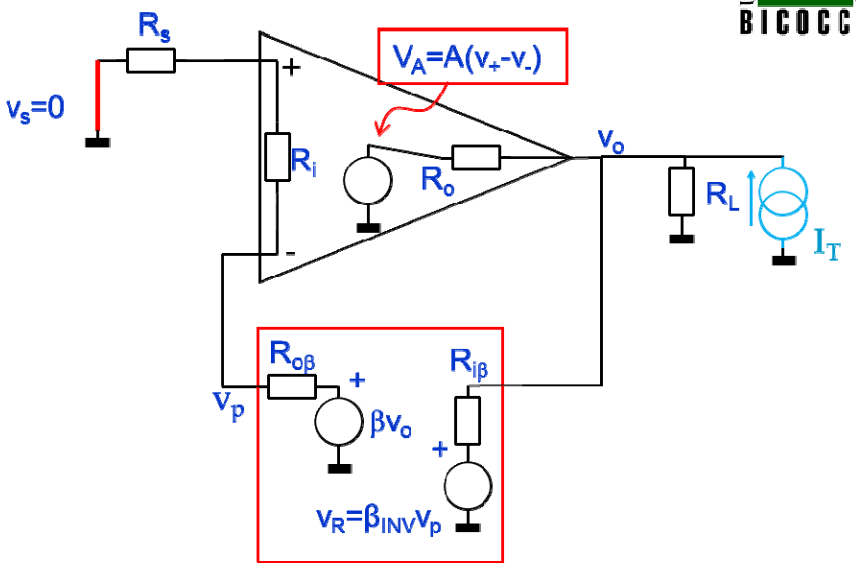
Finally, again:

$$R_{of\infty}=R_{i\beta A}\left\|\frac{R_o}{\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)}\right.$$

cvd!

Output impedance of a v-to-v fed backed OA 14

A last consideration :



An useful utility is to disentangle the load impedance, R_L, from the amplifier output impedance.
In practice the impedance so far evaluated, R_{of}, we would like to be expressed as:

$$R_{of} = R_L \| R_{of\infty}$$

Starting from the zero gain, or open loop, input impedance:

$$R_{ool} = R_L \| R_o \| R_{i\beta A}$$

We can write, obviously:

$$R_{ool} = R_L \| R_{ool\infty}$$

We would be happy to arrive at an expression of this kind:

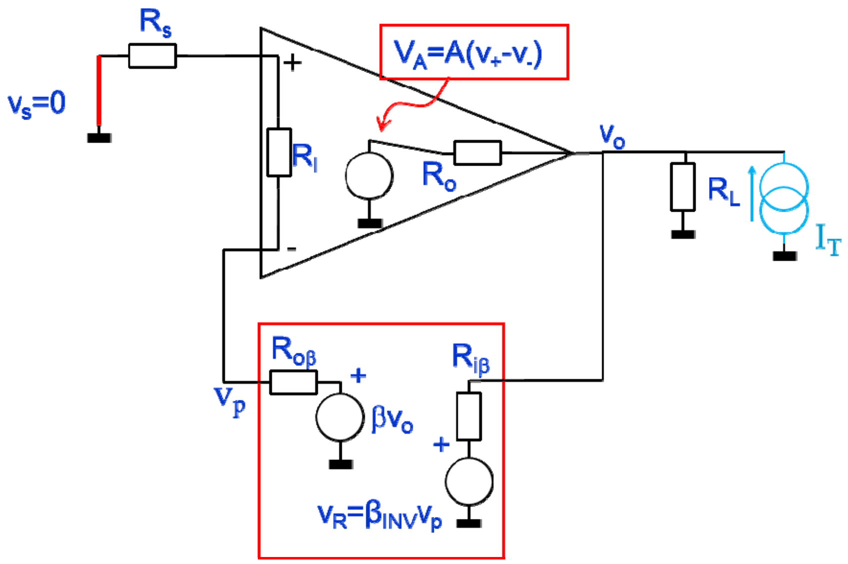
$$R_{of} = R_L \| \frac{R_{ool\infty}}{1 - T_\infty}$$

Where $R_{ool\infty}$ and T_∞ refer to R_{ool} and T evaluated when $R_L=\infty\ \Omega$.

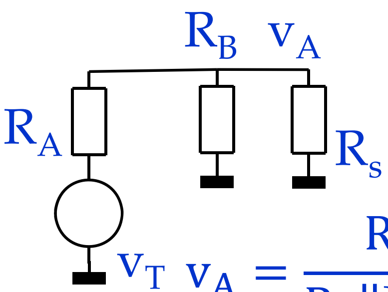
A last consideration :

Let's proceed in a way similar to the consideration done with regard to the input impedance.
We have seen that , in open loop condition, it results:

$$R_{ool} = R_L \| R_o \| R_{i\beta A}$$

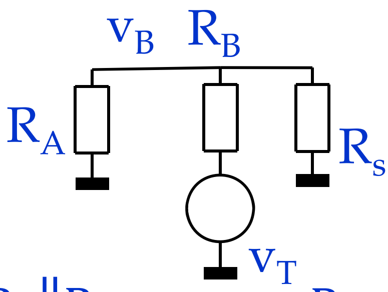


Now, let's study the voltage divider below:



$$R_T = R_A \| R_B \| R_s$$

$$v_A = \frac{R_s \| R_B}{R_s \| R_B + R_A} v_T = \frac{R_T}{R_A} v_T$$

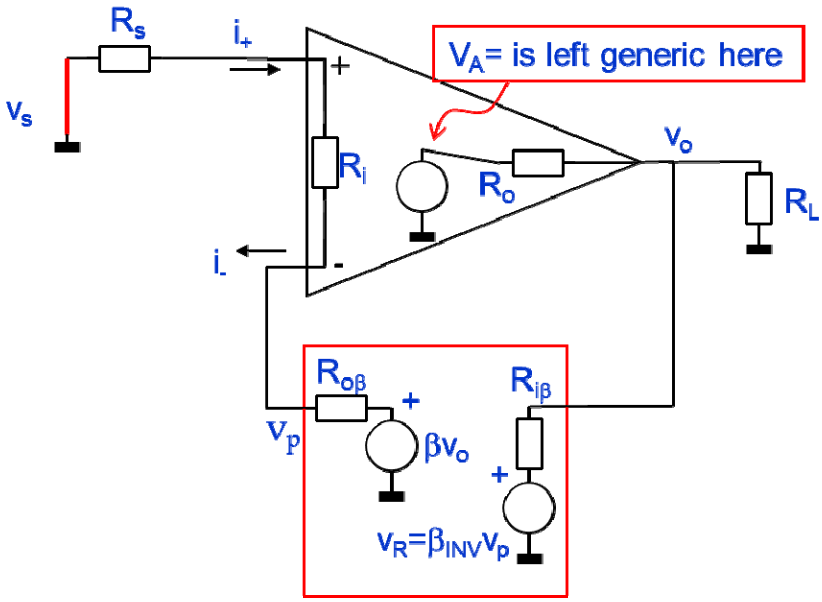


$$v_B = \frac{R_s \| R_A}{R_s \| R_A + R_B} v_T = \frac{R_T}{R_B} v_T$$

We see that (a), irrespectively of where we apply the excitation, the node voltage to which we are interested is proportional to the parallel combination of the resistors, R_T.
(b) The denominator of the voltage drop of interest is proportional only to the resistor to which the source is connected.
This proves that:

$$\frac{v_x}{R_T} = \frac{v_T}{R_x} \quad \text{or} \quad \frac{v_x}{R_T} \quad \text{is dependent only on } R_x$$

Output impedance of a v-to-v fed backed OA 16



As already verified, the loop gain is a cascade of voltage divider and we can apply the last result at the output mesh.
The loop gain T is proportional to the voltage partition of the 3 resistors in parallel R_o, R_L and $R_{i\beta A}$, From which we can therefore write:

$$\frac{T}{R_{ool}} = \text{depend only on the resistor to which } v_A \text{ is connected, } R_o$$

And:

$$\frac{T}{R_{ool}} = \frac{T_\infty}{R_{ool\infty}}$$

Where $R_{ool\infty}$ and T_∞ are R_{ool} and T evaluated when $R_L \rightarrow \infty$

Let's elaborate:

$$\frac{1}{R_{of}} = \frac{1 - T}{R_{ool}}$$

$$= \frac{1}{R_{ool}} - \frac{T}{R_{ool}}$$

$$= \frac{1}{R_{ool}} - \frac{T_{\infty}}{R_{ool\infty}}$$

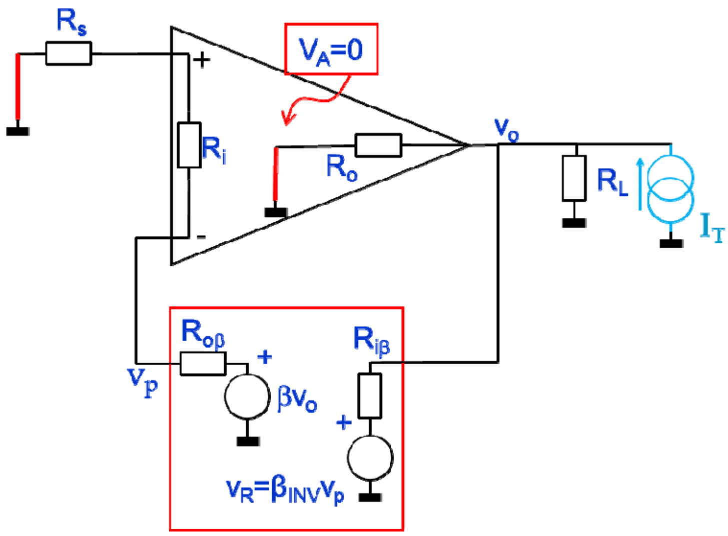
$$= \frac{1}{R_L} + \frac{1}{R_{ool\infty}} - \frac{T_{\infty}}{R_{ool\infty}}$$

$$= \frac{1}{R_L} + \frac{1}{R_{of\infty}}$$

Namely:

$$R_{of} = R_L \| R_{of\infty}$$

We can verify in our practical situation, starting from our loop gain:



$$T=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{R_L\parallel R_{i\beta A}}{R_L\parallel R_{i\beta A}+R_o}$$

And:

$$R_{ool}=R_L\parallel R_o\parallel R_{i\beta A}$$

$$\frac{1}{R_{ool}}=\frac{1}{R_L}+\frac{1}{R_o}+\frac{1}{R_{i\beta A}}$$
$$=\frac{1}{R_L\parallel R_{i\beta A}}+\frac{1}{R_o}$$
$$=\frac{R_L\parallel R_{i\beta A}+R_o}{R_L\parallel R_{i\beta A}\,R_o}$$

$$\frac{T}{R_{ool}}=-\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\frac{1}{R_o}$$ (independent from R_L)

$$\frac{1}{R_{of}}=\frac{1}{R_L}+\frac{1}{R_{i\beta A}}+\frac{1}{R_o}\left(1+\frac{A\beta R_i}{R_i+R_S+R_{o\beta}}\right)$$

Input/Output impedance of a v-to-v fed backed OA

Summary about the input/output impedance of a v-to-v.

Input impedance measured with a voltage source with a series resistance R_s in series:

$$R_{ifs} = R_{iol}(1 - T)$$

$$R_{ifs} = R_s + R_{iol0}(1 - T_o) = R_s + R_{ifs0}$$

Where R_{iolo} and T_o are R_{iol} and T measured when R_s is set to 0 Ω.

If the input impedance is measured using a current generator with in parallel a source resistor R_s we have:

$$R_{ifp} = R_s \| R_{ifso}$$

oooOOooo

Output impedance measured with a current source with in parallel the load resistor R_L:

$$R_{ofp} = \frac{R_{ol}}{1 - T}$$

$$R_{ofp} = R_L \left\| \frac{R_{ol\infty}}{1 - T_\infty} \right. = R_L \| R_{ofp\infty}$$

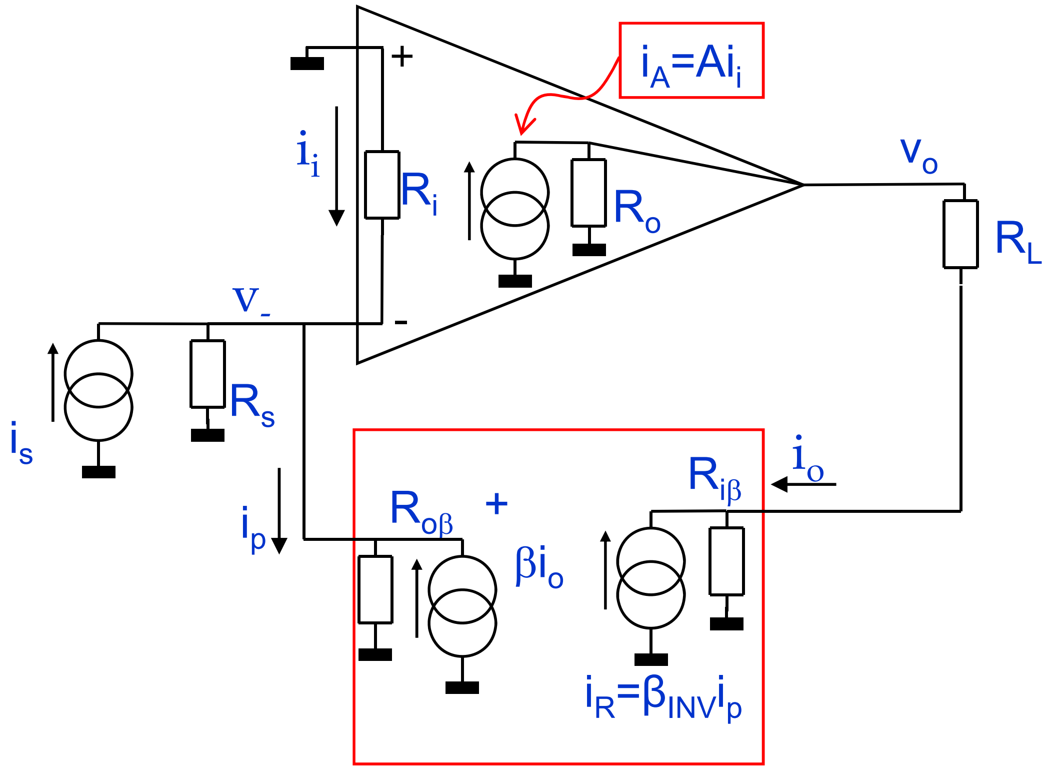
Where $R_{ool\infty}$ and T_∞ are R_{ool} and T measured when R_L is set to ∞ Ω.

If the output impedance is measured using a voltage generator in series to the load R_L we have:

$$R_{ofs} = R_L + R_{ofp\infty}$$

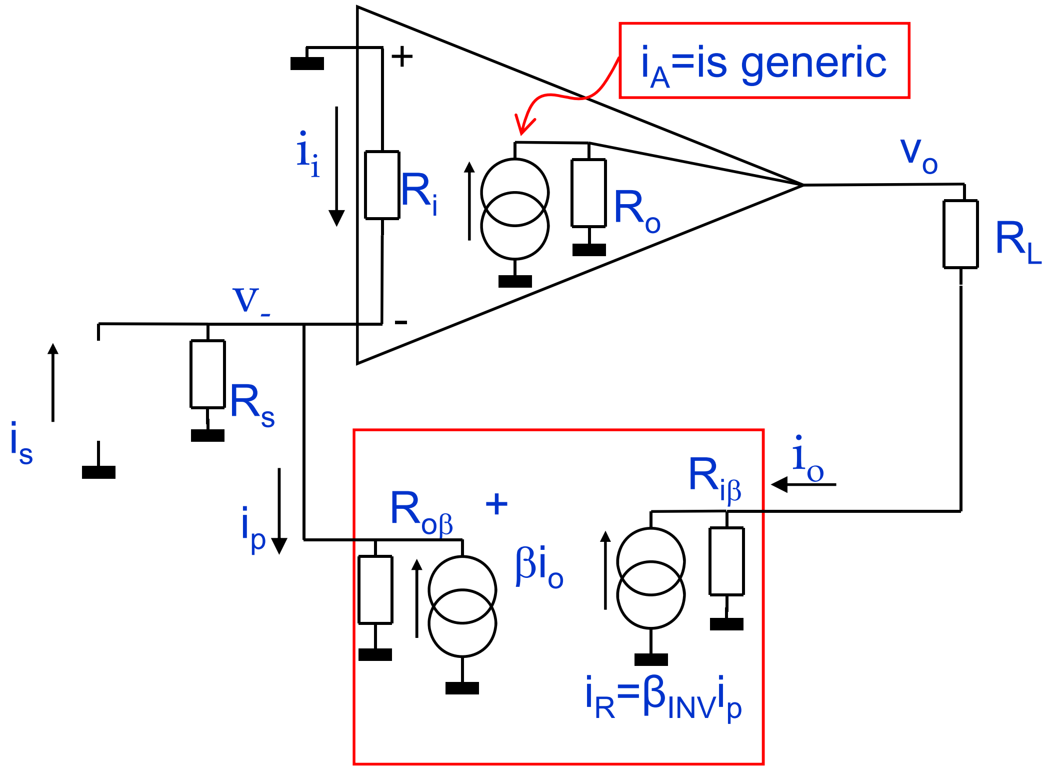
Gain of a c-to-c fed backed current OA 1

c-to-c= current input and current output amplifier

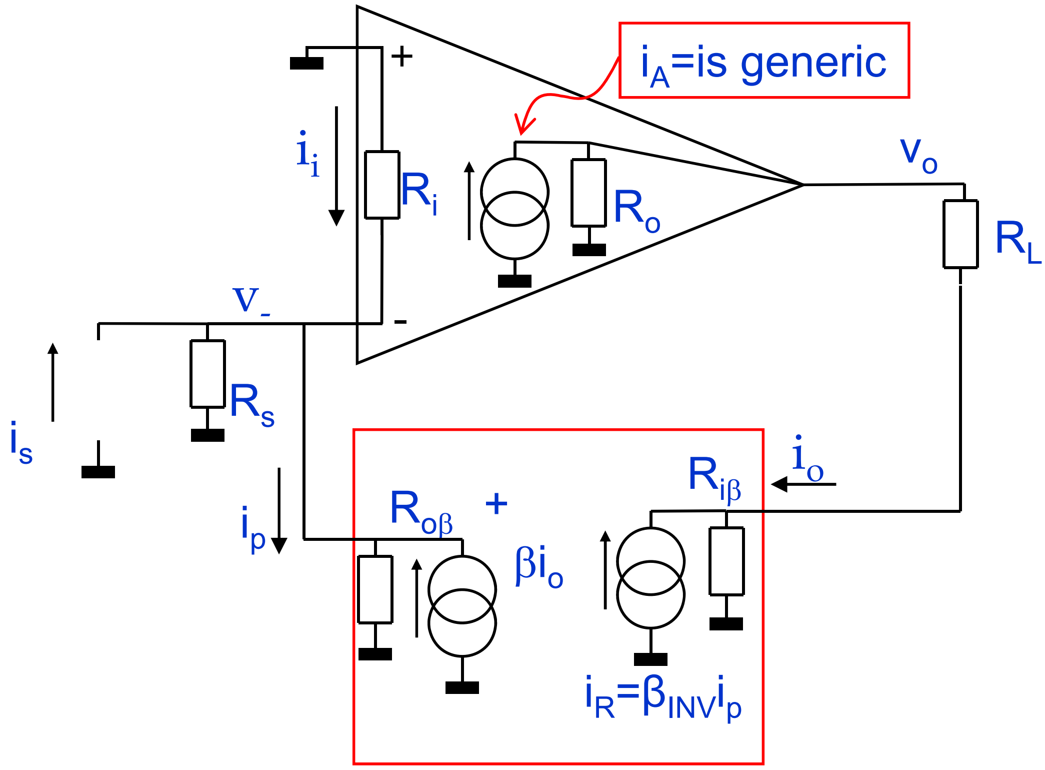


Just for exercise we can evaluate the loop gain and direct transmission.

For loop gain we put i_s=0 A, namely, we open i_s:



Gain of a c-to-c fed backed current OA 2



$$\begin{cases} i_A = \dfrac{v_o}{R_o} + \dfrac{v_o - v_B}{R_L} \\ \dfrac{v_o - v_B}{R_L} + \beta_{INV} i_p = \dfrac{v_B}{R_{i\beta}} \\ i_o = \dfrac{v_o - v_B}{R_L} \\ i_p = -\dfrac{1/R_s + 1/R_i}{1/R_s + 1/R_i + 1/R_{o\beta}} \beta i_o \end{cases}$$

$$\left(i_p = -\frac{R_{o\beta}}{R_{o\beta} + R_s \| R_i} \beta i_o \right)$$

From the second, the third and the last:

$$i_o = \frac{v_B}{R_{i\beta}} - \beta_{INV} i_p$$

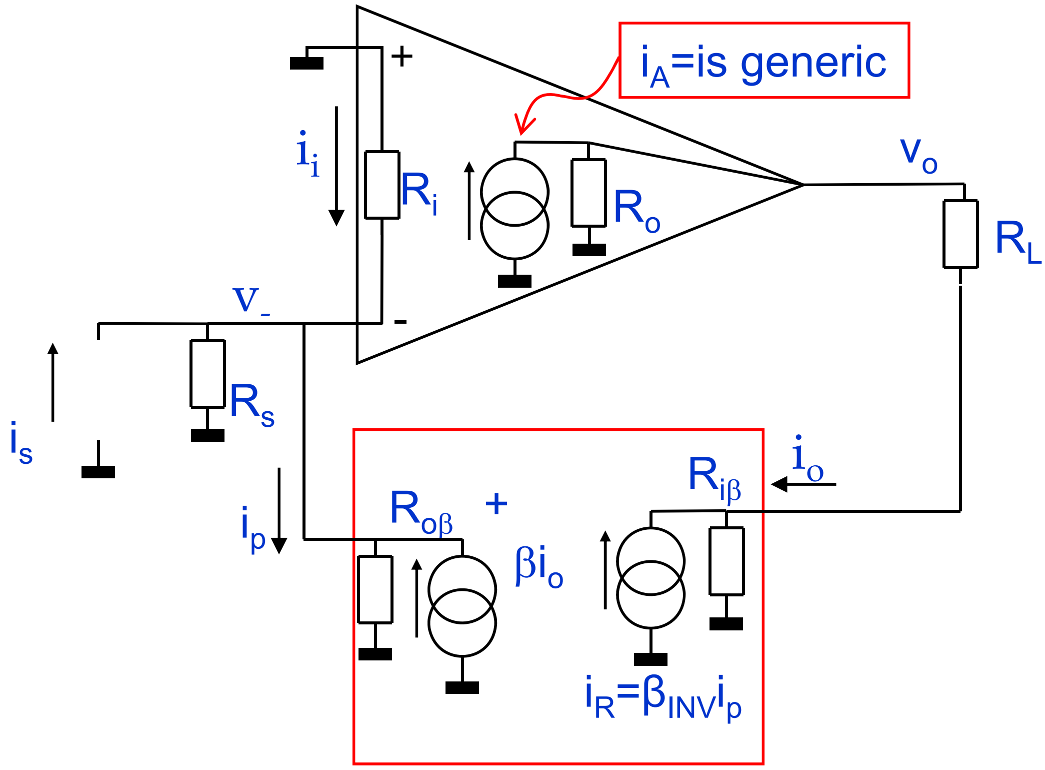
$$i_o - \frac{R_{o\beta}}{R_{o\beta} + R_s \| R_i} \beta i_o \beta_{INV} i_o = \frac{v_B}{R_{i\beta}}$$

$$i_o = \frac{v_B}{R_{i\beta}\left(1 - \dfrac{R_{o\beta}}{R_{o\beta} + R_s \| R_i} \beta i_o \beta_{INV}\right)}$$

$$i_o = \frac{v_B}{R_{i\beta A}}$$

From the third: $v_o = i_o R_L + i_o R_{i\beta A}$ $\quad v_o = (R_L + R_{i\beta A}) i_o$

Gain of a c-to-c fed backed current OA 3



$$\begin{cases} i_A = \dfrac{v_o}{R_o} + \dfrac{v_o - v_B}{R_L} \\ \dfrac{v_o - v_B}{R_L} + \beta_{INV} i_p = \dfrac{v_B}{R_{i\beta}} \\ i_o = \dfrac{v_o - v_B}{R_L} \\ i_p = -\dfrac{R_{o\beta}}{R_{o\beta} + R_s \| R_i} \beta i_o \end{cases}$$

$$v_o = \left(R_L + R_{i\beta A}\right) i_o$$

Finally the first:

$$i_A = \frac{R_L + R_{i\beta A}}{R_o} i_o + i_o$$

$$i_o = \frac{R_o}{R_L + R_o + R_{i\beta A}} i_A$$

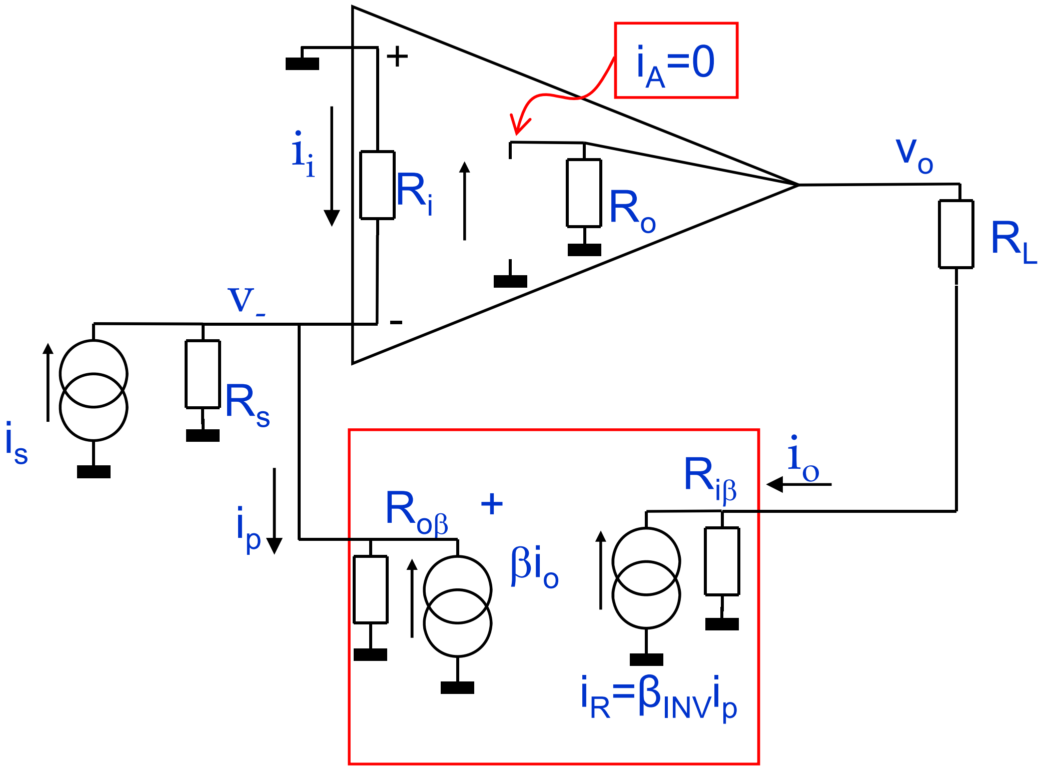
At the input:

$$i_i = \frac{1/R_i}{1/R_i + 1/R_s} i_p = -\frac{R_s \| R_{o\beta}}{R_s \| R_{o\beta} + R_i} \beta i_o$$

To obtain:

$$T = A\frac{i_i}{i_A} = -\frac{A\beta R_s \| R_{o\beta}}{R_s \| R_{o\beta} + R_i} \frac{R_o}{R_L + R_o + R_{i\beta A}}$$

Gain of a c-to-c fed backed current OA 4



Now the direct gain:

$$\begin{cases} i_s = \dfrac{v_-}{R_s} + \dfrac{v_-}{R_i} + i_p \\[2ex] i_p = \dfrac{v_-}{R_{o\beta}} - \beta i_o \\[2ex] i_o = -\dfrac{R_{i\beta}}{R_{i\beta} + R_L + R_o} \beta_{INV} i_p \end{cases}$$

From the last 2 lines:

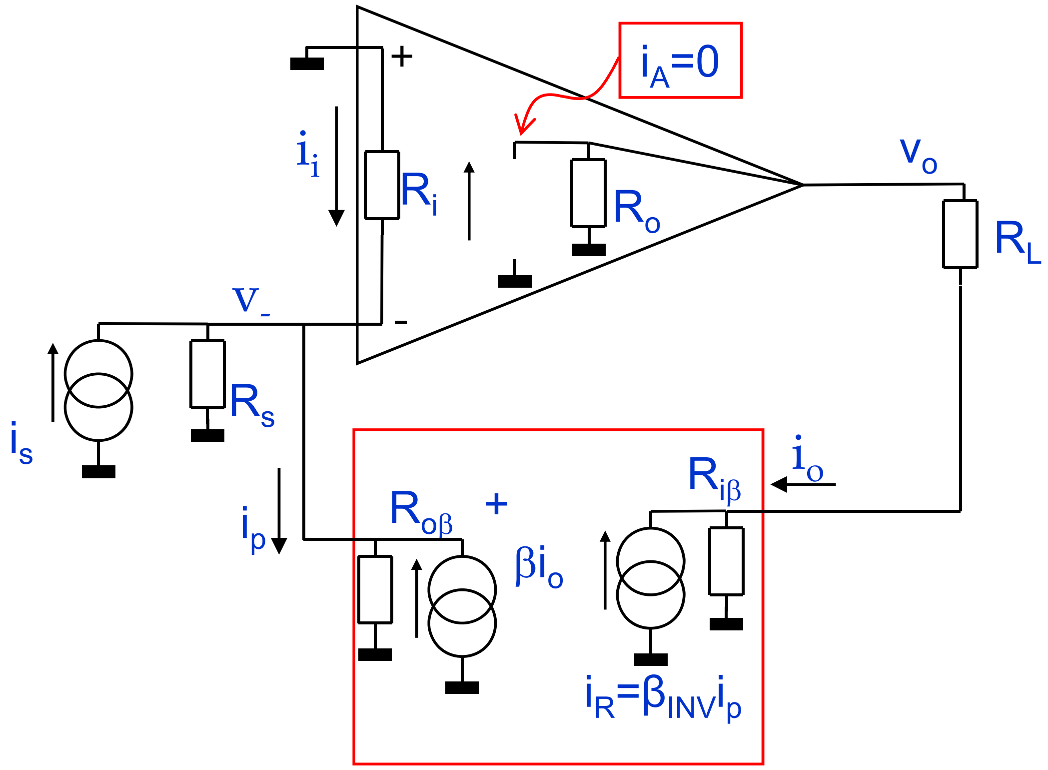
$$i_p = \frac{v_-}{R_{o\beta}} + \frac{R_{i\beta}}{R_{i\beta} + R_L + R_o} \beta\beta_{INV} i_p$$

$$i_p = \frac{v_-}{R_{o\beta}} \frac{1}{1 - \dfrac{R_{i\beta}}{R_{i\beta} + R_L + R_o} \beta\beta_{INV}}$$

$$i_p = \frac{v_-}{R_{o\beta A}}$$

$$\left(R_{o\beta A} = R_{o\beta} \left(1 - \frac{R_{i\beta}}{R_{i\beta} + R_L + R_o} \beta\beta_{INV} \right) \right)$$

Gain of a c-to-c fed backed current OA 5



Now the direct gain:

$$\begin{cases} i_s = \dfrac{v_-}{R_s} + \dfrac{v_-}{R_i} + i_p \\ i_p = \dfrac{v_-}{R_{o\beta}} - \beta i_o \\ i_o = -\dfrac{R_{i\beta}}{R_{i\beta} + R_L + R_o} \beta_{INV} i_p \end{cases}$$

From the last 2 lines:

$$i_p = \frac{v_-}{R_{o\beta A}}$$

$$i_s = \frac{v_-}{R_s \| R_i} + i_p$$

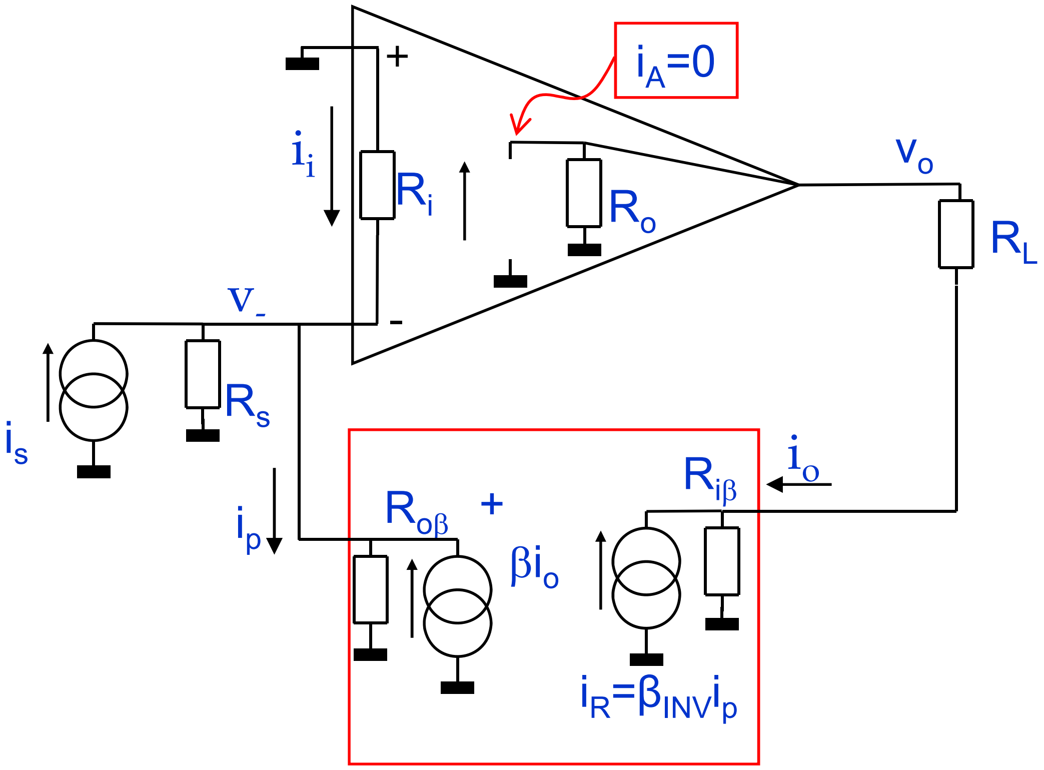
$$i_s = \left(\frac{R_{o\beta A}}{R_s \| R_i} + 1 \right) i_p$$

$$i_p = \frac{R_s \| R_i}{R_s \| R_i + R_{o\beta A}} i_s$$

Finally:

$$i_o = -\frac{R_{i\beta}}{R_{i\beta} + R_L + R_o} \frac{R_s \| R_i}{R_s \| R_i + R_{o\beta A}} \beta_{INV} i_s$$

Gain of a c-to-c fed backed current OA 6



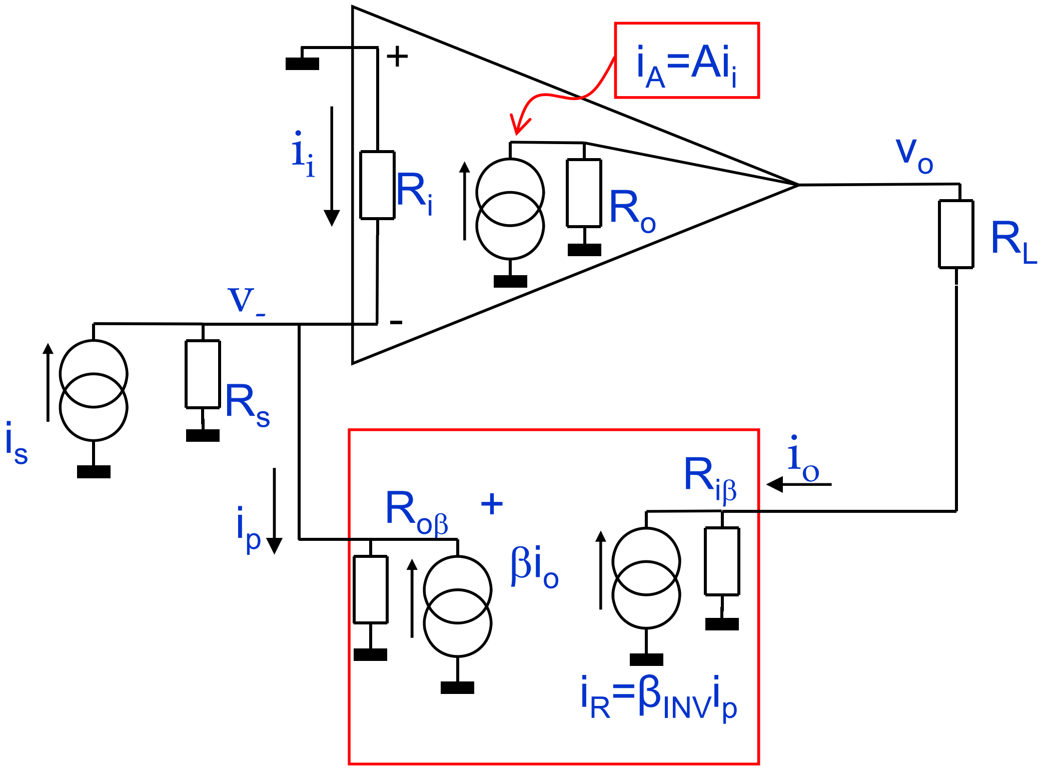
And from the last result, repeated here:

$$i_o = -\frac{R_{i\beta}}{R_{i\beta}+R_L+R_o}\frac{R_s\|R_i}{R_s\|R_i+R_{o\beta A}}\beta_{INV}i_s$$

More compact:

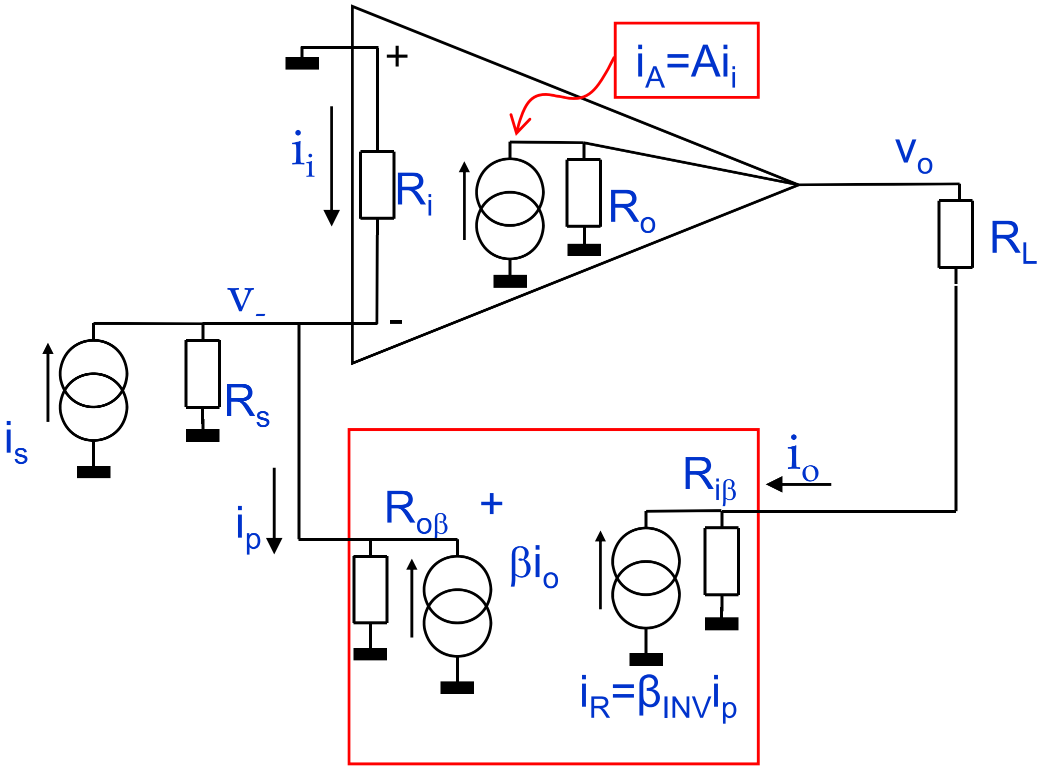
$$i_o = \frac{R_{i\beta}}{R_{i\beta}+R_L+R_o}\beta_{INVA}i_s$$

Gain of a c-to-c fed backed current OA 7



So we can write, for what concerns the direct gain:

$$i_o = \frac{R_{i\beta}}{R_{i\beta} + R_L + R_o}\frac{\beta_{INVA}}{1 - T} i_s = \frac{A_R}{1 - T} i_s$$



The procedure we have adopted for measuring the input/output impedances remains valid for all the other configurations as we will show.
We consider now a current input to current output feedback amplifier when the OA is a current amplifier.
Our input impedance is given by:

$$R_{if} = \frac{v_-}{i_s}$$

where, from the scheme above:

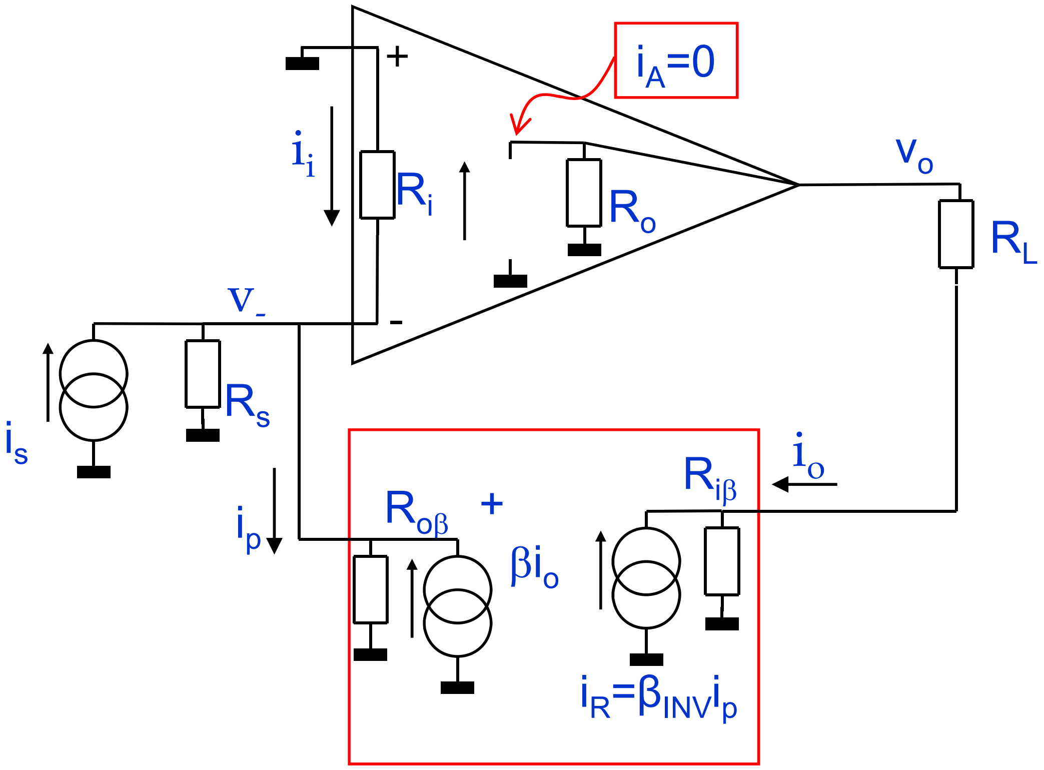
$$v_- = -R_i i_i$$

namely:

$$v_- = -R_i i_i = -\,R_i \frac{i_A}{A}$$

and we need to elaborate i_A.

Input impedance of a c-to-c fed backed current OA 2



Similarly to the previous case:

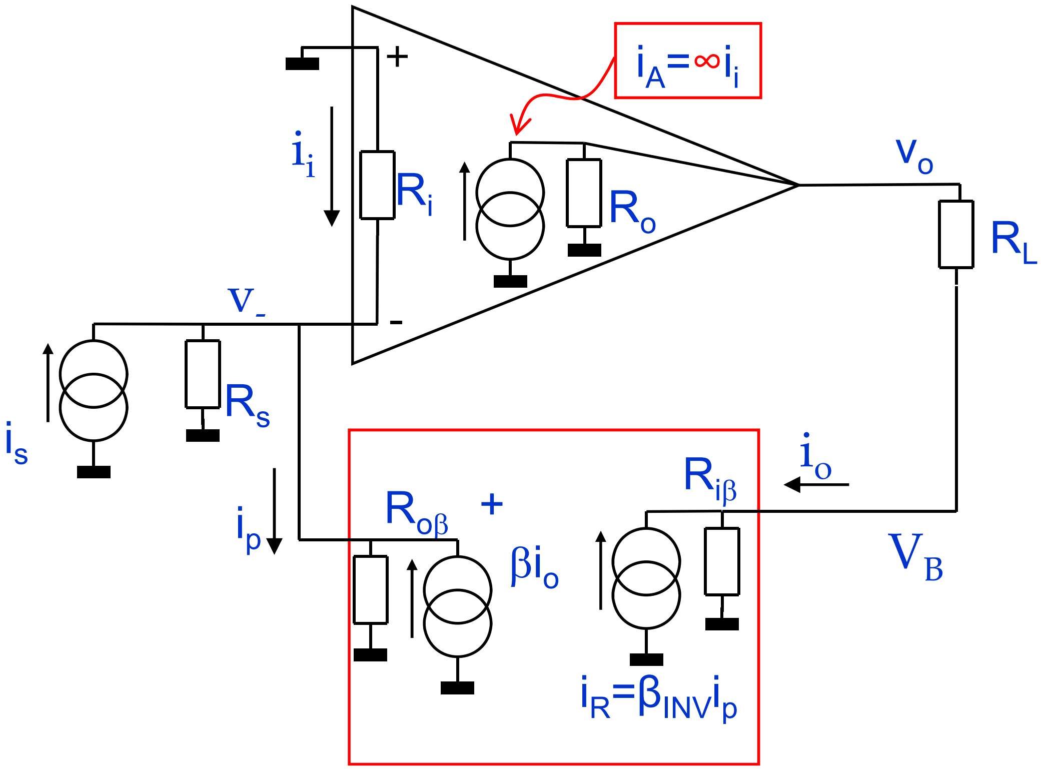
$$i_A = \frac{1}{\beta'}\frac{-T}{1-T}i_s + \frac{A_{R'}}{1-T}i_s$$

from the scheme above we see that the current i_R cannot change the source i_A and, as a consequence, $A_{R'}$=0:

$$i_A = \frac{1}{\beta'}\frac{-T}{1-T}i_s$$

Let's introduce the steps for β' evaluation

Input impedance of a c-to-c fed backed current OA 3



Let's assume A=∞ and determine the current in the amplifier generator i_A. We know that v_- is close to zero, since v_+=0 V:

$$i_s = -\beta i_o$$

At the output mesh:

$$i_o = \frac{v_B}{R_{i\beta}} - \beta_{INV} i_p$$

Where:

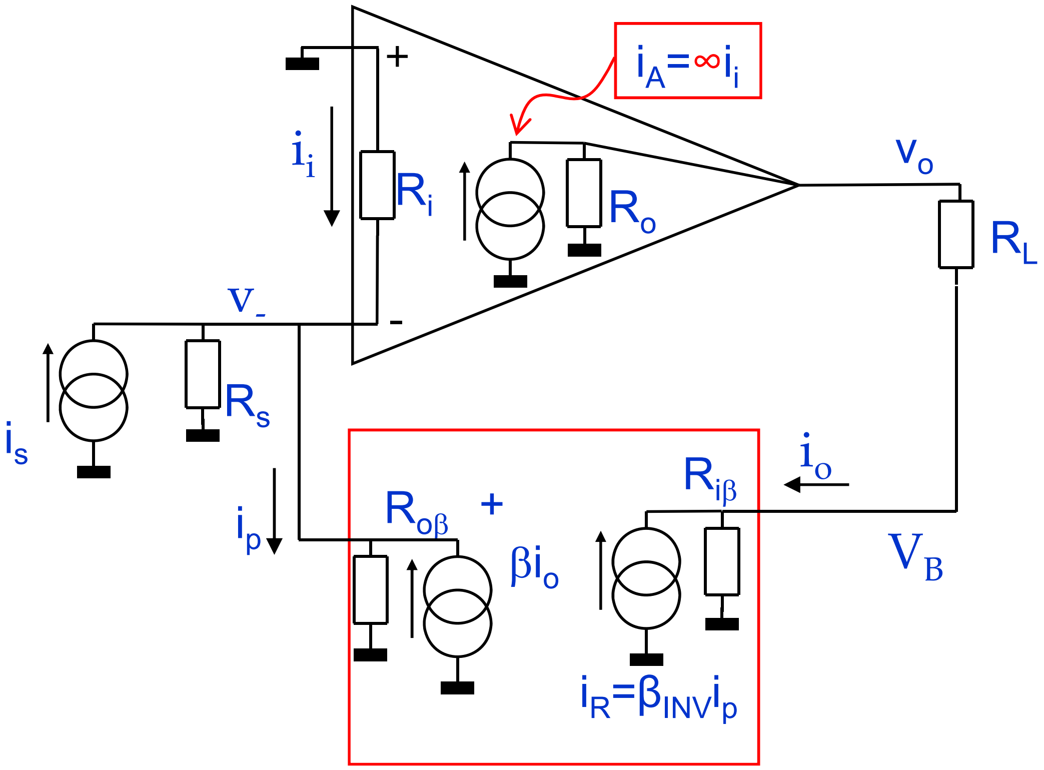
$$i_s = i_p$$

Then:

$$v_B = R_{i\beta}\left(i_o + \beta_{INV} i_p\right) = R_{i\beta}\left(-\frac{1}{\beta} i_s + \beta_{INV} i_s\right)$$

$$v_B = -R_{i\beta}(1 - \beta\beta_{INV})\frac{i_s}{\beta}$$

Input impedance of a c-to-c fed backed current OA 4



$$v_B = -R_{i\beta}(1-\beta\beta_{INV})\frac{i_s}{\beta}$$

Now:

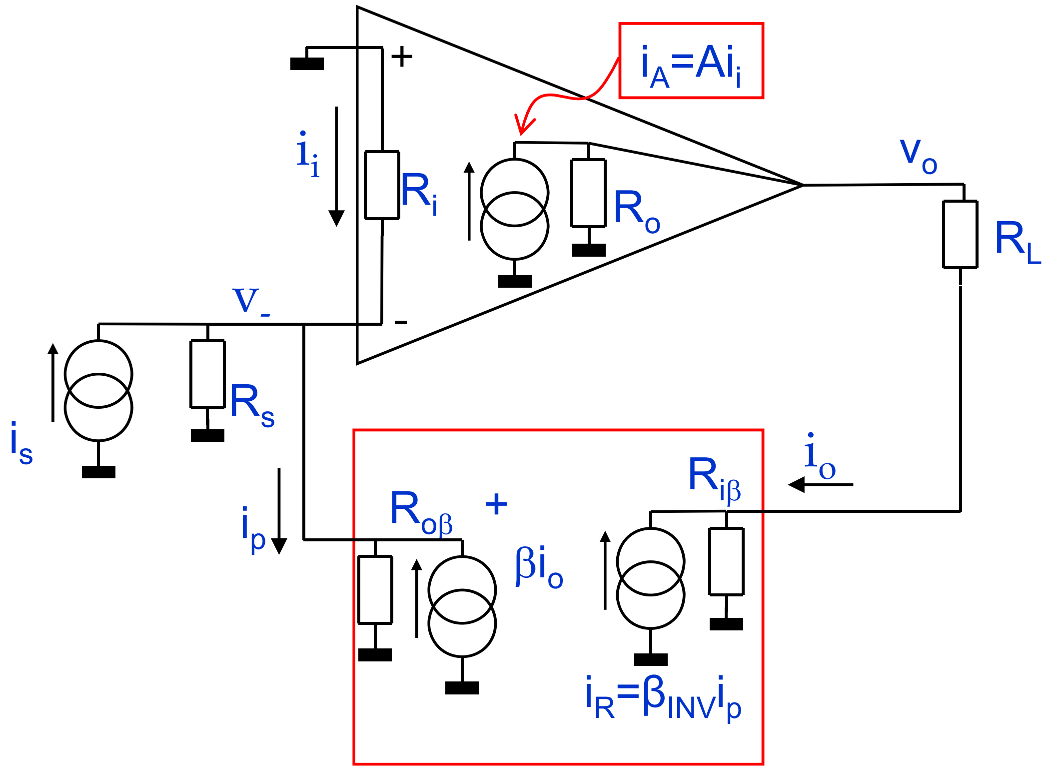
$$v_o = v_B + i_0 R_L = v_B - \frac{i_s R_L}{\beta} = -\frac{i_s}{\beta}\left(R_{i\beta}(1-\beta\beta_{INV}) + R_L\right)$$

Therefore:

$$i_A = \frac{v_o}{R_o} + i_0 = -\frac{i_s}{\beta}\left(\frac{R_{i\beta}(1-\beta\beta_{INV}) + R_L}{R_o} + 1\right)$$

Namely:

$$\beta' = \beta\left(\frac{R_{i\beta}(1-\beta\beta_{INV}) + R_L}{R_o} + 1\right)^{-1}$$



$$i_i = \frac{f(\beta')}{1-T} i_s$$

And:

$$R_{if} = \frac{v_-}{i_s} = \frac{-R_i i_i}{i_s} = R_i \frac{f(\beta')}{1-T}$$

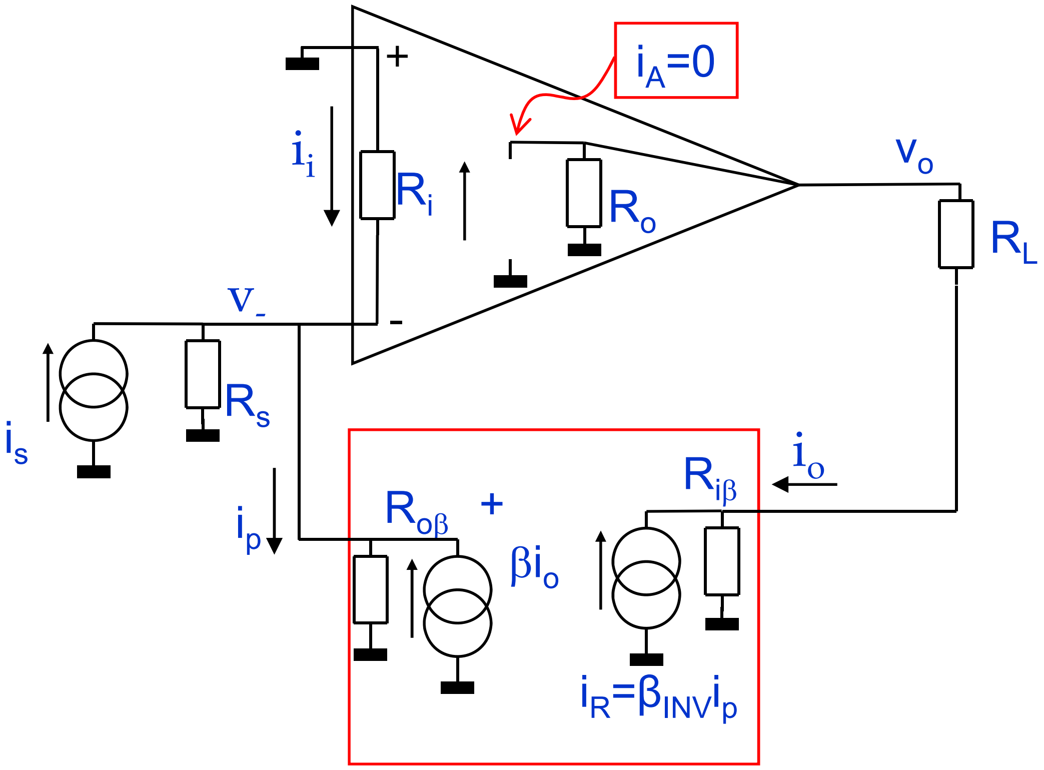
We see that this feedback, which implements in this case a current amplifier, reduces the input impedance measured at zero gain, or open loop, $R_i f(\beta')$, by the factor 1-T, decreasing the input impedance further by a large amount.

Then, the input impedance is:

$$R_{if} \triangleq \frac{R_{iol}}{1-T}$$

Being R_{iol} the impedance evaluated once the gain is set to 0.

Input impedance of a c-to-c fed backed current OA 6



Let determine R_{iol}:

$$\begin{cases} i_s + \beta i_o = \dfrac{v_-}{R_s} + \dfrac{v_-}{R_i} + \dfrac{v_-}{R_{o\beta}} \\ i_p + \beta i_o = \dfrac{v_-}{R_{o\beta}} \\ i_o = -\dfrac{R_{i\beta}}{R_{i\beta} + R_L + R_i}\beta_{INV} i_p \end{cases}$$

From which we know that:

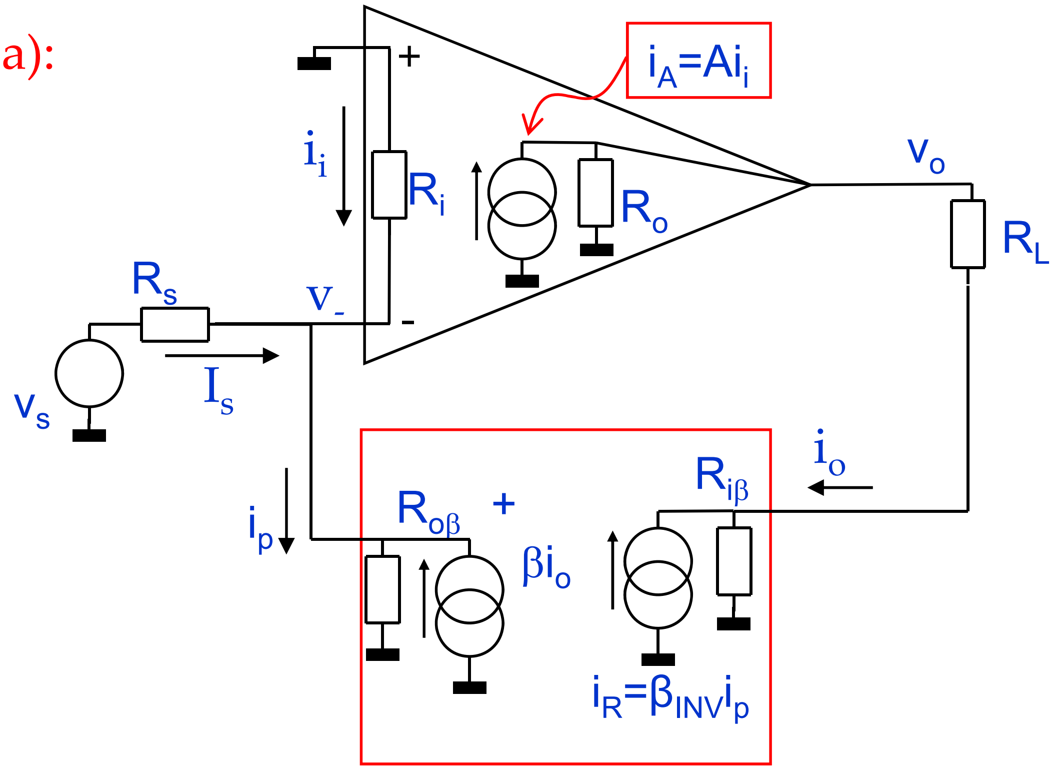
$$i_s = \frac{v_-}{R_s} + \frac{v_-}{R_i} + \frac{v_-}{R_{o\beta A}}$$

$$\left(R_{o\beta A} = R_{o\beta}\left(1 - \frac{R_{i\beta}}{R_{i\beta} + R_L + R_o}\beta\beta_{INV}\right)\right)$$

Therefore:

$$R_{iol} = R_s \,\|\, R_i \,\|\, R_{o\beta A}$$

A consideration (a):



As with the v-to-v configuration we see here that is not convenient to adopt a voltage source at the input to measure the input impedance.

If we short v_s to evaluate the loop gain we find:

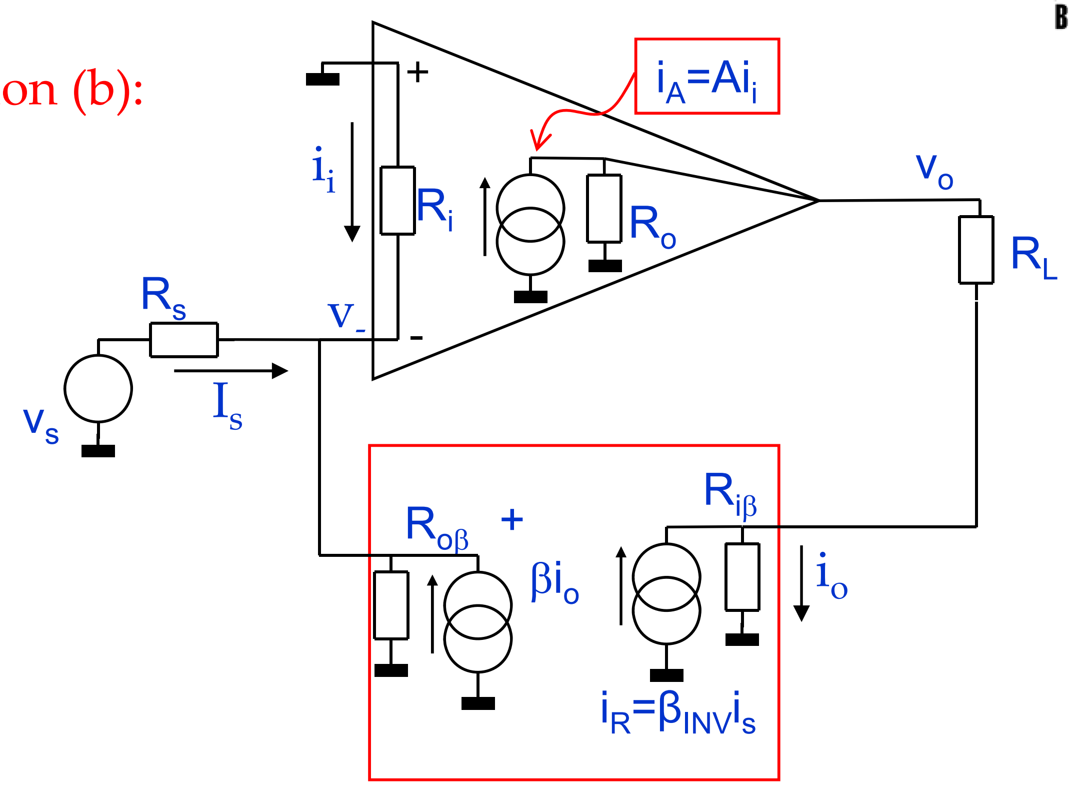
$$T = A\frac{i_i}{i_A} = -\frac{A\beta\left(R_s \| R_{o\beta}\right)}{\left(R_s \| R_{o\beta}\right) + R_i}\frac{R_o}{R_L + R_o + R_{R_{i\beta A}}}$$

We see that $T \rightarrow 0$ if $R_s \rightarrow 0$, and this suggest to use a current source.

Insisting anyway in using a voltage source we obtain (Norton equivalent):

$$i_o = -\frac{1}{\beta}\frac{-T}{1-T}i_s + \frac{A_R}{1-T}i_s = -\frac{1}{\beta}\frac{-T}{1-T}\frac{v_s}{R_s} + \frac{A_R}{1-T}\frac{v_s}{R_s}$$

A consideration (b):



We can anyway use this configuration to measure the input impedance once that we consider that:

$$i_s = \frac{v_s - v_-}{R_s} = \frac{v_s}{R_s} - \frac{1}{R_s} R_i \frac{f(\beta')}{1-T} i_s$$

$$\frac{i_s}{R_s}\left(R_s + R_i \frac{f(\beta')}{1-T}\right) = \frac{v_s}{R_s}$$

That is to say that:

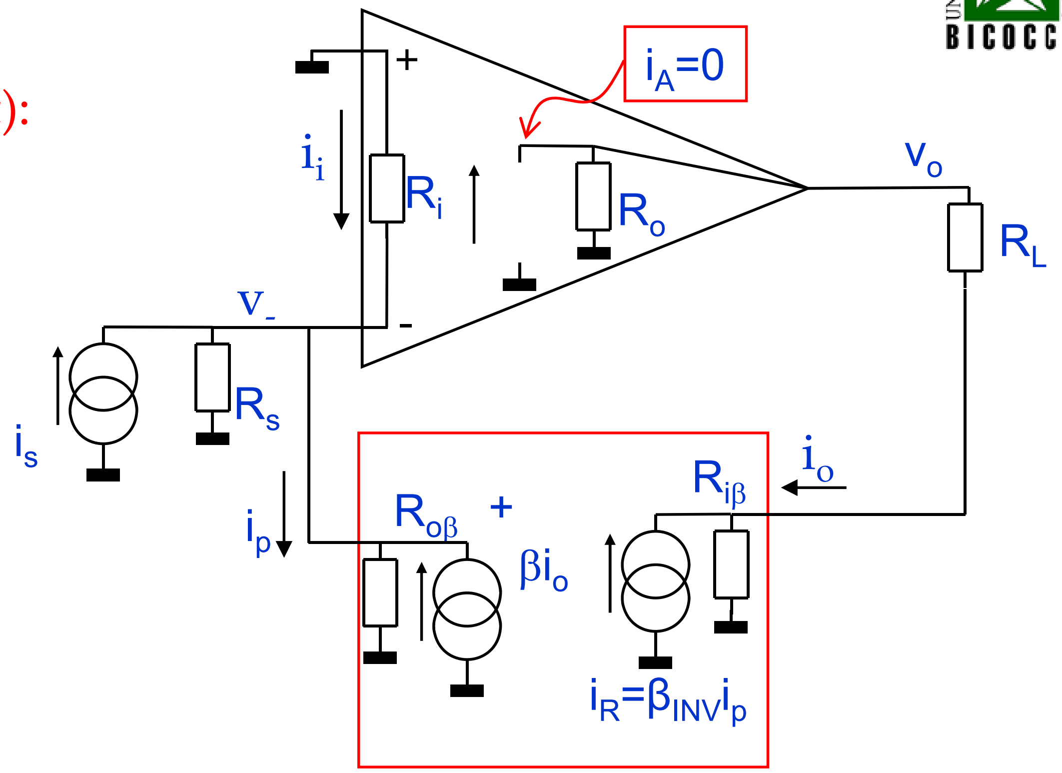
$$R_{ifs} = \frac{v_s}{i_s} = R_s + R_{if\infty}$$

Where, of course, $R_{if\infty}$ is the input impedance measured with the test current source and $R_s \rightarrow \infty$.

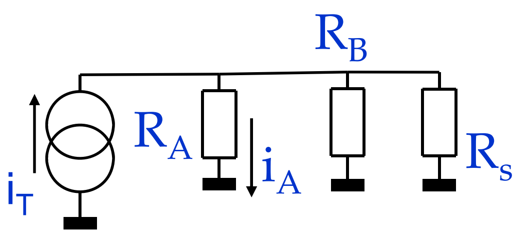
Input impedance of a c-to-c fed backed current OA 9

A consideration (c):

Likewise with the voltage amplifier we would disentangle the input impedance of the fed – backed OA from R_s.



We start from the following consideration about a current partition:



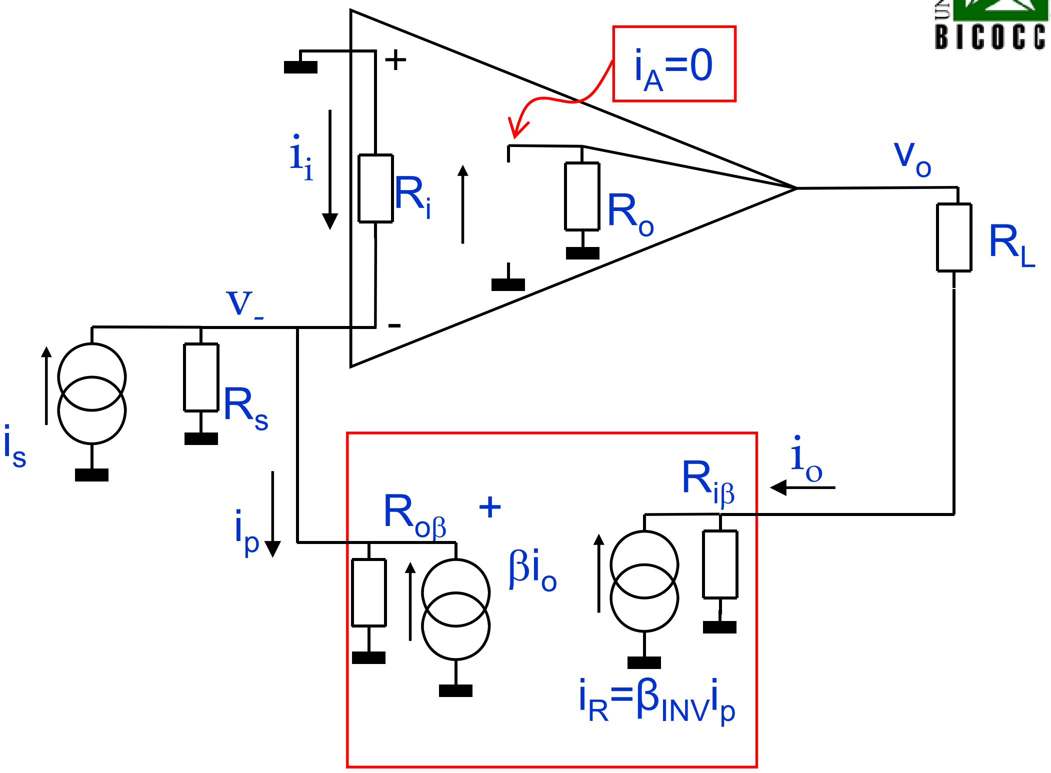
$$i_A = \frac{R_B \| R_s}{R_A + R_B \| R_s} i_T = \frac{R_P}{R_A} i_T$$

$$R_P = R_B \| R_s \| R_A$$

In general: $$i_x = \frac{R_P}{R_x} i_T$$

and: $\dfrac{i_x}{R_P} = \dfrac{i_T}{R_x}$ or $\dfrac{i_x}{R_P}$ Depends only on the current that flows in R_x.

Input impedance of a c-to-c fed backed current OA 10



The open loop input impedance resulted:

$$R_{iol} = R_s \,\|\, R_i \,\|\, R_{o\beta A}$$

At the input mesh the loop gain T is proportional to the current thorough R_i.

At the input mesh T is proportional to the current through R_i, therefore we can write:

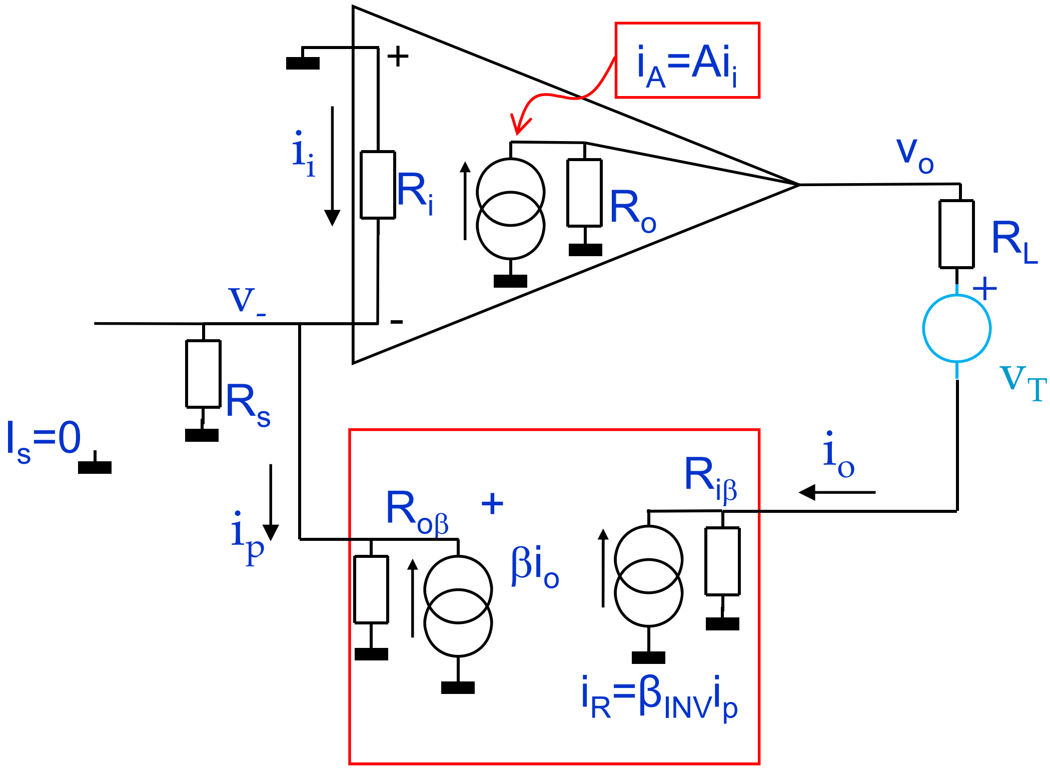
$$\frac{T}{R_{iol}} = \frac{T_\infty}{R_{iol\infty}}$$

(here $R_{iol\infty}$ is R_{iol} evaluated when $R_s \to \infty$.)

and:

$$\frac{1}{R_{if}} = \frac{1-T}{R_{iol}} = \frac{1}{R_{iol}} - \frac{T}{R_{iol}} = \frac{1}{R_s} + \frac{1}{R_{iol\infty}} - \frac{T_\infty}{R_{iol\infty}} = \frac{1}{R_s} + \frac{1-T_\infty}{R_{iol\infty}} = \frac{1}{R_s} + \frac{1}{R_{if\infty}}$$

Output impedance of a c-to-c fed backed current OA 1



By applying v_T we expect:

$$i_o = \frac{1}{\beta'}\frac{-T}{1-T}v_T + \frac{A_{R'}}{1-T}v_T$$

But, at the input mesh, with A=∞ is i_i=0, hence v_-=0:

$$\beta i_o = 0$$

And we reduces to:

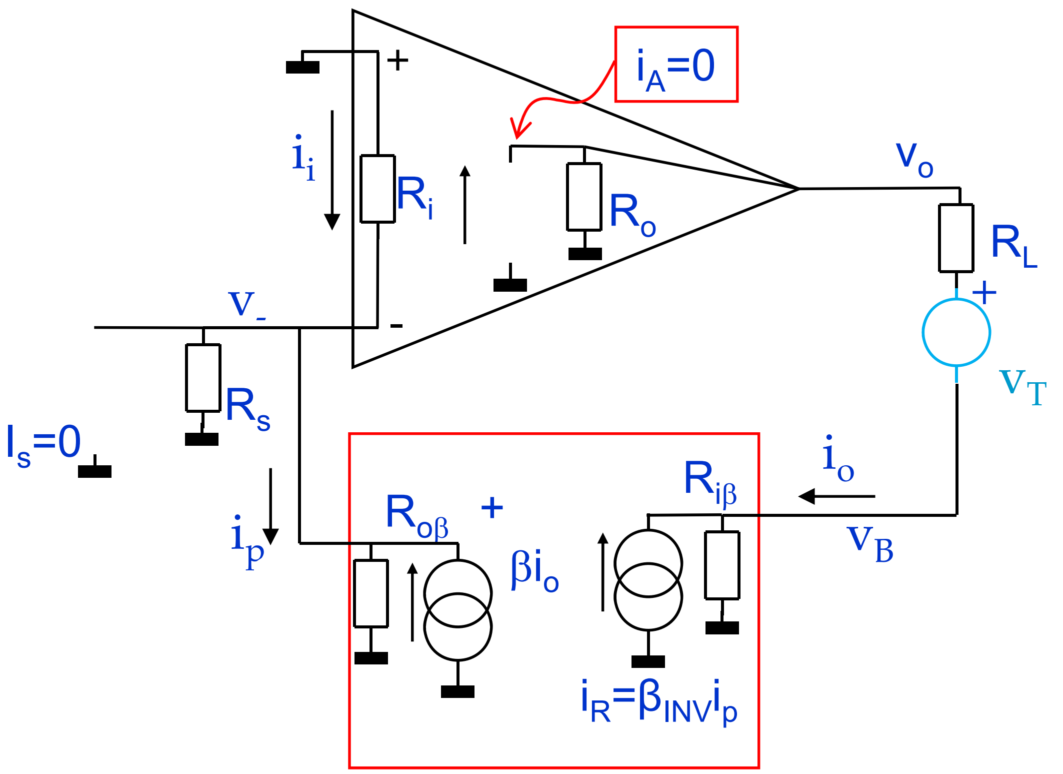
$$i_o = \frac{A_{R'}}{1-T}v_T$$

Or (take care of the direction chosen for the current):

$$R_{ofs} = \frac{v_T}{-i_o} = \frac{1-T}{-A_{R'}}$$

… now let's determine $A_{R'}$…

Output impedance of a c-to-c fed backed current OA 2



$$\begin{cases} i_o + \beta_{INV} i_p = \dfrac{v_B}{R_{i\beta}} \\[2ex] i_o = -\dfrac{v_B + v_T}{R_o + R_L} \\[2ex] i_p = -\dfrac{R_{o\beta}}{R_{o\beta} + R_i + R_s}\beta i_o \end{cases}$$

From the first and the last:

$$i_o - \frac{R_{o\beta}}{R_{o\beta} + R_i + R_s}\beta\beta_{INV} i_o = \frac{v_B}{R_{i\beta}}$$

$$i_o = \frac{v_B}{R_{i\beta}\left(1 - \dfrac{R_{o\beta}}{R_{o\beta} + R_i + R_s}\beta\beta_{INV}\right)}$$

$$i_o = \frac{v_B}{R_{i\beta A}}$$

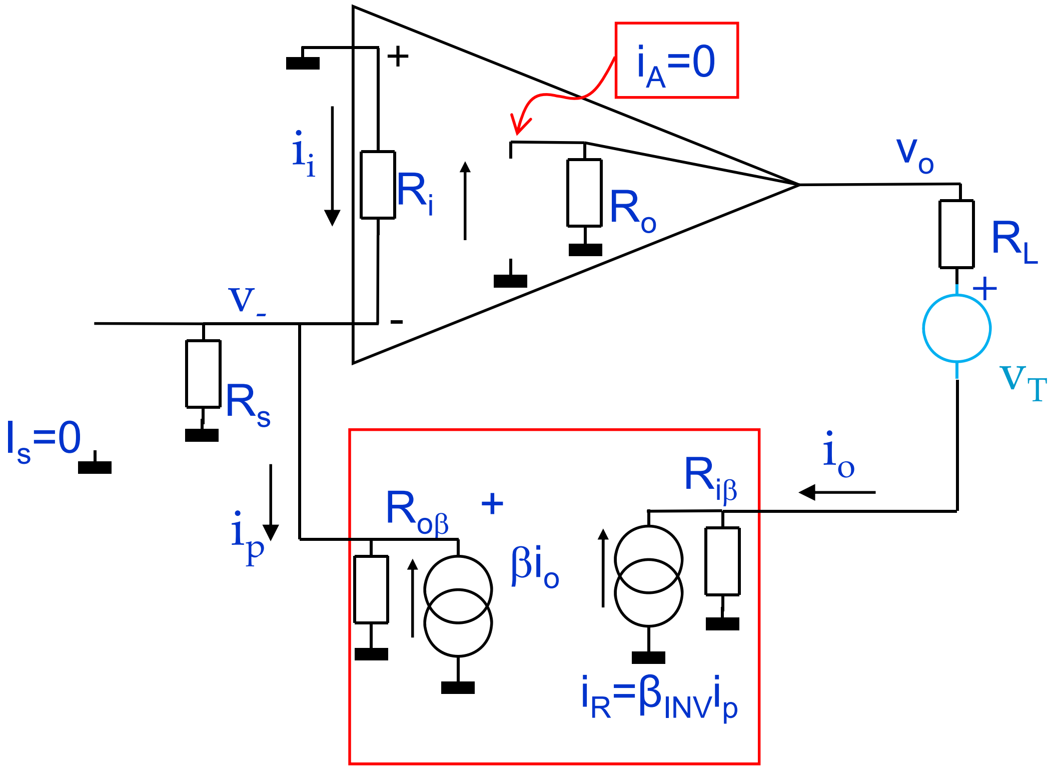
Working with the second:

$$i_o = -\frac{R_{i\beta A} i_o}{R_o + R_L} - \frac{v_T}{R_o + R_L} \quad \Rightarrow \quad i_o = -\frac{v_T}{R_{i\beta A} + R_o + R_L}$$

Therefore:

$$A_{R'} = \frac{i_o}{v_T} - \frac{1}{R_{i\beta A} + R_o + R_L}$$

Output impedance of a c-to-c fed backed current OA 3



$$i_o = -\frac{v_T}{R_{i\beta A} + R_o + R_L}$$

⇨ in closed loop: $$i_o = \frac{A_{R'}}{1-T}v_T = -\frac{1}{R_{i\beta A} + R_o + R_L}\frac{1}{1-T}v_T$$

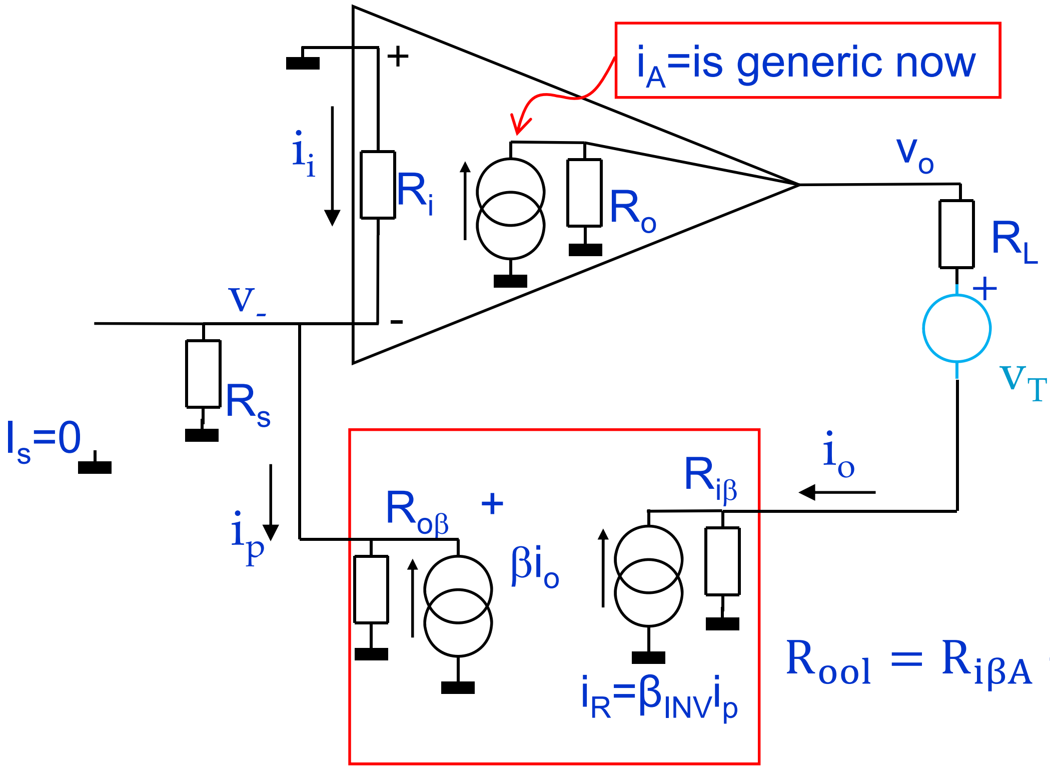
$$R_{ofs} = \frac{1-T}{-A_{R'}} = \frac{v_T}{-i_o} = \left(R_{i\beta A} + R_o + R_L\right)(1-T)$$

Namely:

$$R_{of} \triangleq R_{ool}(1-T) \qquad R_{ool} = \frac{v_T}{-i_o} = R_{i\beta A} + R_o + R_L$$

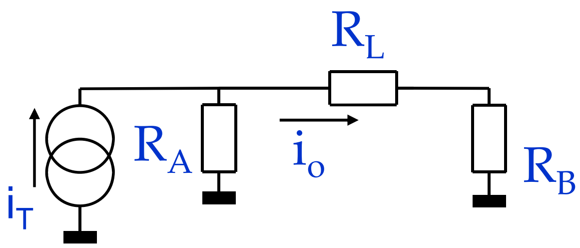
Being R_{ool} the output impedance evaluated once the gain is set to 0.

Output impedance of a c-to-c fed backed current OA 4



$$R_{ool} = R_{i\beta A} + R_o + R_L$$

The output mesh is summarized by the following:



$$i_o = \frac{R_A}{R_A + R_L + R_B} i_T = \frac{R_A}{R_T} i_T$$

$$R_T = R_A + R_L + R_B$$

Namely:

$$R_T i_o = R_A i_T$$

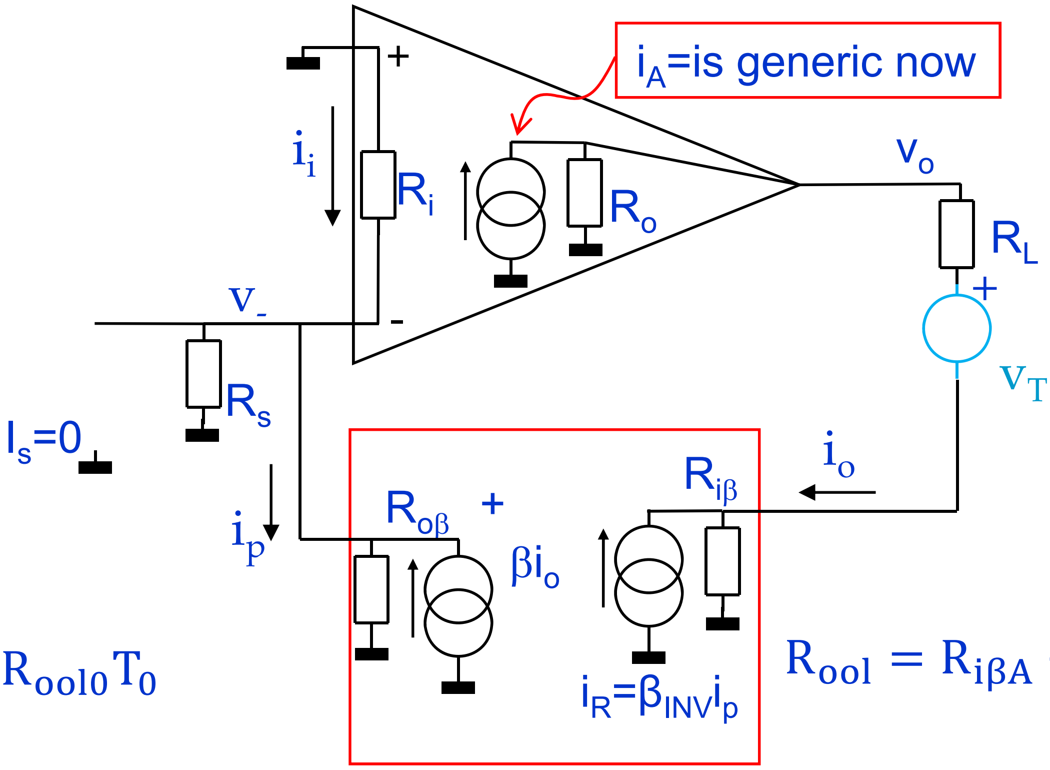
is independent from the resistor in which the measured current flows.

Therefore it can be written that:

$$R_{ool} T = R_{ool0} T_0$$

where R_{ool0} and T_0 are R_{ool} and T when R_L is 0 Ω.

Output impedance of a c-to-c fed backed current OA 5



$R_{ool}T = R_{ool0}T_0$

$R_{ool} = R_{i\beta A} + R_o + R_L$

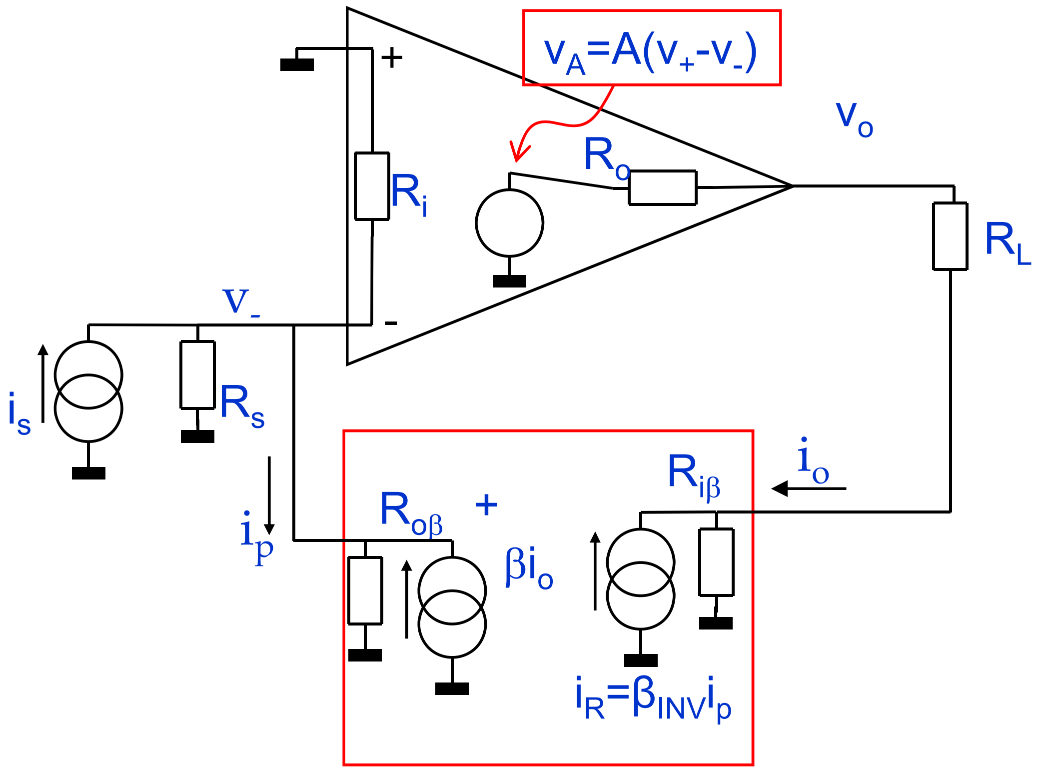
Therefore:

$$
\begin{aligned}
R_{of} &= R_{ool}(1 - T) \\
&= R_{ool} - R_{ool}T \\
&= R_L + R_{ool0} - R_{ool}T \\
&= R_L + R_{ool0} - R_{ool0}T_0 \\
&= R_L + R_{ool0}(1 - T) \\
&= R_L + R_{if0}
\end{aligned}
$$

where R_{ool0} and T_0 are R_{ool} and T when R_L is 0 Ω.

Input impedance of a c-to-c fed backed OA 1

In the previous configuration studied we have used a current OA for implementing a fed backed current amplifier. This choice is the natural, but current OA are not easy to be found off-the-shelf or, if they are available, are expensive. Normally, the fed backed current amplifier is implemented with a standard OA. We will study now also this configuration and put into evidence the differences.



The evaluation procedure is rather similar to that seen before:

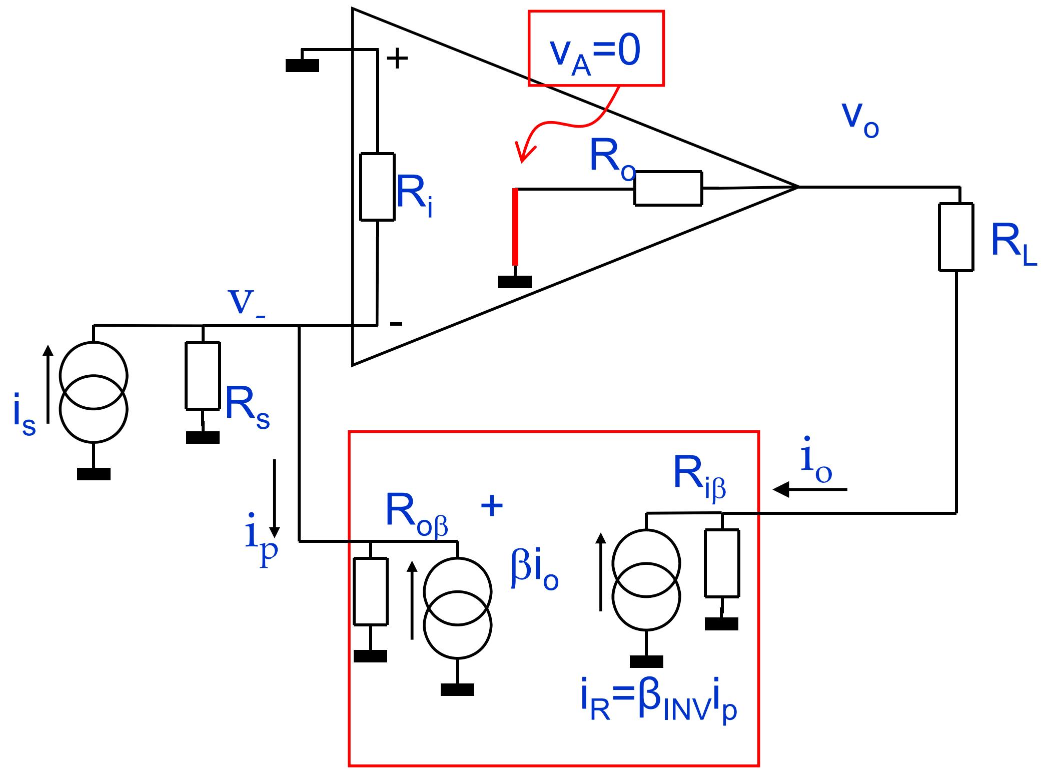
$$R_{if} = \frac{v_-}{i_s}$$

where, from the scheme above:

$$v_- = -\frac{v_A}{A}$$

and we need to elaborate v_A.

Input impedance of a c-to-c fed backed OA 2



Continuing:

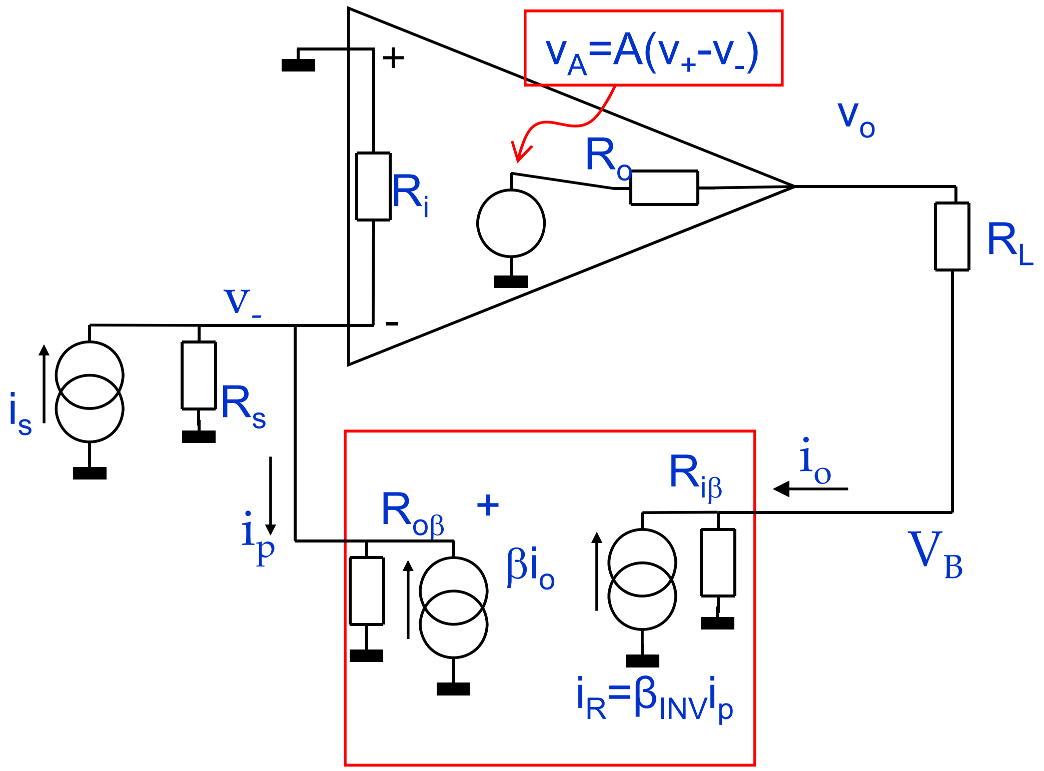
$$v_A = \frac{1}{\beta'}\frac{-T}{1-T}i_s + \frac{A_{R'}}{1-T}i_s$$

from the scheme above we see that the current i_R cannot change the node at v_A when the source is nulled and, as a consequence, $A_{R'}$=0:

$$v_A = \frac{1}{\beta'}\frac{-T}{1-T}i_s \qquad \left[\frac{1}{\beta'}\right] = [\Omega]$$

Let's determine β'…

Input impedance of a c-to-c fed backed OA 3



Let's take A=∞, which gives v_-=0:

$$i_p = i_s \text{ and } i_s + \beta i_o = 0$$

continuing:

$$i_0 + \beta_{INV} i_s = \frac{v_B}{R_{i\beta}} \qquad\qquad v_B = -R_{i\beta}(1 - \beta\beta_{INV})\frac{i_s}{\beta}$$

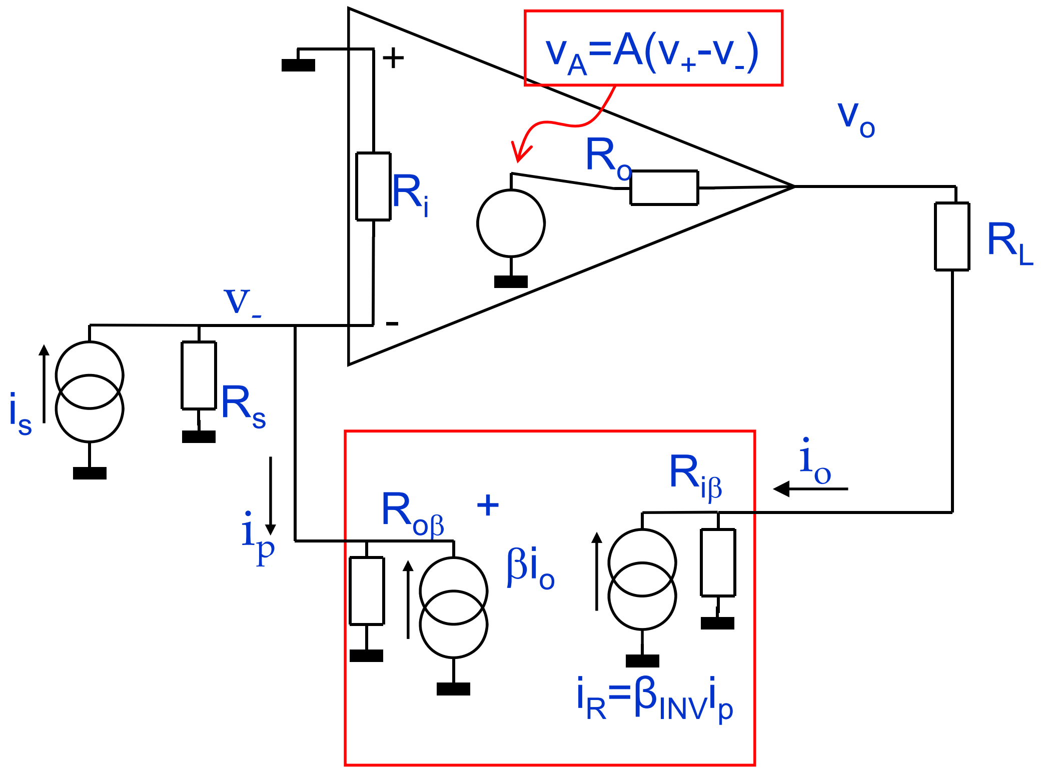
and:

$$v_o = v_B + i_0(R_L + R_o) = -\frac{i_s}{\beta}\left[R_{i\beta}(1 - \beta\beta_{INV}) + R_L + R_o\right]$$

therefore:

$$\beta' = -\frac{\beta}{R_{i\beta}(1 - \beta\beta_{INV}) + R_L + R_o}$$

Input impedance of a c-to-c fed backed OA 4



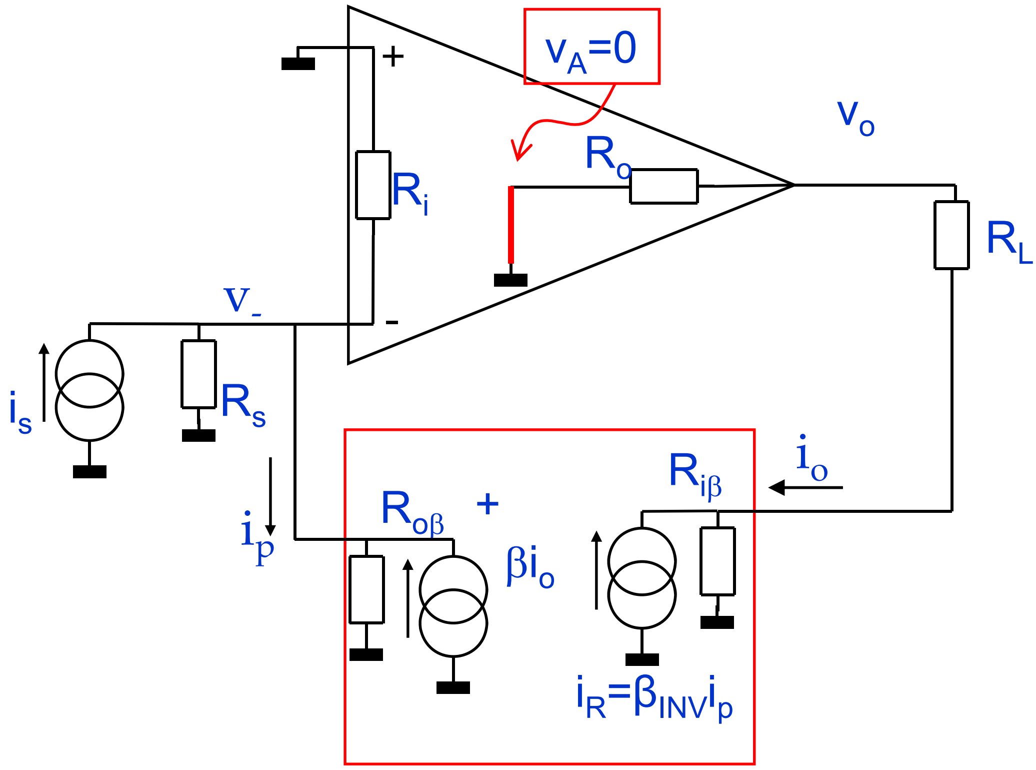
Let's resume:

$$v_A = \frac{1}{\beta'}\frac{-T}{1-T}i_s \qquad \left[\frac{1}{\beta'}\right] = [\Omega]$$

and:

$$v_- = -\frac{v_A}{A} = -\frac{1}{A\beta'}\frac{-T}{1-T}i_s = -\frac{f(\beta')}{1-T}i_s \qquad [f(around_{here})] = [\Omega]$$

Input impedance of a c-to-c fed backed OA 5



$$v_- = -\frac{f(\beta')}{1-T} i_s$$

And:

$$R_{if} = \frac{v_-}{i_s} = -\frac{f(\beta')}{1-T}$$

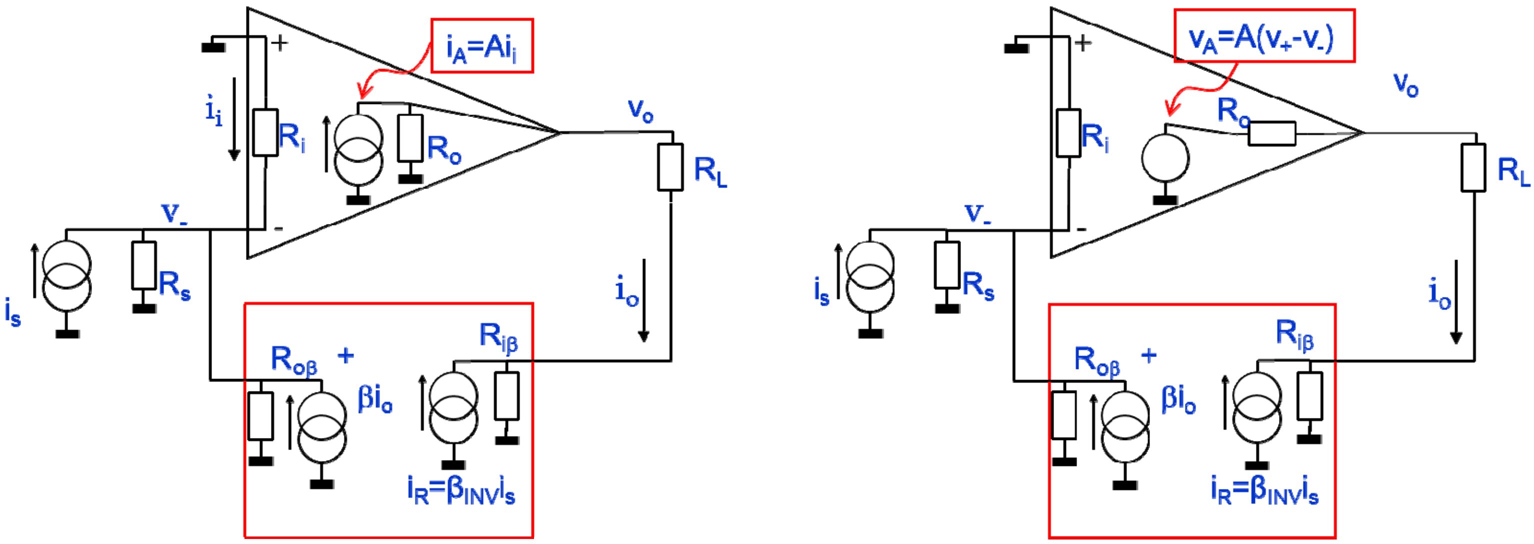
We see that this feedback, that implements a current amplifier, reduces the input impedance measured at zero gain, or open loop, f(β'), by the factor 1-T, decreasing the input impedance further by a large amount.

Then, the input impedance is:

$$R_{if} \triangleq \frac{R_{iol}}{1-T}$$

Being R_{iol} the impedance evaluated once the gain is set to 0.

Input impedance of a c-to-c fed backed OA 6



Let's compare the difference of the results obtained for the 2 kind of OAs.
In both cases the expression for the input impedance is the same, namely the ratio between the open loop input impedance divided by the 1-T term.
What is remarkable different is the open loop input impedance.
If we neglect for a while the effect of β_{INV} in both cases, from the scheme above we have that:

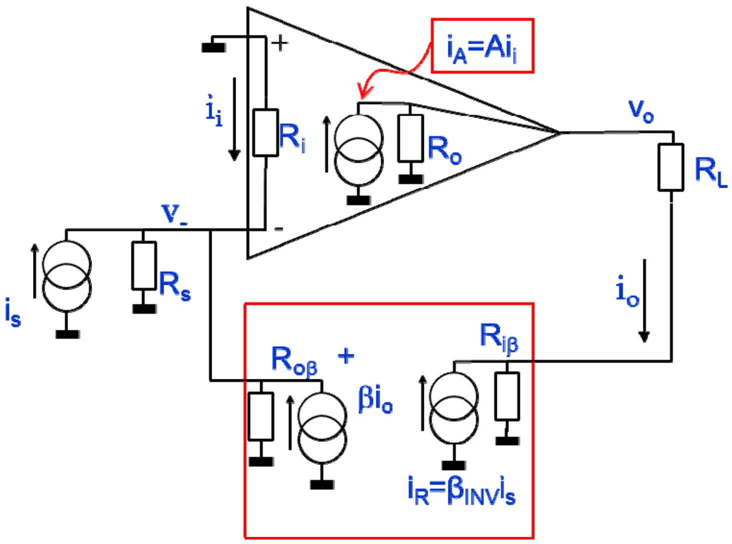
$$R_{iol} = R_s \| R_{o\beta} \| R_i$$

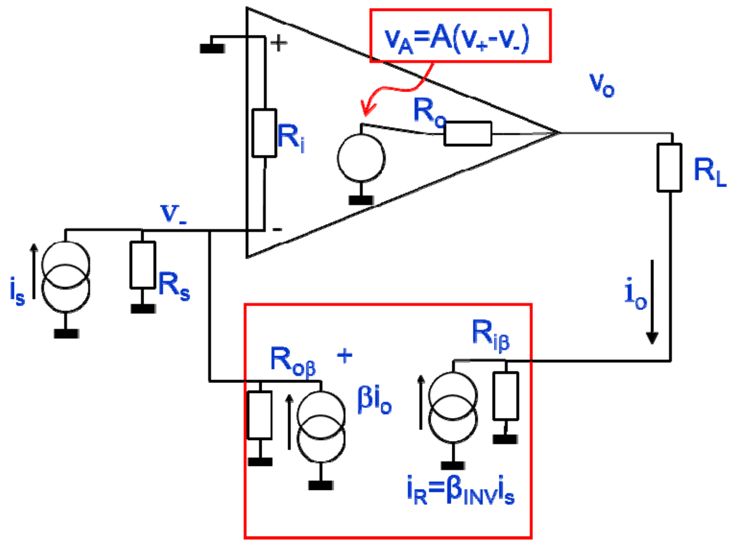
For the current OA R_i is of a few Ω and:

$$R_{iol\_current\_OA} \approx R_i$$

For the standard OA R_i can be very large, or a few MΩ at least and the above approximation does not applies.

Input impedance of a c-to-c fed backed OA 7





Just for giving the order of magnitude let's suppose that our OAs have a voltage/current gain of the order of 10^7, a term close to 1 for f(β') and the close loop gain is 100, then T results of the order of 10^5.

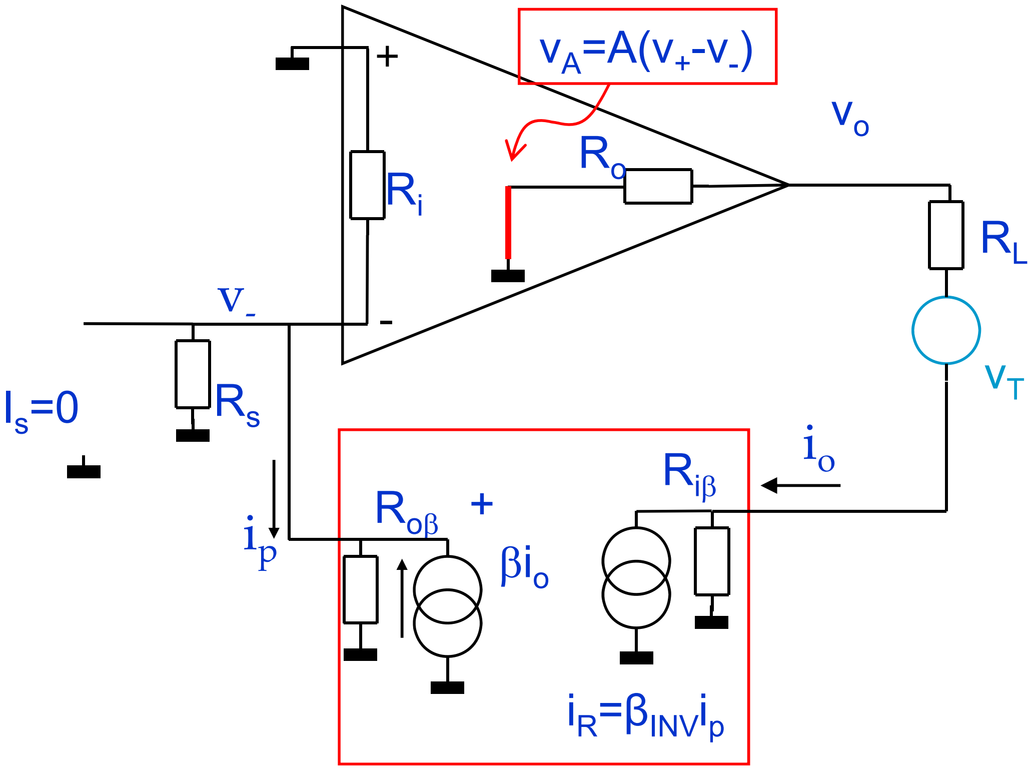
As a result:

$$R_{if} \triangleq \frac{R_{iol_{current_{OA}}}}{1-T} = \frac{1}{10^5} \div 10^{-5}\Omega$$

$$R_{if} \triangleq \frac{R_{iol_{standard_{OA}}}}{1-T} = \frac{1\ \mathrm{M\Omega}}{10^5} \div 10\ \Omega$$

That is the difference.

Output impedance of a c-to-c fed backed OA 1



$$i_o = -\frac{1}{\beta'}\frac{-T}{1-T}v_T + \frac{A_{R'}}{1-T}v_T$$

But, at the input mesh:

$$\beta i_o = 0$$

And we reduces to: $$i_o = \frac{A_{R'}}{1-T}v_T$$

Where, by setting A=0, we know that:

$$A_{R'} = -\frac{1}{R_o + R_L + R_{i\beta A}} \quad \left(R_{i\beta A} = R_{i\beta}\left(1 - \frac{R_{o\beta}}{R_{o\beta} + R_i + R_s}\beta i_o \beta_{INV}\right)\right)$$

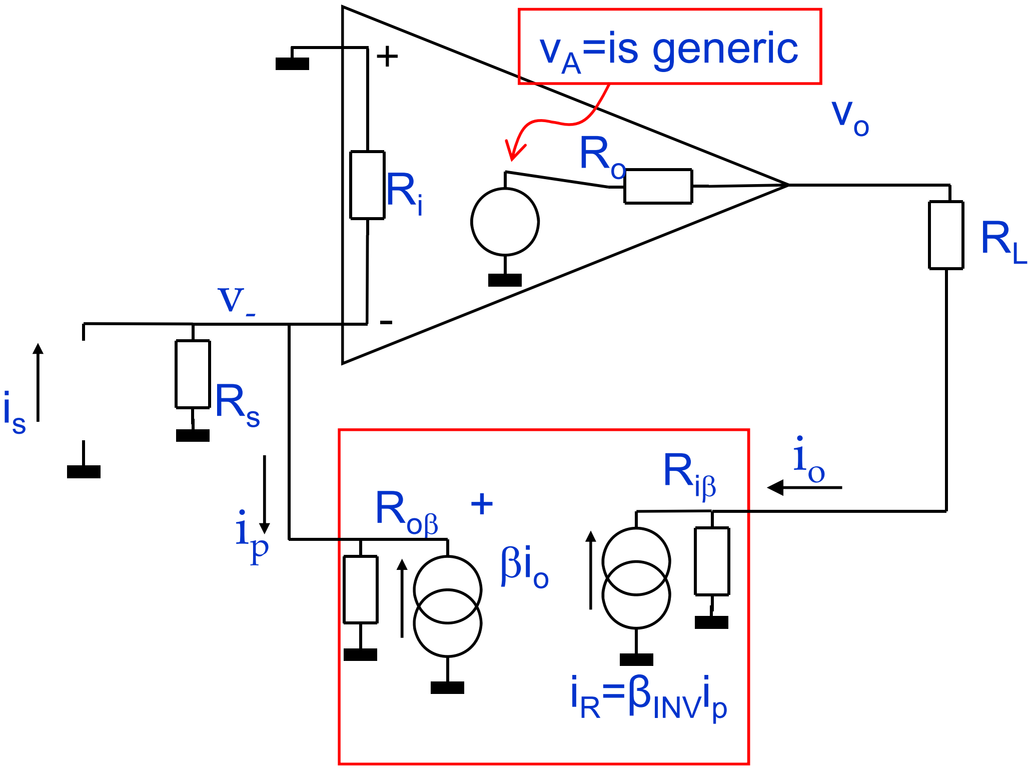
And finally:

$$R_{ofs} = \frac{v_T}{i_o} = \frac{1-T}{A_{R'}} = (R_o + R_L + R_{i\beta A})(1-T)$$

$$R_{ofs} = R_{ool}(1-T)$$

Where R_{ool} is the impedance that is measured when the gain is set to 0, or in the open loop condition.

Output impedance of a c-to-c fed backed OA 2



Let's determine T:

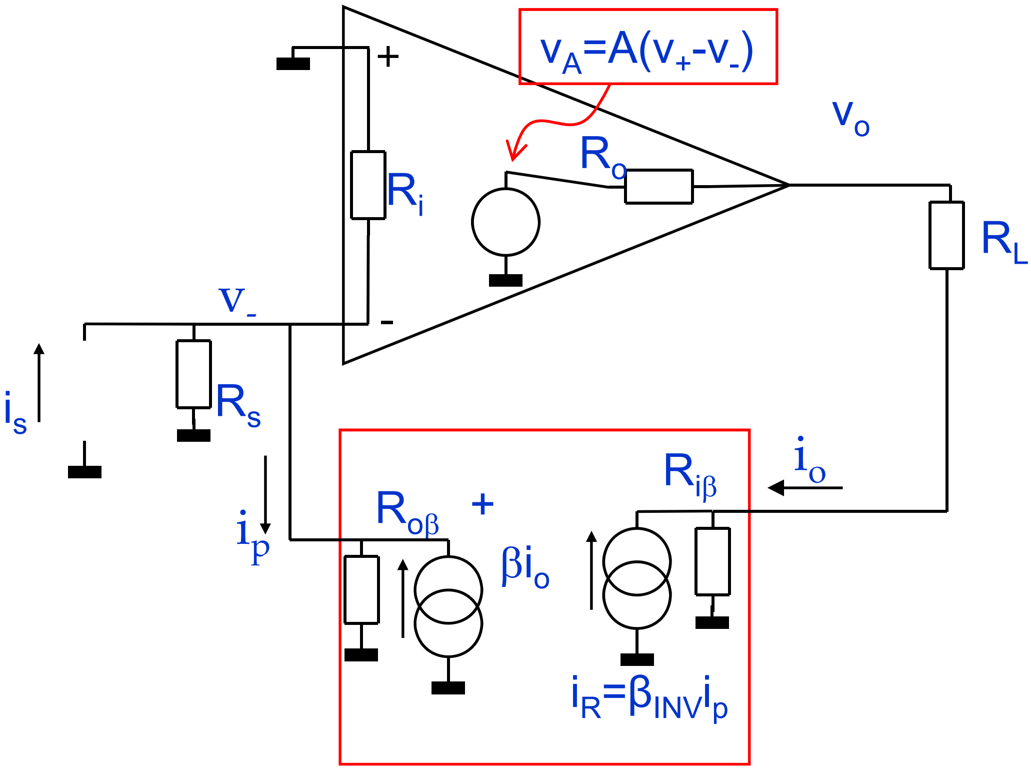
$$i_o = \frac{v_A}{R_o + R_L + R_{i\beta A}}$$

$$v_- = \beta i_o \left(\frac{1}{R_{o\beta}} + \frac{1}{R_s} + \frac{1}{R_i}\right)^{-1} = \beta i_o \frac{R_{o\beta} R_s R_i}{R_{o\beta} R_s + R_s R_i + R_{o\beta} R_i}$$
$$= R_s \| R_{o\beta} \frac{R_i}{R_s \| R_{o\beta} + R_i} \beta i_o$$

And finally:

$$T = A\frac{v_+ - v_-}{v_A} = -A\beta \frac{R_s \| R_{o\beta}}{R_s \| R_{o\beta} + R_i} \frac{R_i}{R_o + R_L + R_{i\beta A}}$$

Output impedance of a c-to-c fed backed OA 3



$$T = -A\beta\left(R_s \| R_{o\beta}\right)\frac{1}{R_s \| R_{o\beta} + R_i}\frac{R_i}{R_o + R_L + R_{i\beta A}}$$

Finally:

$$R_{ofs} = R_L + R_{of'} = \left(R_o + R_L + R_{i\beta A}\right)(1 - T)$$

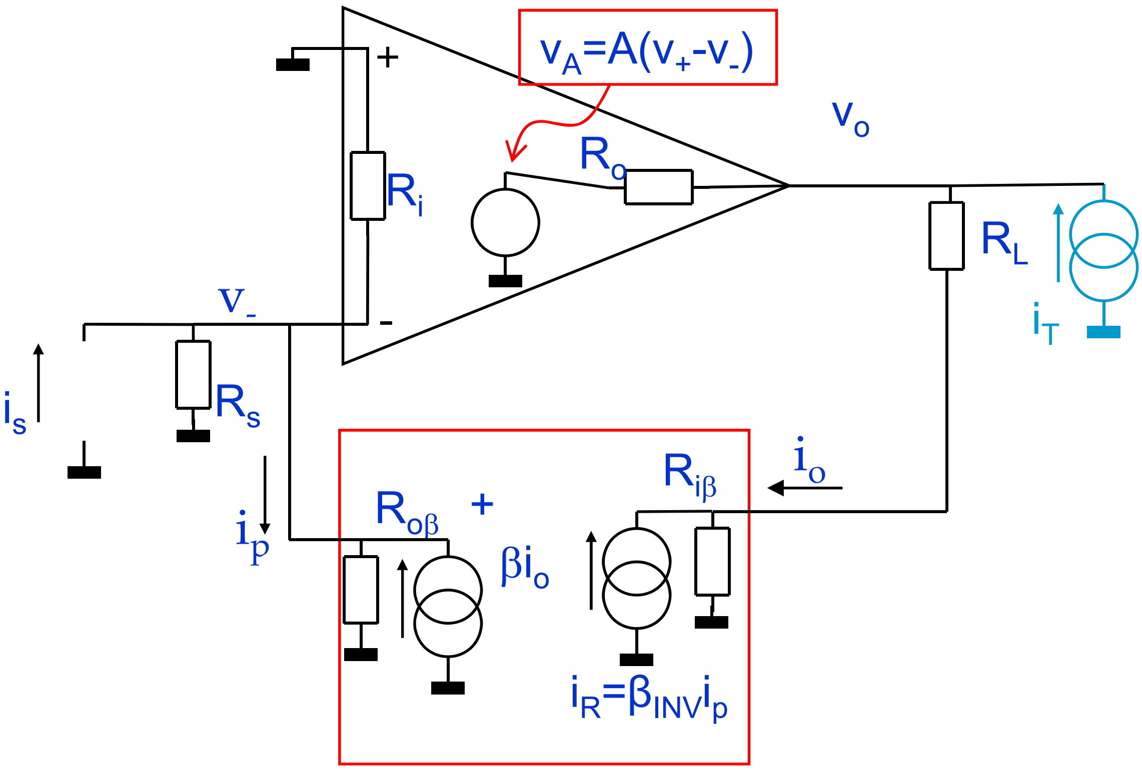
$$R_{ofs} = R_o + R_L + R_{i\beta A} + A\beta\frac{R_s \| R_{o\beta}}{R_s \| R_{o\beta} + R_i}$$

And:

$$R_{of0} = R_{ofs} - R_L = R_o + R_{i\beta A} + A\beta\frac{R_s \| R_{o\beta}}{R_s \| R_{o\beta} + R_i}$$

Output impedance of a c-to-c fed backed OA 4

As with the v-to-v amplifier we can try to measure the output impedance using the complementary test source, a current source in this case.
We have to take care the way we connect the source. The connection shown below does not allow to measure the output impedance.



Indeed, considering that $1/\beta'=0$:

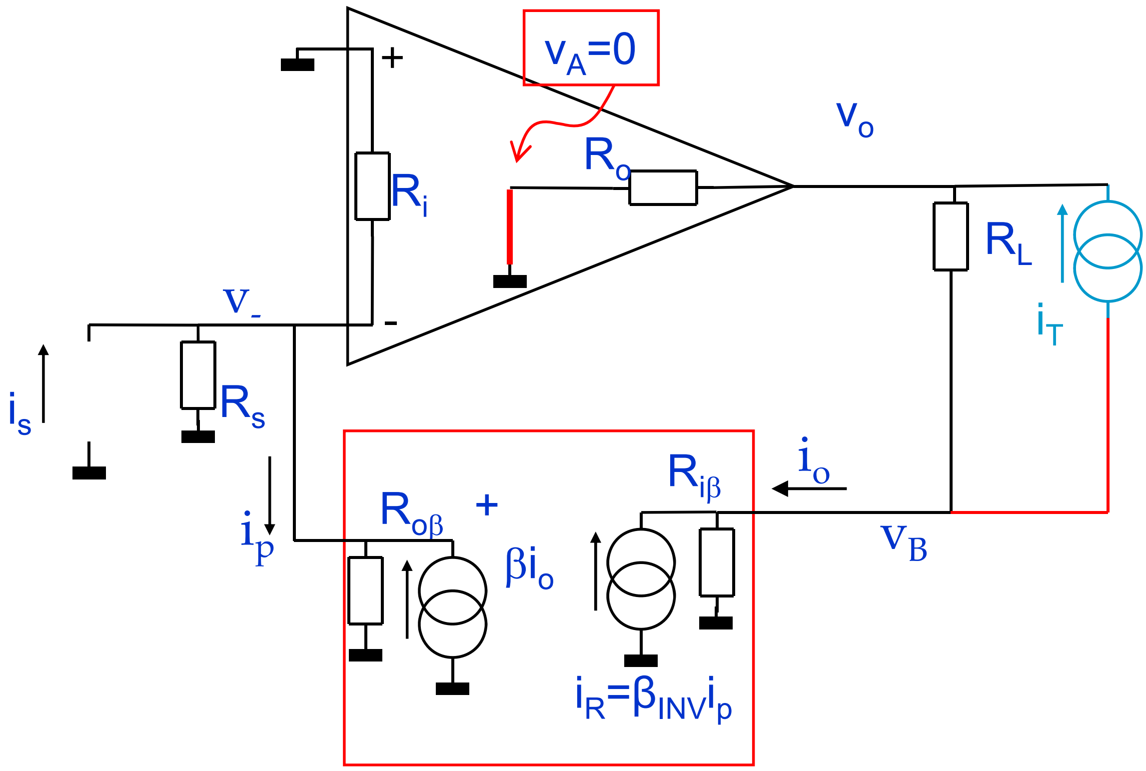
$$v_o = \left(R_L + R_{i\beta A}\right) i_o = \left(R_L + R_{i\beta A}\right) \frac{A_{R'}}{1-T} i_T$$
$$= \left(R_L + R_{i\beta A}\right) \frac{R_o}{R_o + R_L + R_{i\beta A}} \frac{1}{1-T} i_T$$

$$R_{o\_wrong} = \frac{v_o}{i_T} = \frac{\left(R_L + R_{i\beta A}\right) R_o}{R_o + R_L + R_{i\beta A}} \frac{1}{1-T}$$

A very small value: that is correct as the feedback wish to force a negligible current into R_L and, as a consequence, its dropout voltage is negligible: the current i_T flows into v_A.

Output impedance of a c-to-c fed backed OA 5

The impedance must be characterized in parallel to the load R_L as it is shown below: the terminal of i_T previously connected to ground is now connected to the second terminal of R_L.



Again, $1/\beta'=0$ (actually $1/\beta'$ is always 0 when the signal is input at the output), and:

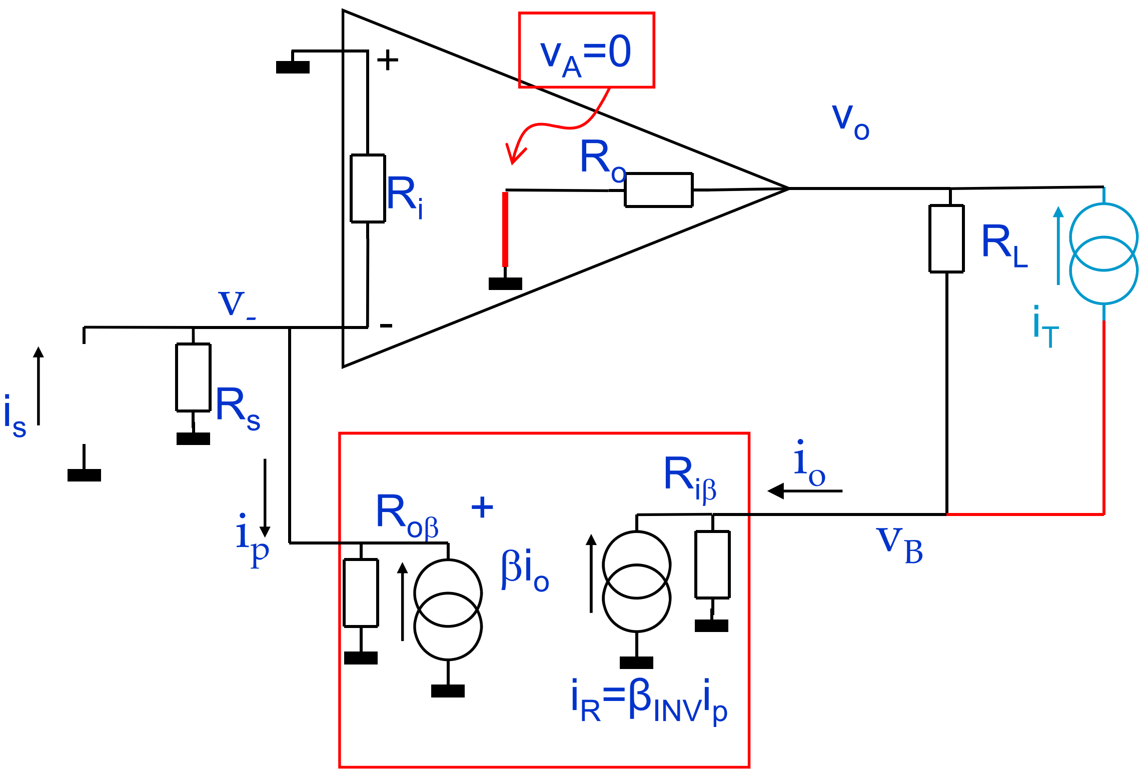
$$\begin{cases} i_T = \dfrac{v_o}{R_o} + \dfrac{v_o - v_B}{R_L} \\ i_o = \dfrac{v_o - v_B}{R_L} - i_T \\ i_o = \dfrac{v_B}{R_{i\beta}} - \beta_{INV}\, i_p \\ i_p = -\dfrac{R_{o\beta}}{R_{o\beta} + R_i + R_s} \beta i_o \end{cases}$$

From the third and last:

$$i_o = \frac{v_B}{R_{i\beta}\left(1 - \dfrac{R_{o\beta}}{R_{o\beta} + R_i + R_s}\beta\beta_{INV}\right)}$$

$$i_o = \frac{v_B}{R_{i\beta A}}$$

Output impedance of a c-to-c fed backed OA 6



… continuing:

$$\begin{cases} i_T = \dfrac{v_o}{R_o} + \dfrac{v_o - v_B}{R_L} \\ i_o = \dfrac{v_o - v_B}{R_L} - i_T \\ i_o = \dfrac{v_B}{R_{i\beta}} - \beta_{INV}\, i_p \\ i_p = -\dfrac{R_{o\beta}}{R_{o\beta} + R_i + R_s} \beta i_o \end{cases}$$

$$i_o = \frac{v_B}{R_{i\beta A}}$$

From the second:

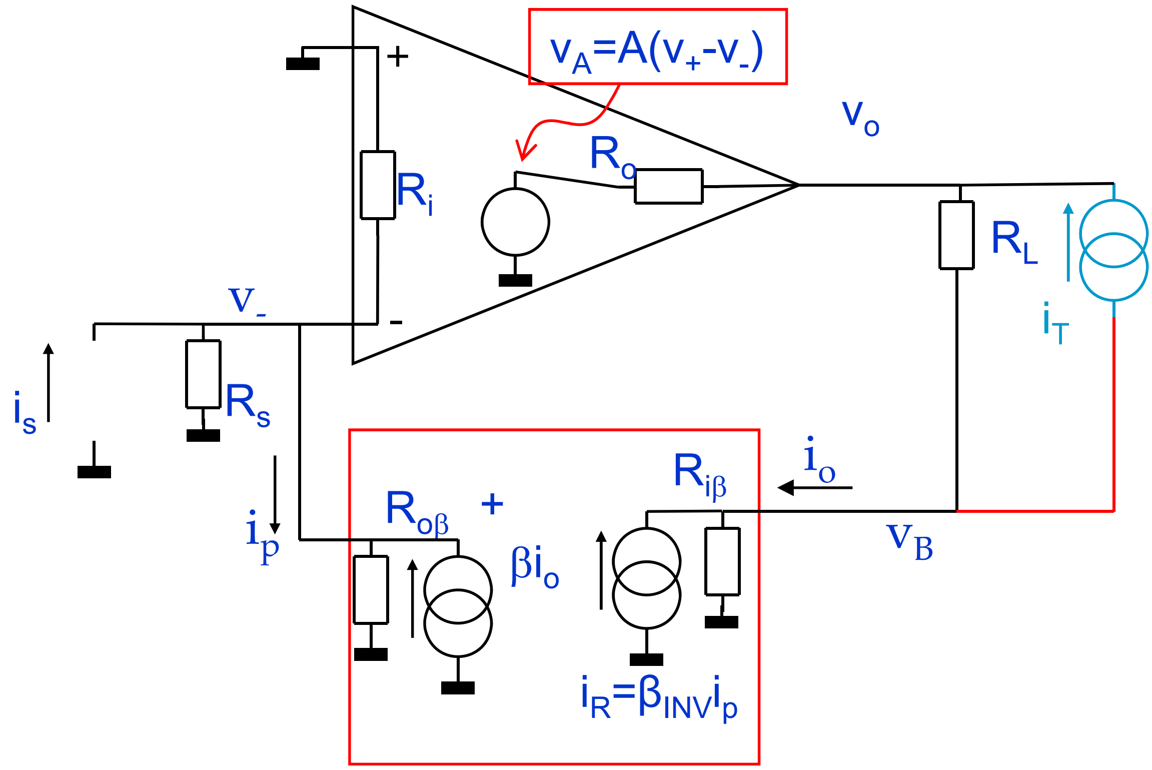
$$\frac{R_L + R_{i\beta A}}{R_L} i_o + i_T = \frac{v_o}{R_L}$$

From the second and first:

$$i_T = \frac{R_L}{R_o}\left(\frac{R_L + R_{i\beta A}}{R_L} i_o + i_T\right) + i_o + i_T$$

$$i_o = -\frac{R_L}{R_L + R_o + R_{i\beta A}} i_T = A_{R''} i_T$$

Output impedance of a c-to-c fed backed OA 7



Now is:

$$i_o = \frac{A_{R''}}{1-T} i_T = -\frac{R_L}{R_L + R_o + R_{i\beta A}} \frac{1}{1-T} i_T$$

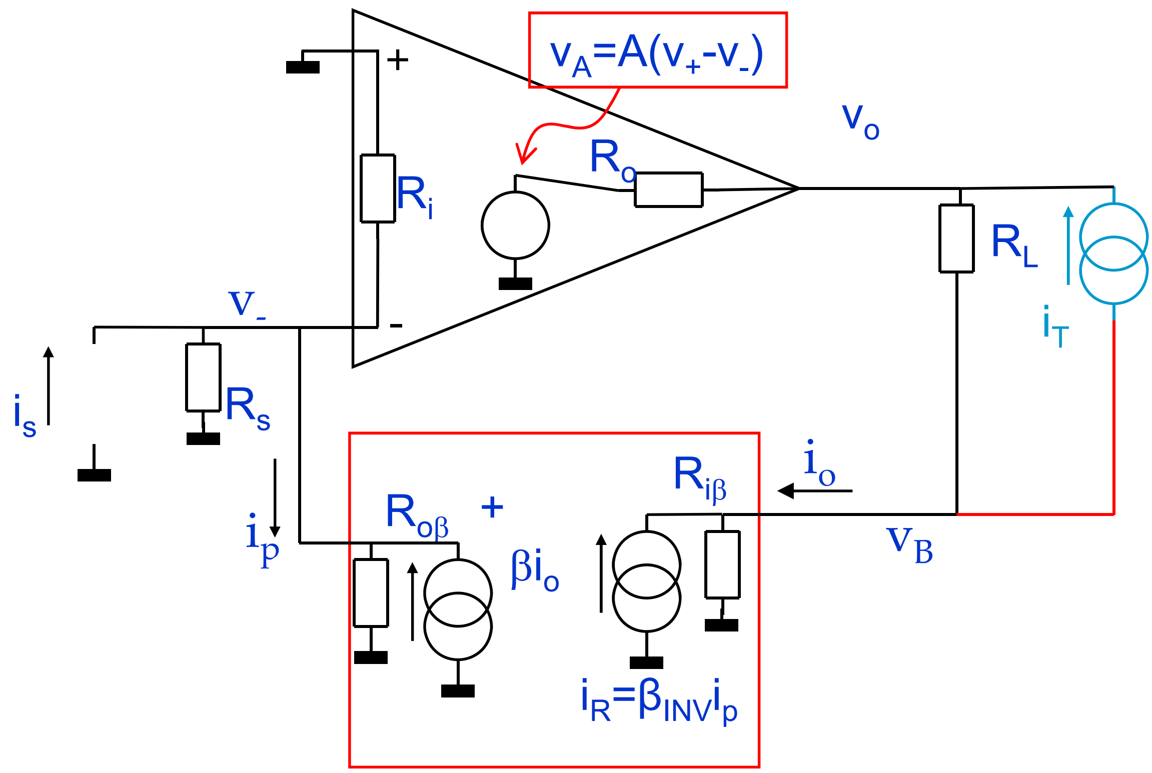
$$R_{ofp} = \frac{v_o - v_B}{i_T} = \frac{R_L(i_o + i_T)}{i_T} = R_L\left(\frac{A_{R''}}{1-T} + 1\right)$$

$$R_{ofp} = \frac{R_L}{1-T}\left(-\frac{R_L}{R_L + R_o + R_{i\beta A}} + 1 + \frac{A\beta\left(R_s \| R_{o\beta}\right)}{R_s \| R_{o\beta} + R_i} \frac{R_i}{R_o + R_L + R_{i\beta A}}\right)$$

$$\left(T = -A\beta\left(R_s \| R_{o\beta}\right) \frac{1}{R_s \| R_{o\beta} + R_i} \frac{R_i}{R_o + R_L + R_{i\beta A}}\right)$$

$$R_{ofp} = \frac{R_L}{\left(R_L + R_o + R_{i\beta A}\right)(1-T)}\left(R_o + R_{i\beta A} + \frac{A\beta\left(R_s \| R_{o\beta}\right) R_i}{R_s \| R_{o\beta} + R_i}\right)$$

Output impedance of a c-to-c fed backed OA 8

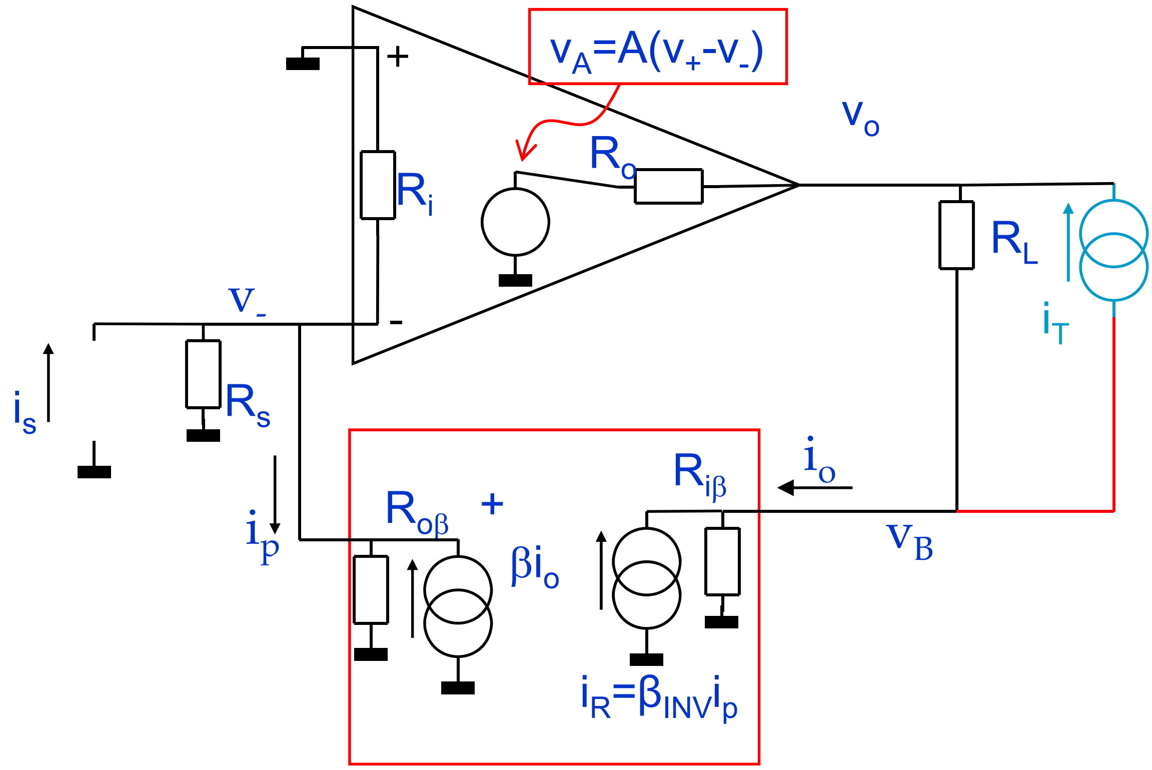


$$R_{ofp}=\frac{R_L}{\left(R_L+R_o+R_{i\beta A}\right)(1-T)}\left(R_o+R_{i\beta A}+\frac{A\beta\left(R_s\|R_{o\beta}\right)R_i}{R_s\|R_{o\beta}+R_i}\right)$$

$$\left(T=-A\beta\left(R_s\|R_{o\beta}\right)\frac{1}{R_s\|R_{o\beta}+R_i}\frac{R_i}{R_o+R_L+R_{i\beta A}}\right)$$

$$R_{ofp}=\frac{R_L\left(R_o+R_{i\beta A}+\frac{A\beta\left(R_s\|R_{o\beta}\right)R_i}{R_s\|R_{o\beta}+R_i}\right)}{R_o+R_L+R_{i\beta A}+\frac{A\beta\left(R_s\|R_{o\beta}\right)R_i}{R_s\|R_{o\beta}+R_i}}$$

Output impedance of a c-to-c fed backed OA 9



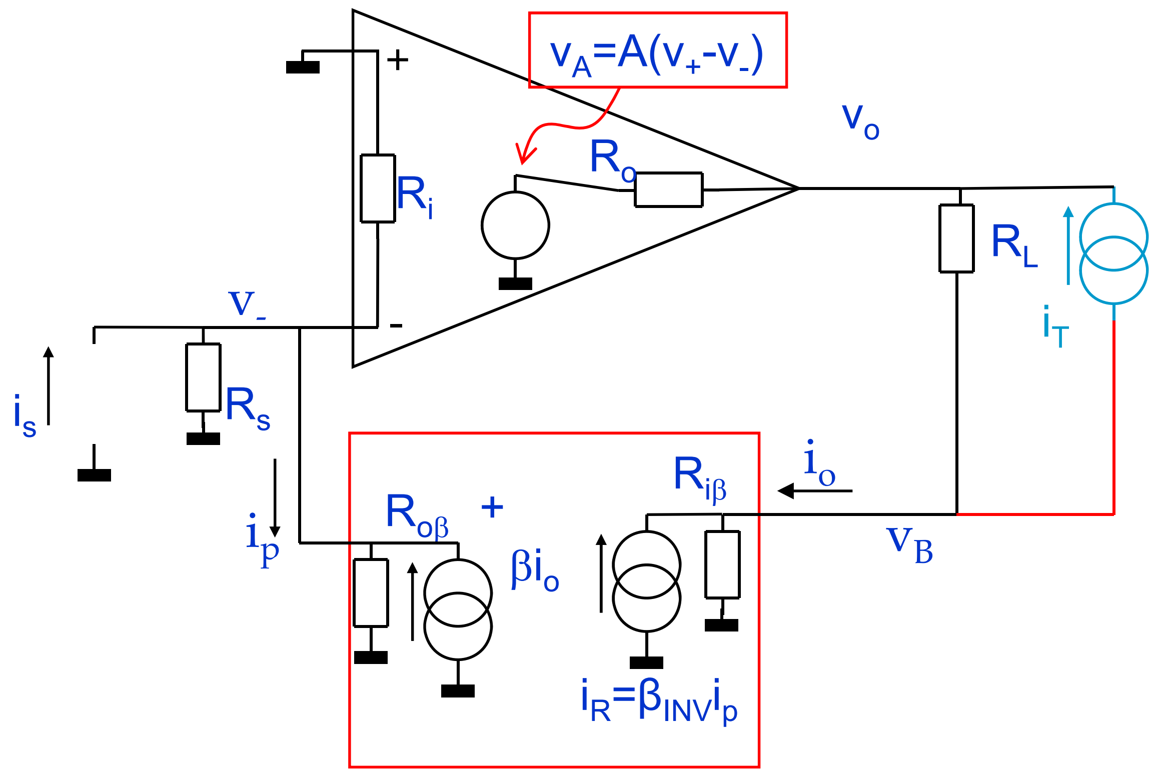
$$R_{ofp} = \frac{R_L\left(R_o + R_{i\beta A} + \dfrac{A\beta\left(R_s \| R_{o\beta}\right) R_i}{R_s \| R_{o\beta} + R_i}\right)}{R_o + R_L + R_{i\beta A} + \dfrac{A\beta\left(R_s \| R_{o\beta}\right) R_i}{R_s \| R_{o\beta} + R_i}}$$

Let's define:

$$\frac{1}{R_{of'}} = \frac{1}{R_{of}} - \frac{1}{R_L}$$

$$= \frac{1}{R_L}\frac{R_o + R_L + R_{i\beta A} + \dfrac{A\beta\left(R_s \| R_{o\beta}\right) R_i}{R_s \| R_{o\beta} + R_i}}{R_o + R_{i\beta A} + \dfrac{A\beta\left(R_s \| R_{o\beta}\right) R_i}{R_s \| R_{o\beta} + R_i}} - \frac{1}{R_L}$$

Output impedance of a c-to-c fed backed OA 10



$$\frac{1}{R_{of'}}=\frac{1}{R_{of}}-\frac{1}{R_L}$$

$$=\frac{1}{R_L}\frac{R_o+R_L+R_{i\beta A}+\dfrac{A\beta\left(R_s\|R_{o\beta}\right)R_i}{R_s\|R_{o\beta}+R_i}}{R_o+R_{i\beta A}+\dfrac{A\beta\left(R_s\|R_{o\beta}\right)R_i}{R_s\|R_{o\beta}+R_i}}-\frac{1}{R_L}$$

$$\frac{1}{R_{of'}}=\frac{1}{R_o+R_{i\beta A}+\dfrac{A\beta\left(R_s\|R_{o\beta}\right)R_i}{R_s\|R_{o\beta}+R_i}}$$

Or:

$$\mathbf{R_{of'}=R_o+R_{i\beta A}+\frac{A\beta\left(R_s\|R_{o\beta}\right)R_i}{R_s\|R_{o\beta}+R_i}\equiv R_{of0}\ !}$$

Input/Output impedance of a c-to-c fed backed OA

Summary about the input/output impedance of a c-to-c.

Input impedance measured with a current source with a parallel resistance R_s :

$$R_{ifp} = \frac{R_{iol}}{1 - T}$$

$$R_{ifp} = R_s \| R_{ifp\infty}$$

Where $R_{iol\infty}$ and T_∞ are R_{iol} and T measured when R_s is set to (open) ∞ Ω.

If the input impedance is measured using a voltage generator with in series a source resistor R_s we have:

$$R_{ifs} = R_s + R_{ifp\infty}$$

oooOOooo

Output impedance measured with a voltage source with in series the load resistor R_L:

$$R_{ofs} = R_{ol}(1 - T)$$

$$R_{ofs} = R_L + R_{ofs0}$$

Where R_{ools0} and T_0 are R_{ool} and T measured when R_L is set to 0 Ω.

If the output impedance is measured using a current generator in parallel to the load R_L we have:

$$R_{ofp} = R_L \| R_{ofs0}$$

Other configurations

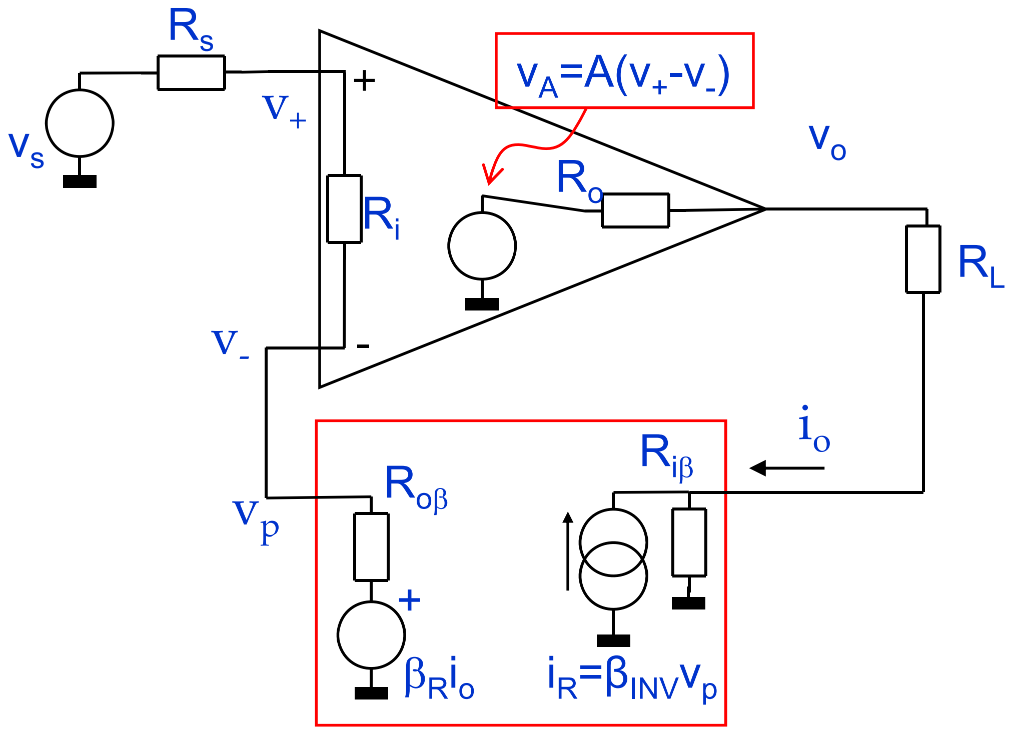
So far we have studied the v-to-v and c-to-c configurations.

There are other 2 possible and affordable feedback networks: the v-to-c (voltage input, current output) and the c-to-v (current input, voltage output).

There is no reason why these configurations should not behave as the two already studied. Indeed, this is what it happens and we will investigate this shortly.

The v-to-c fed backed OA 1

Since it is the most widely diffused the voltage OA is considered, anyway we know that this is not a limitation.



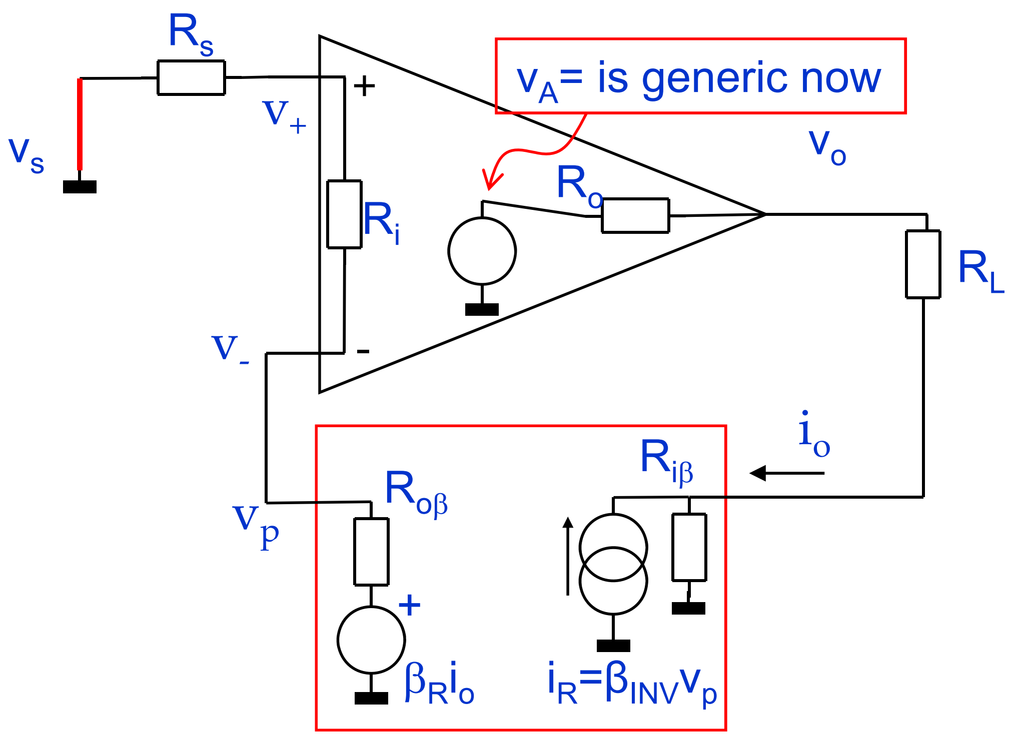
We expect to obtain:

$$i_0 = \frac{1}{\beta_R}\frac{-T}{1-T}v_s + \frac{A_R}{1-T}v_s$$

Where we must not forget that T is dimensionless, but, this time, β_R has dimension: Ω, as well as A_R: Ω^{-1}.

The v-to-c fed backed OA 2

Let' determine T:



$$\begin{cases} i_0 = \dfrac{1}{R_o + R_L + R_{i\beta A}} v_A \\ v_+ - v_- = -\dfrac{R_i}{R_i + R_{o\beta} + R_s}\beta_R i_o \end{cases}$$

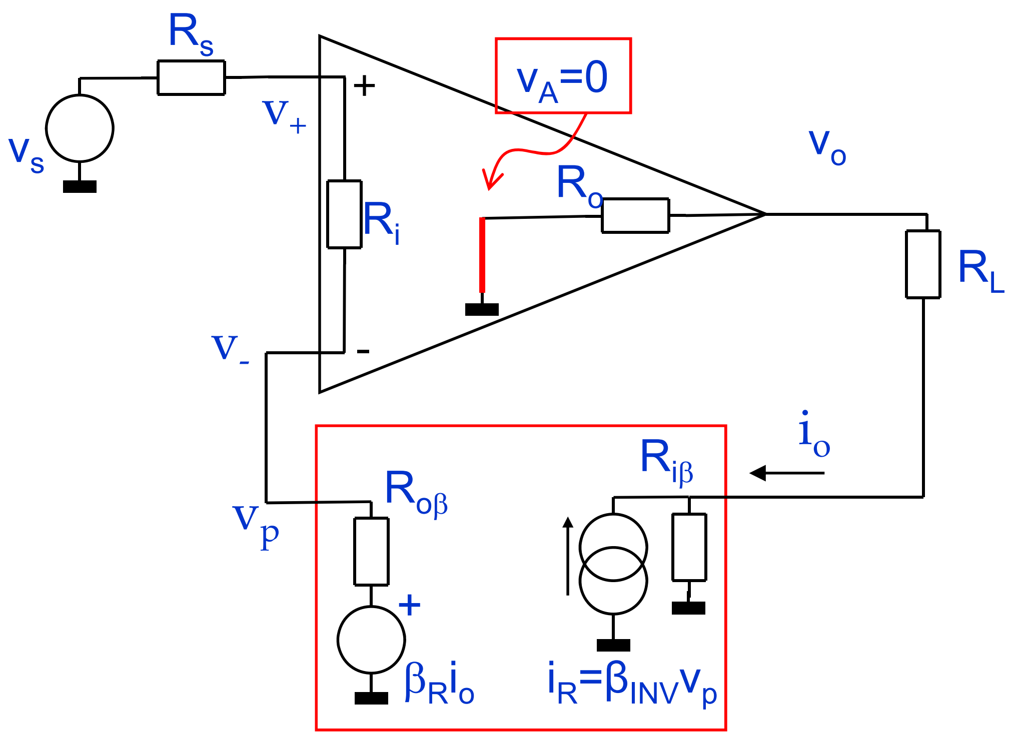
$$\left(R_{i\beta A} = R_{i\beta}\left(1 + \frac{R_i + R_s}{R_i + R_s + R_{o\beta}}\beta_R\beta_{INV}\right)\right)$$

Therefore:

$$T = A\frac{v_+ - v_-}{v_A} = -\frac{A\beta_R R_i}{R_i + R_{o\beta} + R_s}\frac{1}{R_o + R_L + R_{i\beta A}}$$

The v-to-c fed backed OA 3

…and now A_R:



$$i_0 = \frac{R_{i\beta}}{R_o + R_L + R_{i\beta}} \beta_{INVA} v_s$$

$$\left(\beta_{INVA} = \frac{R_{o\beta A}}{R_{o\beta A} + R_i + R_s} \beta_{INV} \right)$$

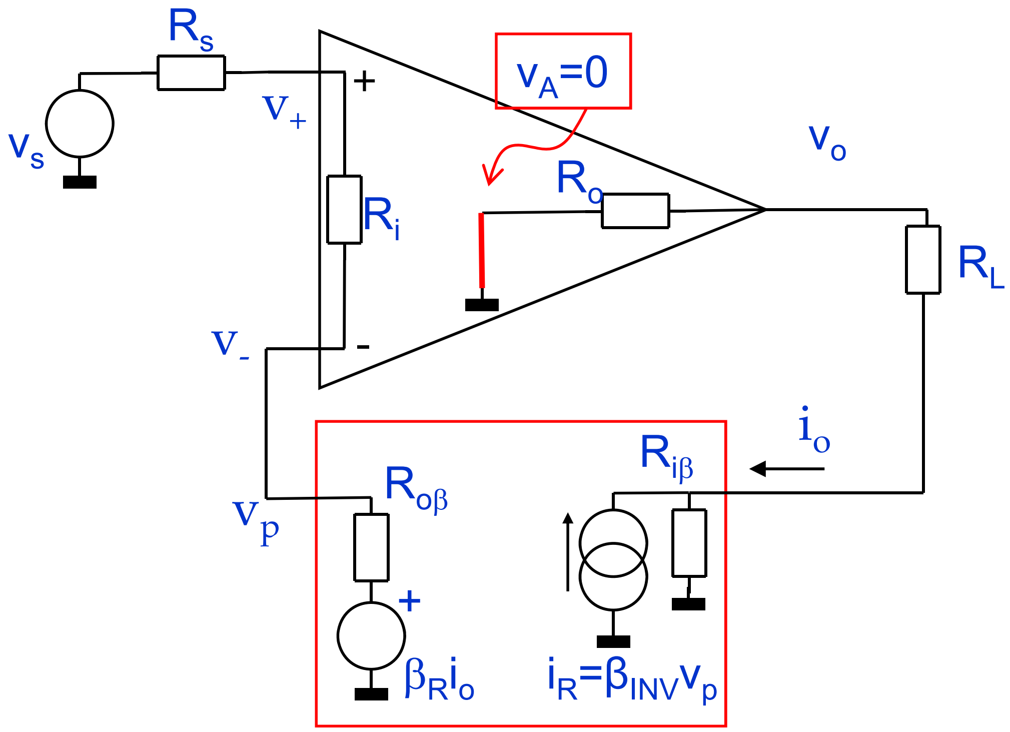
$$\left(R_{o\beta A} = \frac{R_{o\beta}}{1 + \frac{R_{i\beta}}{R_{i\beta} + R_o + R_L} \beta_R \beta_{INV}} \right)$$

Therefore:

$$A_R = \frac{i_o}{v_s} = \frac{R_{i\beta}}{R_o + R_L + R_{i\beta}} \beta_{INVA}$$

The v-to-c fed backed OA 4

Input impedance:



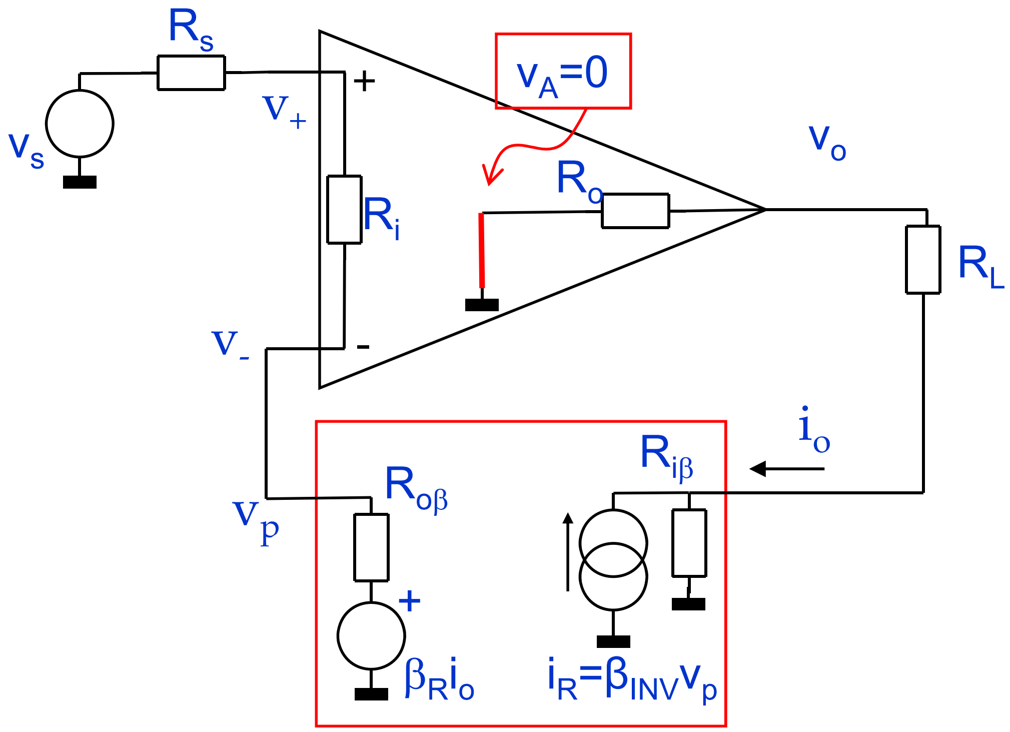
$$\begin{cases} i_s = \dfrac{v_s - v_p}{R_s + R_i} \\ v_p = \dfrac{R_{o\beta}}{R_i + R_s + R_{o\beta}}(v_s - \beta_R i_o) + \beta_R i_o \\ i_o = -\dfrac{R_{i\beta}}{R_o + R_L + R_{i\beta}}\beta_{INV} v_p \end{cases}$$

From the second and third we know that:

$$v_p = \frac{R_{o\beta A}}{R_{o\beta A} + R_i + R_s} v_s \qquad R_{o\beta A} = \frac{R_{o\beta}}{1 + \dfrac{R_{i\beta}}{R_o + R_L + R_{i\beta}}\beta_R \beta_{INV}}$$

The v-to-c fed backed OA 5

Input impedance:



$$\begin{cases} i_s = \dfrac{v_s - v_p}{R_s + R_i} \\ v_p = \dfrac{R_{o\beta}}{R_i + R_s + R_{o\beta}}(v_s - \beta_R i_o) \\ i_o = \dfrac{R_{i\beta}}{R_o + R_L + R_{i\beta}}\beta_{INV} v_p \end{cases}$$

$$v_p = \frac{R_{o\beta A}}{R_{o\beta A} + R_i + R_s} v_s$$

Taking the first:

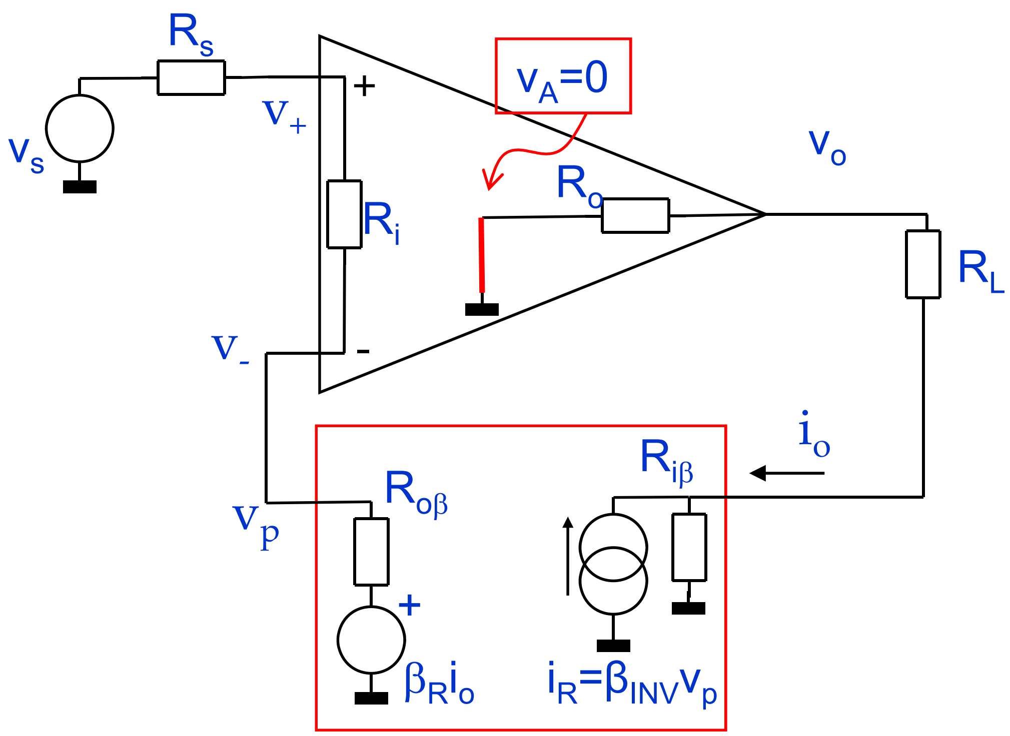
$$i_s = \frac{v_s}{R_s + R_i}\left(1 - \frac{R_{o\beta A}}{R_{o\beta A} + R_i + R_s}\right)$$

$$i_s = \frac{v_s}{R_{o\beta A} + R_i + R_s}$$

$$\Rightarrow \quad R_{iol} = \frac{v_s}{i_s} = R_s + R_i + R_{o\beta A}$$

The v-to-c fed backed OA 6

Input impedance:



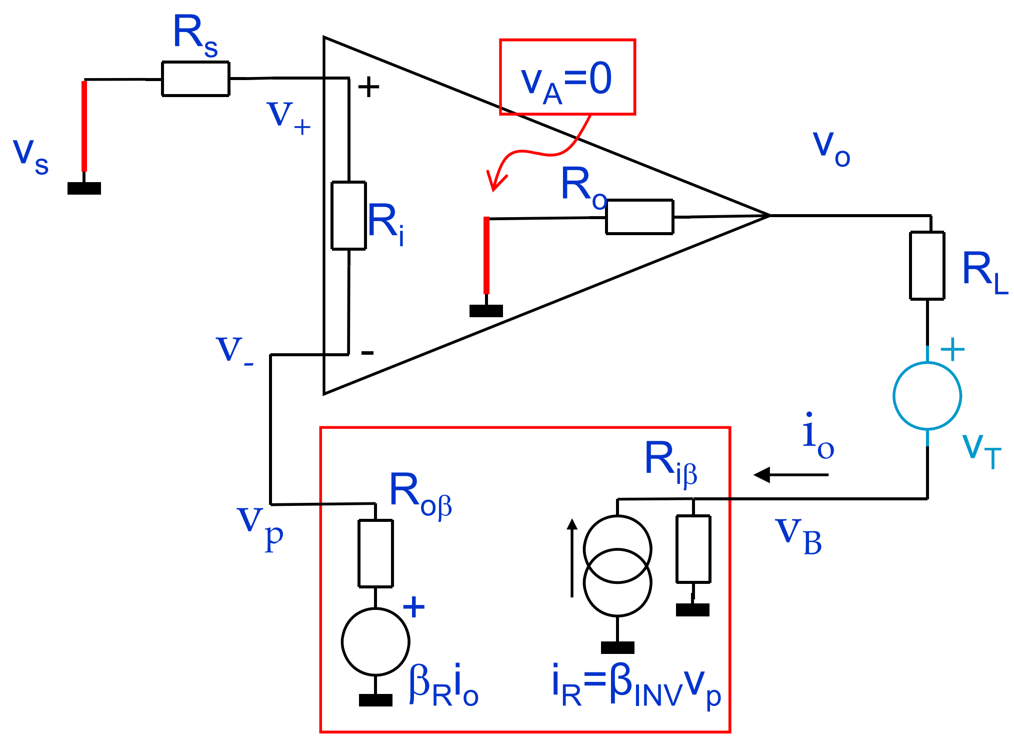
$$R_{iol} = \frac{v_s}{i_s} = R_s + R_i + R_{o\beta A}$$

And:

$$R_{ifs} = R_{iol}(1 - T)$$
$$= (R_s + R_i + R_{o\beta A})\left(1 + \frac{A\beta_R R_i}{R_i + R_{o\beta} + R_s}\frac{1}{R_o + R_L + R_{i\beta A}}\right)$$

The v-to-c fed backed OA 7

Output impedance:

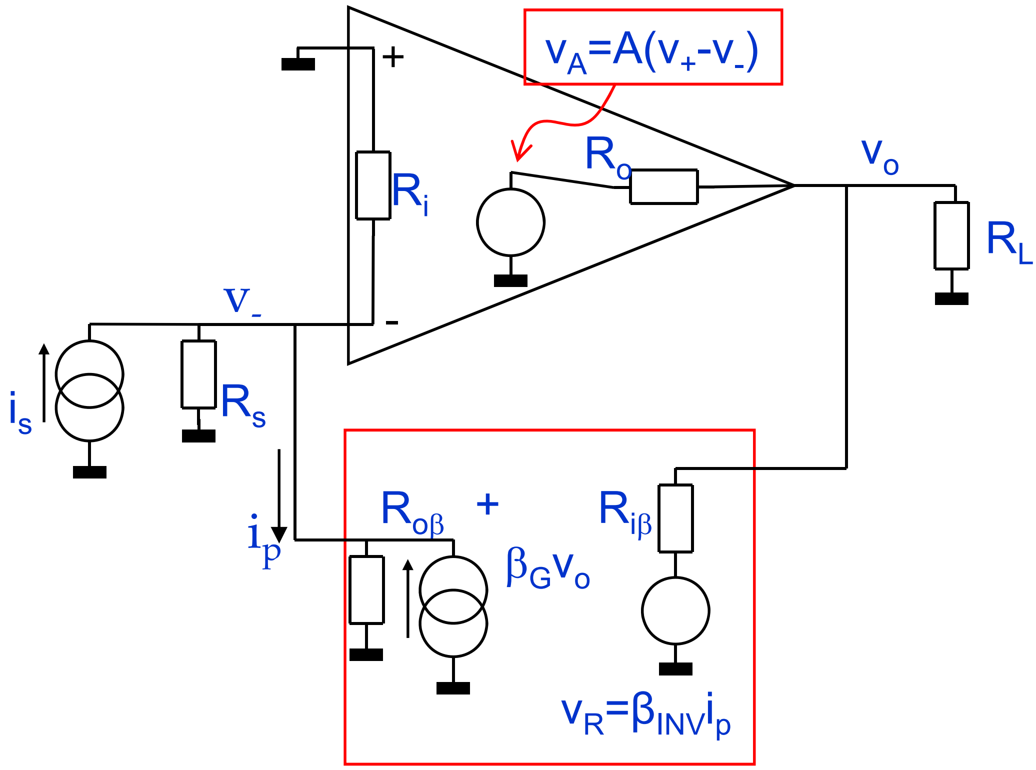


$$R_{ool} = R_o + R_L + R_{i\beta A} \qquad \left(R_{i\beta A} = R_{i\beta}\left(1 + \frac{R_i + R_s}{R_{o\beta} + R_i + R_s}\beta_R\beta_{INV}\right) \right)$$

Therefore:

$$R_{ofs} = R_{ool}(1 - T)$$
$$= \left(R_o + R_L + R_{i\beta A}\right)\left(1 + \frac{A\beta_R R_i}{R_i + R_{o\beta} + R_s}\frac{1}{R_o + R_L + R_{i\beta A}}\right)$$
$$= R_L + R_o + R_{i\beta A} + \frac{A\beta_R R_i}{R_i + R_{o\beta} + R_s}$$

The c-to-v fed backed OA 1

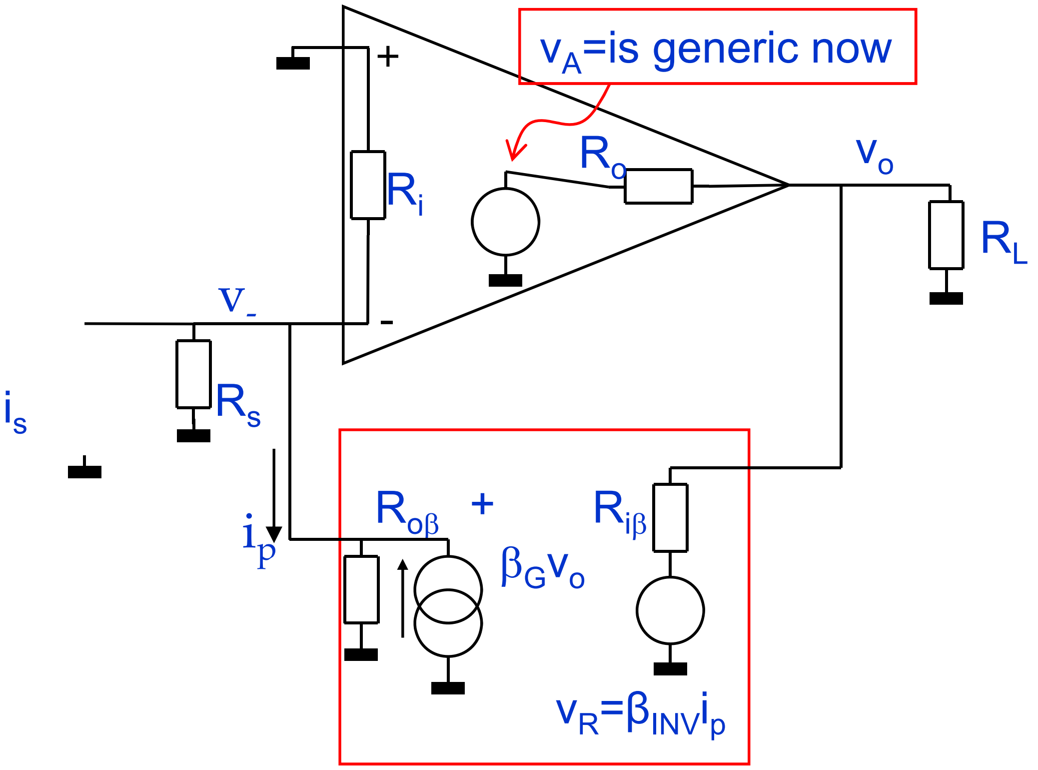


We expect to obtain:

$$v_0 = \frac{1}{\beta_G}\frac{-T}{1-T} i_s + \frac{A_R}{1-T} i_s$$

Again, we must not forget that T is dimensionless, but, this time, β_G has dimension: Ω^{-1}, as well as A_R: Ω.

The c-to-v fed backed OA 2

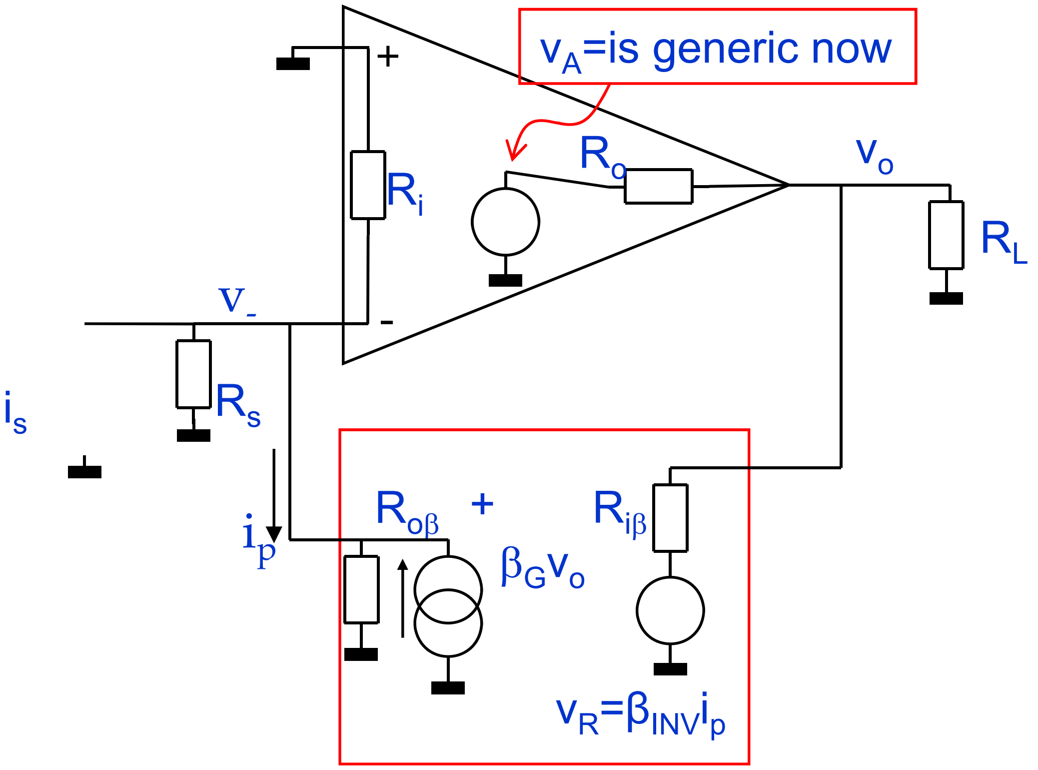


$$\begin{cases} \dfrac{v_A - v_o}{R_o} = \dfrac{v_0}{R_L} + \dfrac{v_0 - \beta_{INV} i_p}{R_{i\beta}} \\ i_p = -\dfrac{R_{o\beta}}{R_{o\beta} + R_s + R_s} \beta_G v_o \end{cases}$$

$$\frac{v_A - v_o}{R_o} = \frac{v_0}{R_L} + \frac{v_0}{R_{i\beta}} \left(1 + \frac{R_{o\beta}}{R_{o\beta} + R_s + R_i} \beta_G \beta_{INV}\right)$$

$$\frac{v_A - v_o}{R_o} = \frac{v_0}{R_L} + \frac{v_0}{R_{i\beta A}} \quad \Rightarrow \quad v_A = \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o} v_0$$

The c-to-v fed backed OA 3



$$v_A = \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o} v_0$$

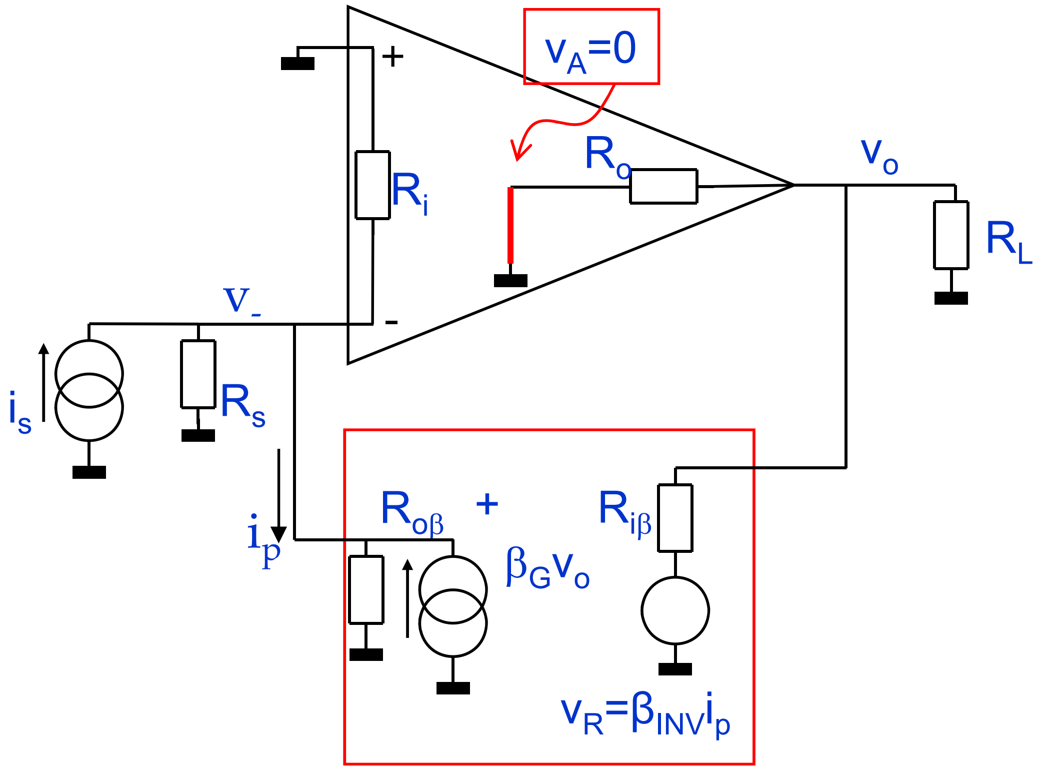
And:

$$v_+ - v_- = -\frac{R_i R_s R_{o\beta}}{R_{o\beta} R_i + R_{o\beta} R_s + R_i R_s} \beta_G v_o$$

In the end:

$$T = A\frac{v_+ - v_-}{v_A} = -\frac{A\beta_G R_i R_s R_{o\beta}}{R_{o\beta} R_i + R_{o\beta} R_s + R_i R_s} \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o}$$

The c-to-v fed backed OA 4



Direct transmission:

$$\begin{cases} i_s + \beta_G v_o = \dfrac{v_-}{R_s} + \dfrac{v_-}{R_i} + \dfrac{v_-}{R_{o\beta}} \\ i_p = \dfrac{v_-}{R_{o\beta}} - \beta_G v_o \\ v_o = \dfrac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta_{INV} i_p \end{cases}$$

From the second and last:

$$i_p = \frac{v_-}{R_{o\beta}} - \frac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta_{INV} \beta_G i_p$$

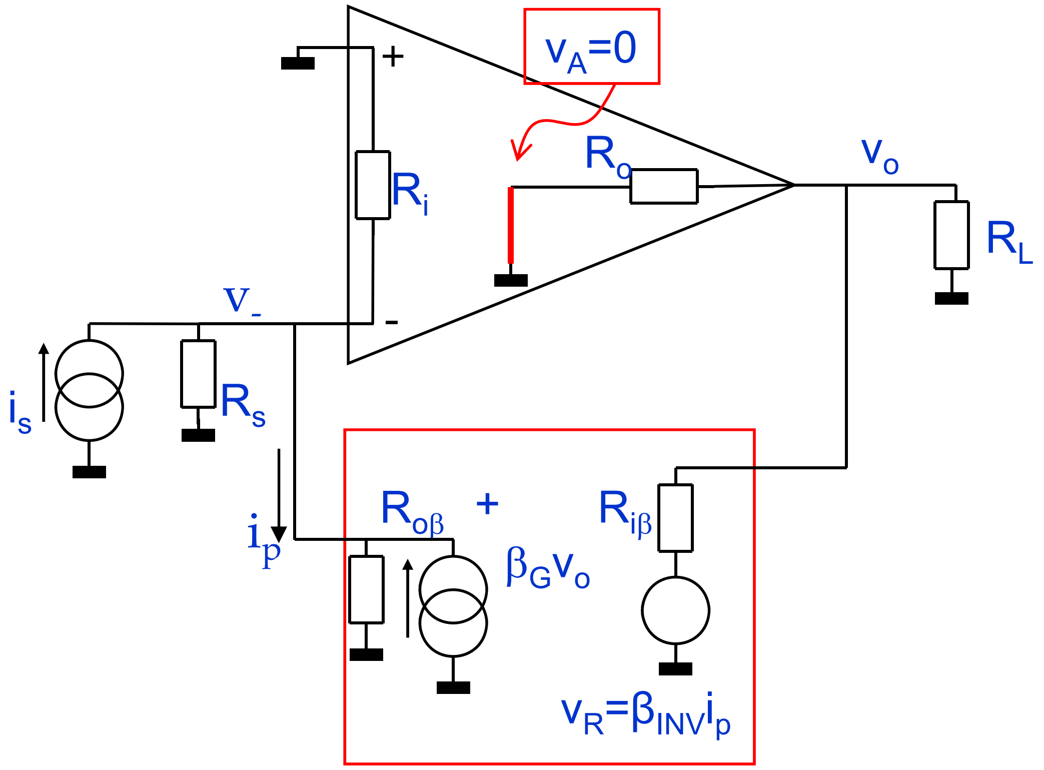
$$i_p = \frac{v_-}{R_{o\beta}\left(1 + \dfrac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta_{INV} \beta_G\right)}$$

$$i_p = \frac{v_-}{R_{o\beta A}}$$

The second in the first:

$$i_s = \frac{v_-}{R_s} + \frac{v_-}{R_i} + i_p \qquad i_s = \frac{R_s + R_i}{R_i R_s} R_{o\beta A} i_p + i_p \qquad i_p = \frac{i_s}{\dfrac{R_s + R_i}{R_i R_s} R_{o\beta A} + 1}$$

The c-to-v fed backed OA 5



$$\begin{cases} i_s + \beta_G v_o = \dfrac{v_-}{R_s} + \dfrac{v_-}{R_i} + \dfrac{v_-}{R_{o\beta}} \\ i_p = \dfrac{v_-}{R_{o\beta}} - \beta_G v_o \\ v_o = \dfrac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta_{INV} i_p \end{cases}$$

$$i_p = \frac{i_s}{\frac{R_s + R_i}{R_i R_s} R_{o\beta A} + 1}$$

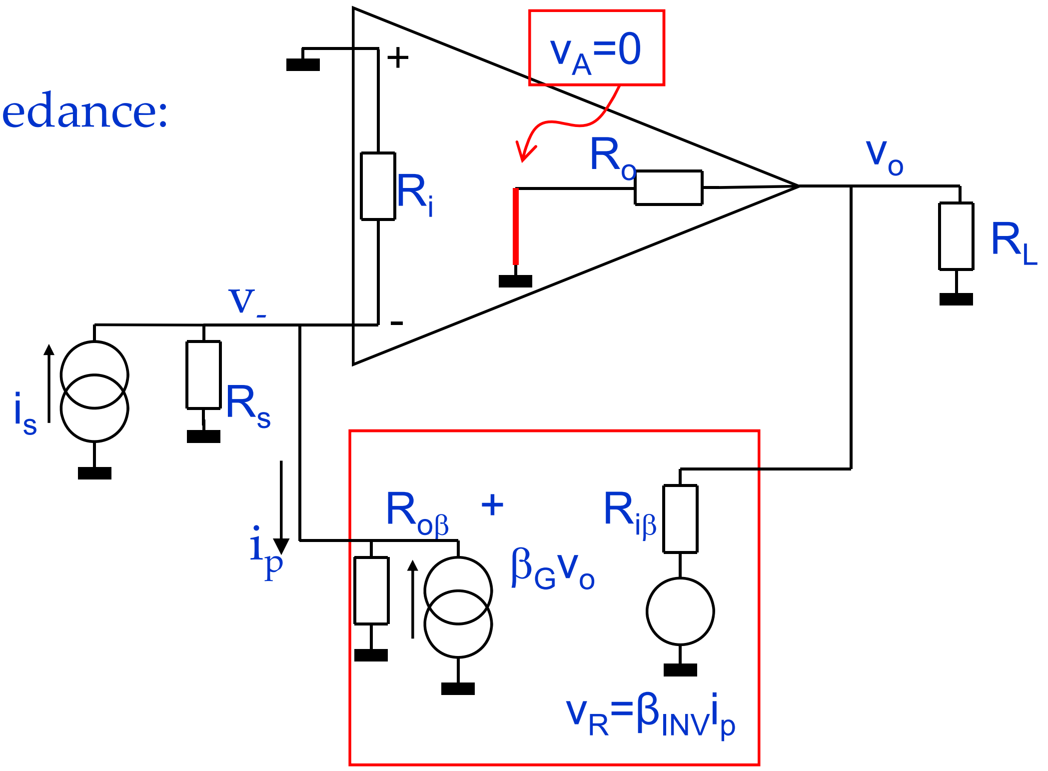
$$i_p = \frac{R_i \| R_s}{R_i \| R_s + R_{o\beta A}} i_s$$

To obtain, from the last:

$$v_o = \frac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \frac{R_i \| R_s}{R_i \| R_s + R_{o\beta A}} \beta_{INV} i_s = A_R i_s$$

The c-to-v fed backed OA 6

Input impedance:



$$R_{iol} = R_s \left\| R_i \right\| R_{o\beta A} = \frac{R_i R_s R_{o\beta A}}{R_{o\beta A} R_i + R_{o\beta A} R_s + R_i R_s}$$

$$\left(R_{o\beta A} = R_{o\beta} \left(1 + \frac{R_o \| R_L}{R_o \| R_L + R_{i\beta}} \beta_{INV} \beta_G \right) \right)$$

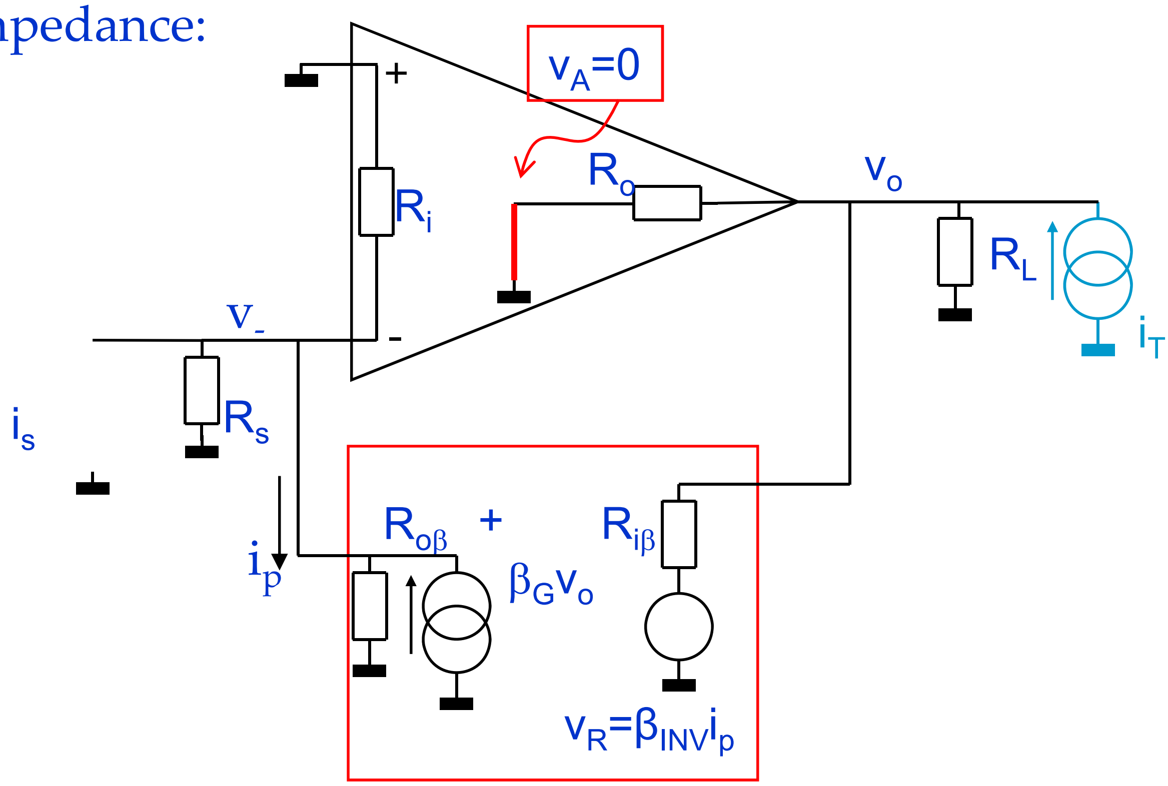
And:

$$R_{ifp} = \frac{R_{iol}}{1 - T}$$

$$= \frac{\left(\frac{R_i R_s R_{o\beta A}}{R_{o\beta A} R_i + R_{o\beta A} R_s + R_i R_s} \right)}{1 + \frac{A \beta_G R_i R_s R_{o\beta}}{R_{o\beta} R_i + R_{o\beta} R_s + R_i R_s} \frac{R_L \left\| R_{i\beta A} \right.}{R_L \left\| R_{i\beta A} + R_o \right.}}$$

The c-to-v fed backed OA 7

Output impedance:



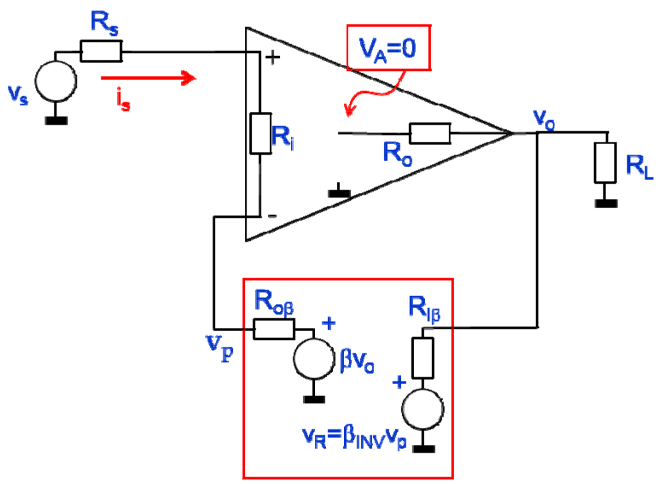
$$R_{ool} = R_o \| R_L \| R_{i\beta A} = \frac{R_o R_L R_{i\beta A}}{R_{i\beta A} R_o + R_{i\beta A} R_L + R_L R_o}$$

$$\left(R_{i\beta A} = R_{i\beta} \left(1 + \frac{R_{o\beta}}{R_{o\beta} + R_s + R_i} \beta_G \beta_{INV} \right)^{-1} \right)$$

$$R_{ofp} = \frac{R_{ool}}{1 - T}$$

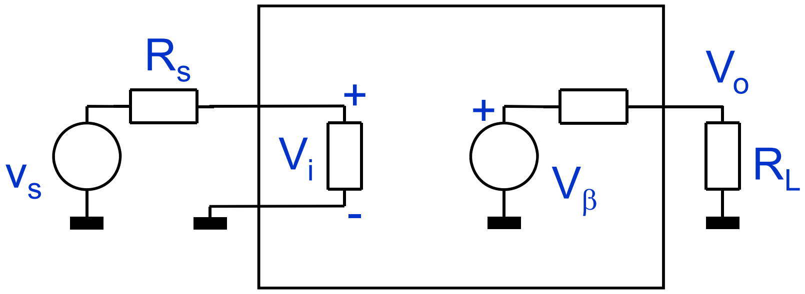
$$= \frac{\left(\frac{R_o R_L R_{i\beta A}}{R_{i\beta A} R_o + R_{i\beta A} R_L + R_L R_o} \right)}{1 + \frac{A \beta_G R_i R_s R_{o\beta}}{R_{o\beta} R_i + R_{o\beta} R_s + R_i R_s} \frac{R_L \| R_{i\beta A}}{R_L \| R_{i\beta A} + R_o}}$$

Quadrupole model of an amplifier 1



We are now in a condition of giving a simpler representation of our fed backed amplifier, useful in several applications and particularly important when we design a system that for an user.

The model we would like to obtain is a quadrupole:



The matter is to give the input and output parameters starting from our knowledge of the circuit above.

We know that:

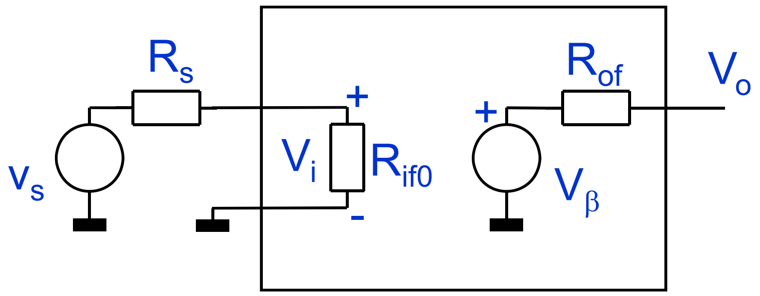
$$\begin{cases} R_{if} = R_s + R_{iol0}(1 - T_0) = R_s + R_{if0} \\ R_{of} = R_L \left\| \left[\dfrac{R_{ool\infty}}{(1 - T_\infty)} \right] \right. = R_L \| R_{of\infty} \\ v_\beta = \dfrac{1}{\beta} \dfrac{-T}{1 - T} v_s + \dfrac{A_R}{1 - T} v_s \end{cases}$$

Where, in general, it is valid that:

$R_{iol0} = R_{iol0}(R_L)$, $T_0 = T_0(R_L)$ and $R_{ool\infty} = R_{ool\infty}(R_s)$, $T_\infty = T_\infty(R_s)$ and $T = T(R_s, R_L)$, $A_R = A_R(R_s, R_L)$.

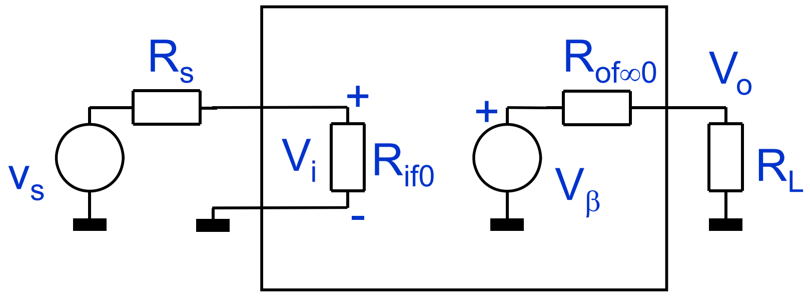
Quadrupole model of an amplifier 2

A first step is to disentangle, for instance, the source from the input:



$$v_\beta = \frac{1}{\beta}\frac{-T_0}{1-T_0}v_s + \frac{A_{R0}}{1-T_0}v_s$$

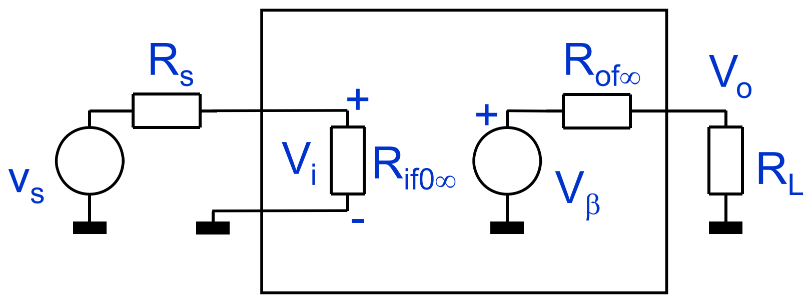
At this point we can disentangle the output, when the input source impedance R_s is considered nulled:



$$v_\beta = \frac{1}{\beta}\frac{-T_{\infty 0}}{1-T_{\infty 0}}v_s + \frac{A_{R\infty 0}}{1-T_{\infty 0}}v_s$$

Where we intend that the condition R_L=∞ Ω is applied after that the position R_s=0 Ω was applied.

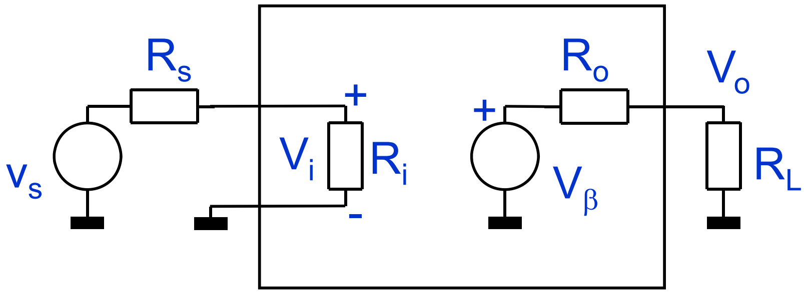
In a similar way we can apply the condition R_L=∞ Ω before the position R_s=0 Ω, with obvious meaning of the symbols:



$$v_\beta = \frac{1}{\beta}\frac{-T_{0\infty}}{1-T_{0\infty}}v_s + \frac{A_{R0\infty}}{1-T_{0\infty}}v_s$$

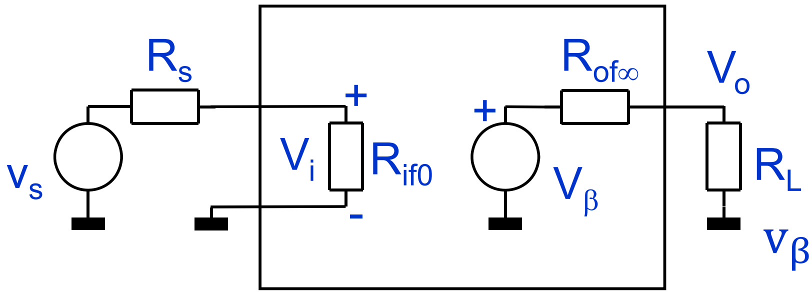
Quadrupole model of an amplifier 3

If we are designing for our customer the datasheet we will provide will be:



$$v_\beta = G v_s$$

In practical cases, and far from limited conditions, is quite a good approximation to not take care of the dependence from the output impedance of T_0 and from the input impedance of T_∞ and the approximation we obtain is:



$$v_\beta = \frac{1}{\beta}\frac{-T_{\infty\leftrightarrow 0}}{1 - T_{\infty\leftrightarrow 0}} v_s + \frac{A_{R\infty\leftrightarrow 0}}{1 - T_{\infty\leftrightarrow 0}} v_s$$

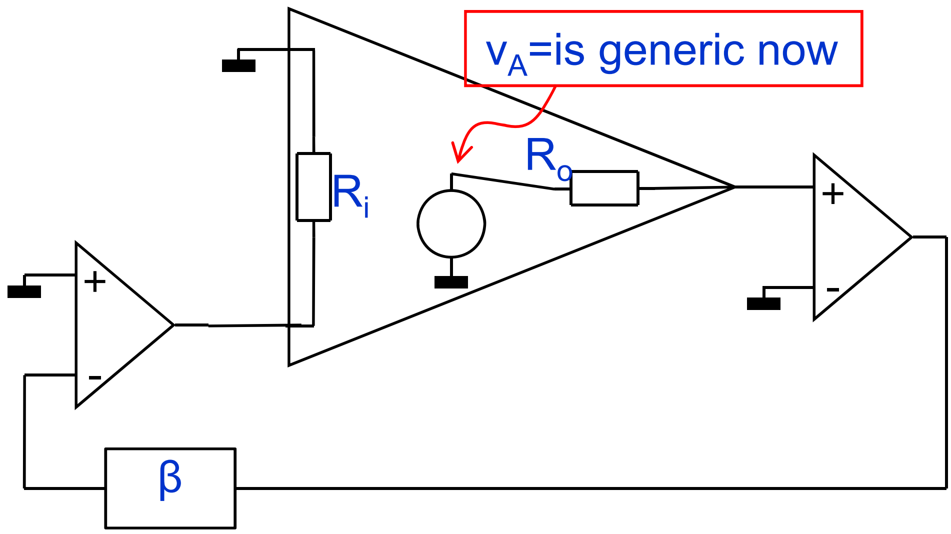
Where $\infty\leftrightarrow 0$ means that T is evaluated once it is set $R_L=\infty\ \Omega$ and $R_s=0\ \Omega$ in a whatever order.

This last approximation is valid as long as $R_s \ll R_{if0}$ and $R_L \gg R_{of\infty}$.

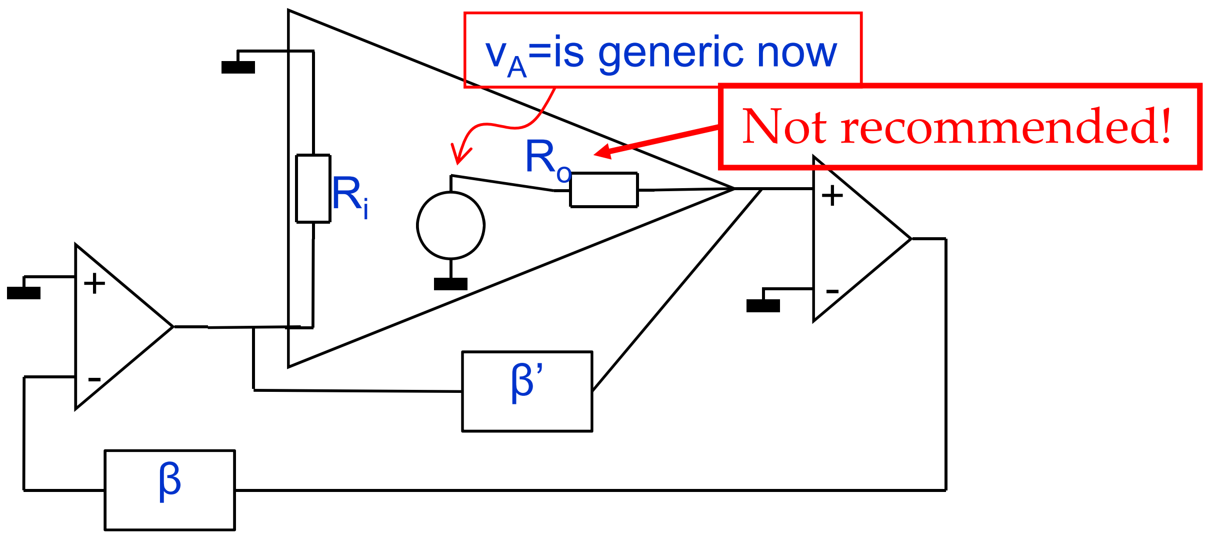
A similar approach can be used when the fed backed OA is a current amplifier, or transimpedance amplifier or transconducatnce amplifier.

Loop gain evaluation of cascaded structures 1

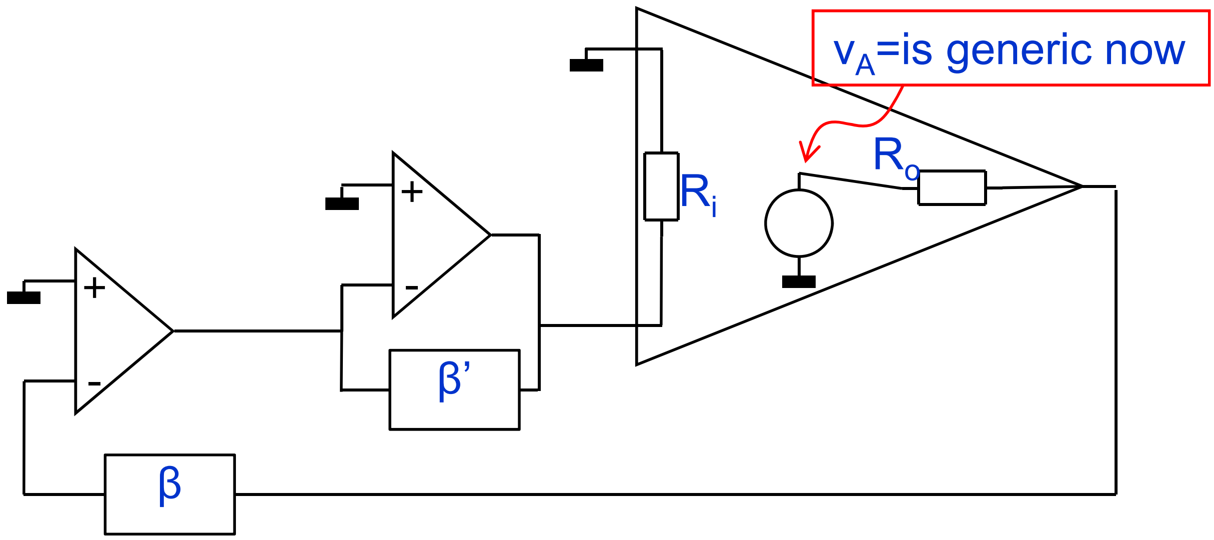
Sometimes the fed backed amplifier is composed by the cascade of several amplifiers. In this case there are 2 possible configurations. The first is when every stage is an open loop stage itself. In this case there is no need for a particular precaution in choosing the source where to open the loop and find the loop gain. Here an example:



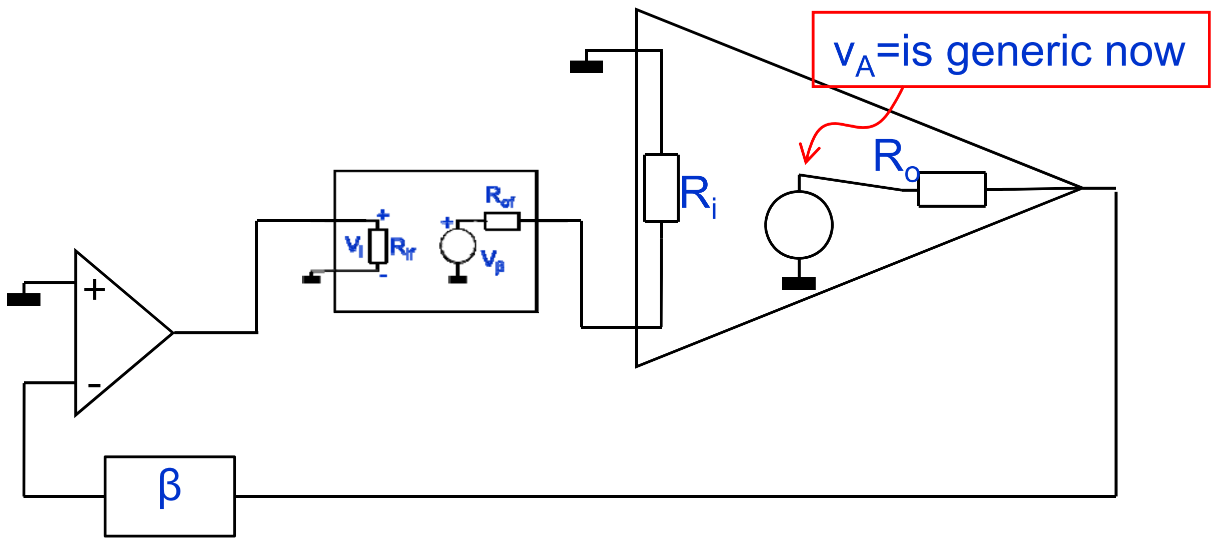
Some precautions are instead to be taken when inside the loop there is one or more closed loop amplifier. In this case it is not recommended to make the choose of the source to be generic ate the nested amplifier as being that amplifier subjected to a double feedback is not obvious the interpretation of the result:



To avoid any possible misleading it is a good choice to make generic a source at a stage not fed backed, first:



A further, useful step is to replace the inner local closed loop amplifier(s) with its model:



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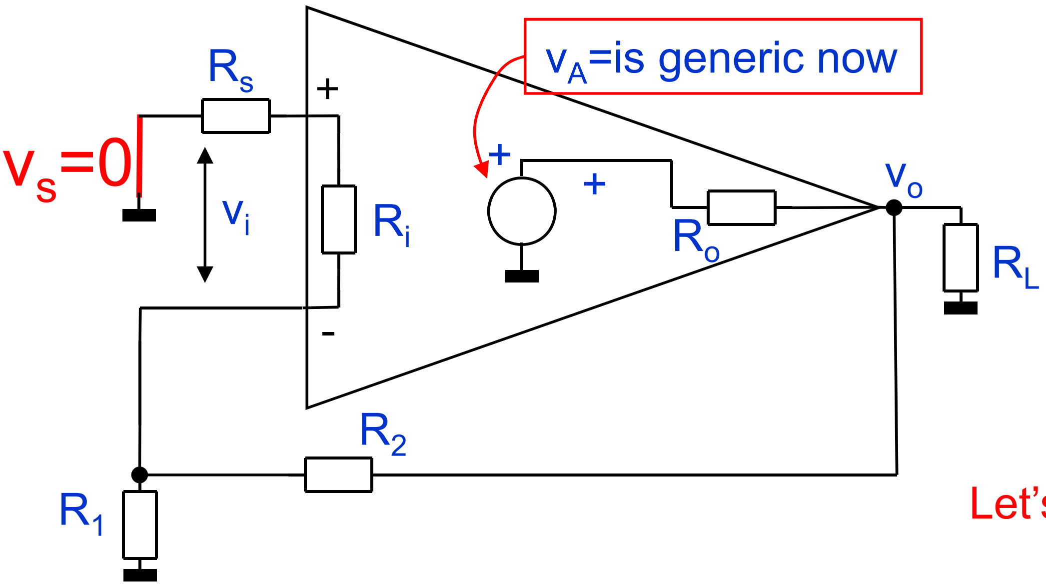
Appendix A (1)

The aim here is to measure the input impedance R_{if} and the output impedance R_{Of} of the below network, for which we already studied the gain:

$A=10^4$, $R_i=10^6$, $R_s=R_O=100\ \Omega$, $R_1=1000\ \Omega$, $R_2=9\ K\Omega$, $R_L=200\ \Omega$.



Let's study T.

Let's write the node equations:

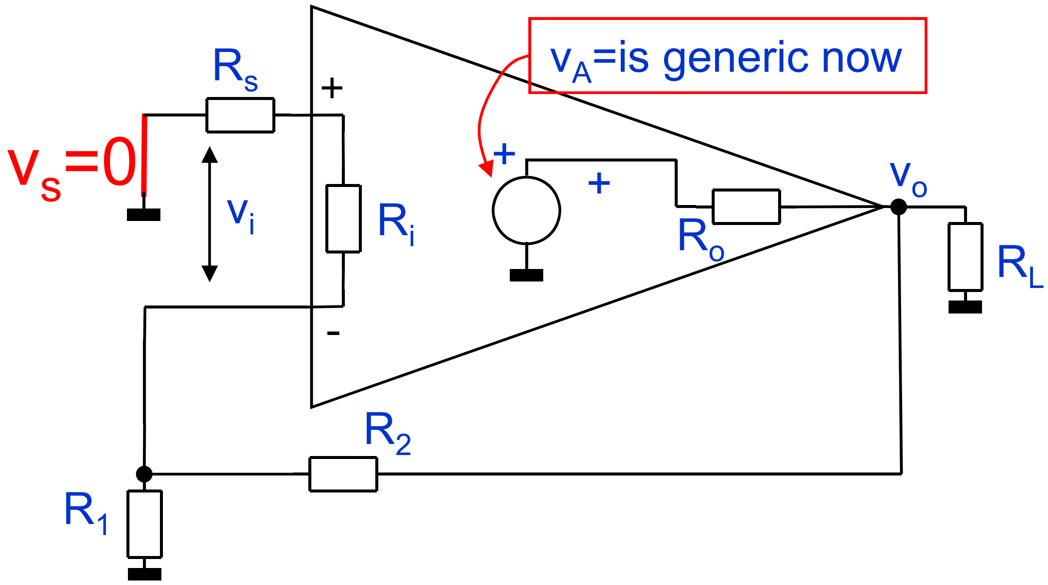
$$\begin{cases} \dfrac{v_A - v_o}{R_o} = \dfrac{v_o}{R_L} + \dfrac{v_o - v_-}{R_2} \\ \dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1} + \dfrac{v_-}{R_s + R_i} \\ v_i = v_+ - v_- \end{cases} \Rightarrow \begin{cases} \dfrac{v_A}{R_o} = \left(\dfrac{1}{R_o} + \dfrac{1}{R_L} + \dfrac{1}{R_2}\right) v_o - \dfrac{v_-}{R_2} \\ \dfrac{v_o}{R_2} = \left(\dfrac{1}{R_2} + \dfrac{1}{R_1} + \dfrac{1}{R_s + R_i}\right) v_- \\ v_i = v_+ - v_- \end{cases}$$

With:

$R_{1'}=R_1||(R_s+R_i)=999\ \Omega$

$$\begin{cases} \dfrac{v_A}{R_o} = \left(\dfrac{1}{R_o} + \dfrac{1}{R_L} + \dfrac{1}{R_2}\right) v_o - \dfrac{v_-}{R_2} \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = v_+ - v_- \end{cases}$$

Appendix A (2)



$$\begin{cases} \dfrac{v_A}{R_o} = \left(\dfrac{1}{R_o} + \dfrac{1}{R_L} + \dfrac{1}{R_2}\right) v_o - \dfrac{v_-}{R_2} \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = v_+ - v_- \end{cases} \Rightarrow \begin{cases} \dfrac{v_A}{R_o} = \left(\dfrac{1}{R_o} + \dfrac{1}{R_L} + \dfrac{1}{R_2}\left(1 - \dfrac{R_{1'}}{R_2 + R_{1'}}\right)\right) v_o \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = v_+ - v_- \end{cases}$$

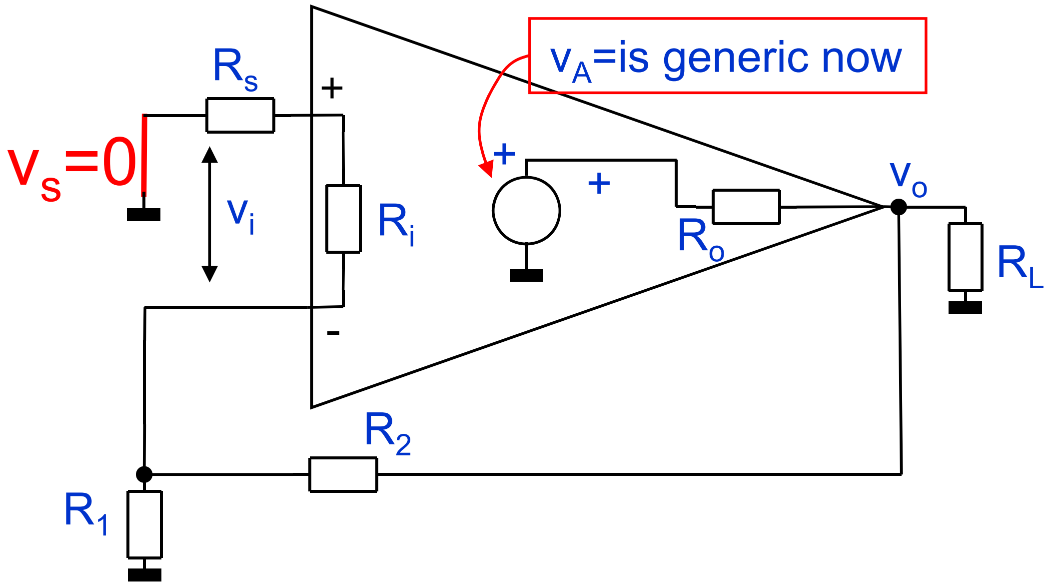
$$\begin{cases} \dfrac{v_A}{R_o} = \left(\dfrac{1}{R_o} + \dfrac{1}{R_L} + \dfrac{1}{R_2 + R_{1'}}\right) v_o \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = v_+ - v_- \end{cases}$$

In addition:

$R_{1'}=R_1||(R_s+R_i)=999\ \Omega$ e
$R_E=R_L||(R_2+R_{1'})$.

$$\begin{cases} \dfrac{v_A}{R_o} = \left(\dfrac{1}{R_o} + \dfrac{1}{R_E}\right) v_o \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = v_+ - v_- \end{cases} \qquad \begin{cases} v_o = \dfrac{R_E}{R_E + R_o} v_A \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = v_+ - v_- \end{cases}$$

Appendix A (3)



$$\begin{cases} v_o = \dfrac{R_E}{R_E + R_o} v_A \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = v_+ - v_- \end{cases} \Rightarrow \begin{cases} v_o = \dfrac{R_E}{R_E + R_o} v_A \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = \dfrac{R_s}{R_s + R_i} v_- - v_- \end{cases} \Rightarrow \begin{cases} v_o = \dfrac{R_E}{R_E + R_o} v_A \\ v_- = \dfrac{R_{1'}}{R_2 + R_{1'}} v_o \\ v_i = -\dfrac{R_i}{R_s + R_i} v_- \end{cases}$$

Therefore:

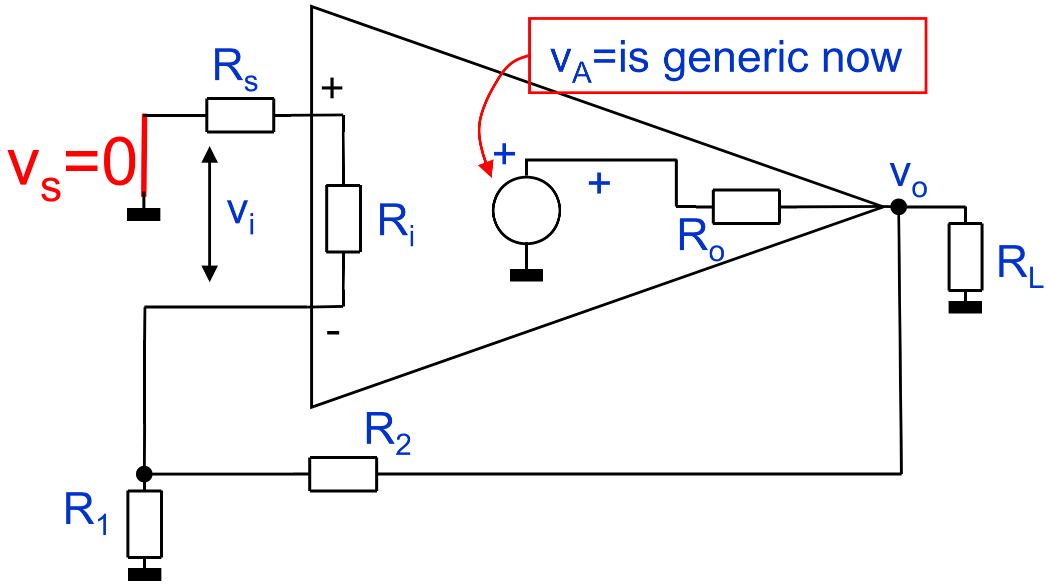
$$v_+ - v_- = -\frac{R_i}{R_s + R_i} \frac{1'_{R_1'+R_2}}{R_{1'} + R_2} \frac{R_E}{R_E + R_o} v_A = \beta f(\beta) v_A$$

And then:

$$T = \frac{A(v_+ - v_-)}{v_A} = -A \frac{R_i}{R_s + R_i} \frac{R_{1'}}{R_{1'} + R_2} \frac{R_E}{R_E + R_o} = -A_R \beta = -661.6$$

$$A_f = \frac{1}{\beta} \frac{-T}{1-T} = \frac{1}{\beta} 0.998$$ With an error of the order of 0.2%

Appendix A (4)



It is interesting to verify which values takes $T_{0\infty}$:

The effect of R_s on T is marginal when becomes negligible, as

$R_{1'}=R_1||(R_s+R_i)=999\ \Omega$ and $R_{1'0}=R_1||(R_i)=999\ \Omega$.

Different is the effect of R_L when it takes very large values:

$R_E=R_L||(R_2+R_{1'})= 196\ \Omega$, while $R_{E\infty}= R_2+R_{1'}=9999\ \Omega$.

The loop gain T, from the value of -661.6, becomes, after that R_s is set to 0 and R_L to ∞:

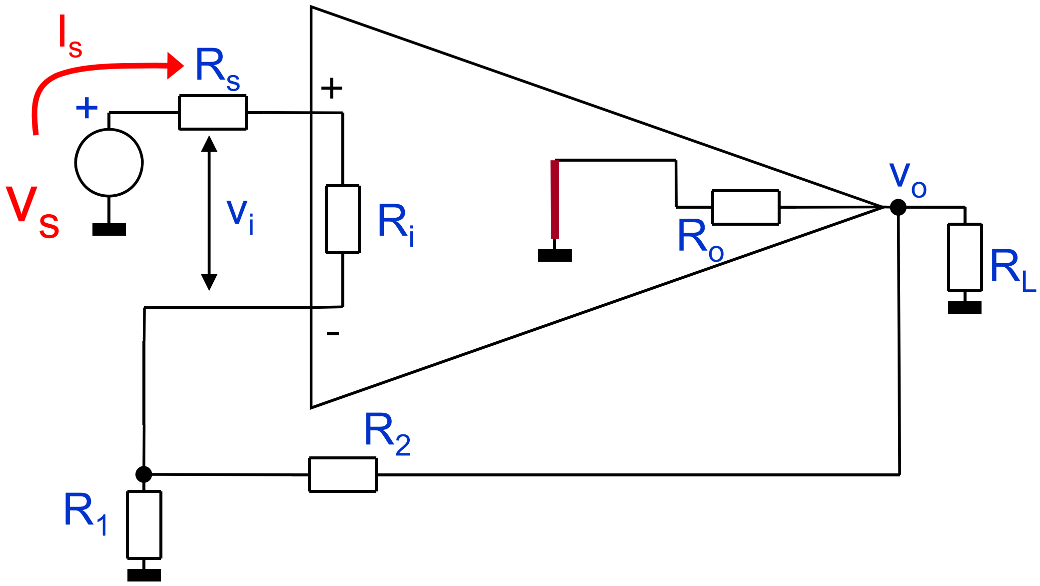
$$T = -A\frac{R_{1'0}}{R_{1'0}+R_2}\frac{R_{E\infty}}{R_{E\infty}+R_o} = -989.2$$

The factor –T/(1-T) from the value 0.998 now takes:

$$\frac{T_{0\infty}}{1-T_{0\infty}} = 0.999$$

Appendix A (5)

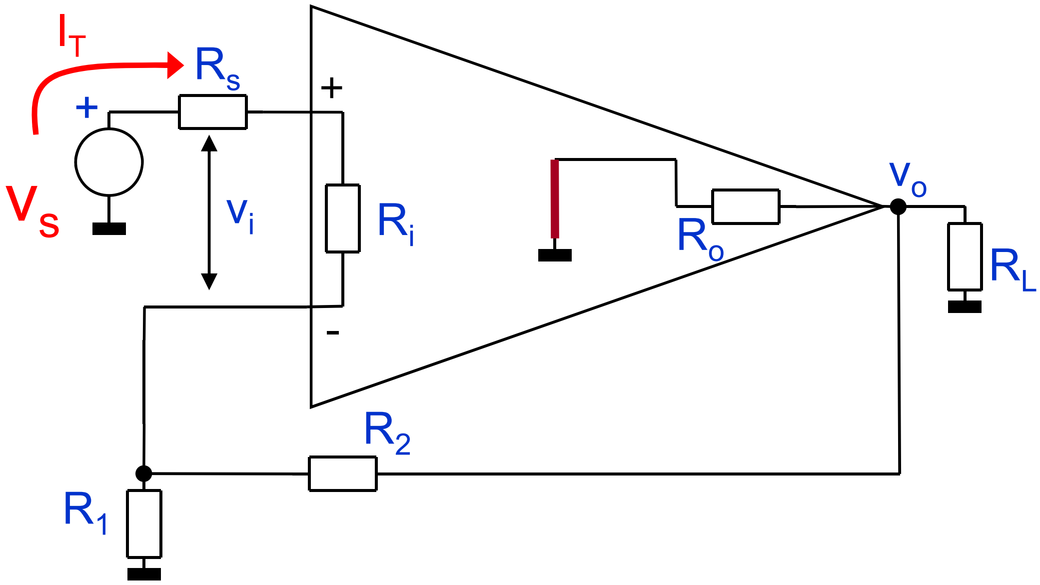
Input impedance is determined starting by nulling the gain and measuring the input current consequent to a source excitation:



Input current I_s comes from:

$$\begin{cases} \dfrac{v_s - v_-}{R_s + R_i} = \dfrac{v_-}{R_1} + \dfrac{v_- - v_o}{R_2} \\ \dfrac{v_- - v_o}{R_2} = \dfrac{v_o}{R_L} + \dfrac{v_o}{R_o} \\ I_s = \dfrac{v_s - v_-}{R_s + R_i} \end{cases} \Rightarrow \begin{cases} \dfrac{v_s - v_-}{R_s + R_i} = \dfrac{v_-}{R_1} + \dfrac{v_- - v_o}{R_2} \\ \dfrac{v_-}{R_2} = \dfrac{v_o}{R_2} + \dfrac{v_o}{R_L \| R_o} \\ I_s = \dfrac{v_s - v_-}{R_s + R_i} \end{cases}$$

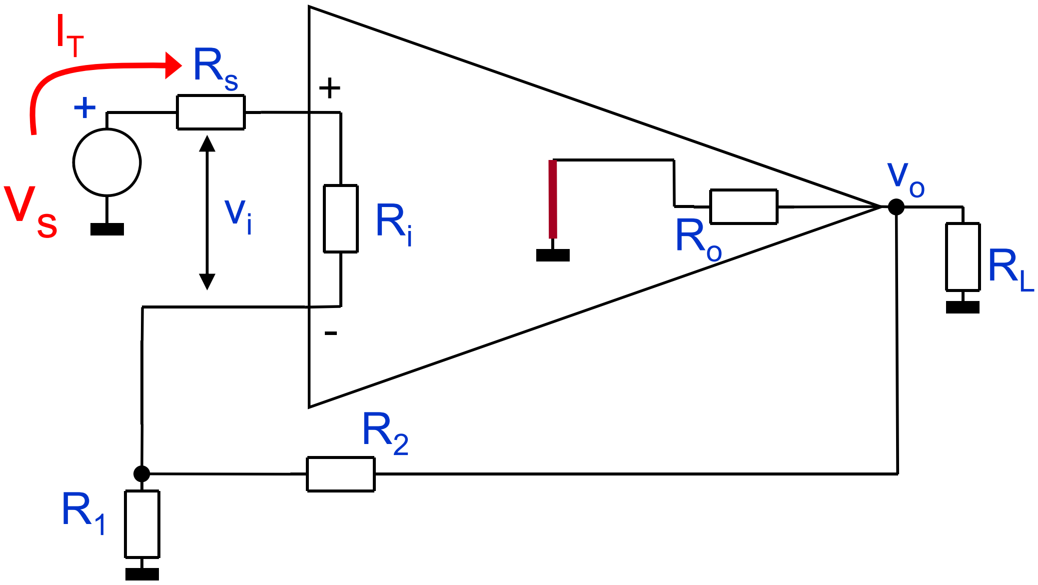
$$\begin{cases} \dfrac{v_s - v_-}{R_s + R_i} = \dfrac{v_-}{R_1} + \dfrac{v_- - v_o}{R_2} \\ v_o = \dfrac{R_L \| R_o}{R_2 + R_L \| R_o} v_- \\ I_s = \dfrac{v_s - v_-}{R_s + R_i} \end{cases} \begin{cases} \dfrac{v_s}{R_s + R_i} = \dfrac{v_-}{R_s + R_i} + \dfrac{v_-}{R_1} + \dfrac{v_-}{R_2}\left(1 - \dfrac{R_L \| R_o}{R_2 + R_L \| R_o}\right) \\ v_o = \dfrac{R_L \| R_o}{R_2 + R_L \| R_o} v_- \\ I_s = \dfrac{v_s - v_-}{R_s + R_i} \end{cases}$$



$$\begin{cases} \dfrac{v_s}{R_s + R_i} = \dfrac{v_-}{R_s + R_i} + \dfrac{v_-}{R_1} + \dfrac{v_-}{R_2}\left(1 - \dfrac{R_L \| R_o}{R_2 + R_L \| R_o}\right) \\ v_o = \dfrac{R_L \| R_o}{R_2 + R_L \| R_o} v_- \\ I_s = \dfrac{v_s - v_-}{R_s + R_i} \end{cases}$$

$$\begin{cases} \dfrac{v_s}{R_s + R_i} = \dfrac{v_-}{R_s + R_i} + \dfrac{v_-}{R_1 \| (R_2 + R_L \| R_o)} \\ v_o = \dfrac{R_L \| R_o}{R_2 + R_L \| R_o} v_- \\ I_s = \dfrac{v_s - v_-}{R_s + R_i} \end{cases}$$

$$\begin{cases} v_- = \dfrac{R_1 \| (R_2 + R_L \| R_o)}{R_s + R_i + R_1 \| (R_2 + R_L \| R_o)} v_s \\ v_o = \dfrac{R_L \| R_o}{R_2 + R_L \| R_o} v_- \\ I_s = \dfrac{v_s - v_-}{R_s + R_i} \end{cases}$$



$$\begin{cases} v_- = \dfrac{R_1 \| (R_2 + R_L \| R_o)}{R_s + R_i + R_1 \| (R_2 + R_L \| R_o)} v_s \\ \\ v_o = \dfrac{R_L \| R_o}{R_2 + R_L \| R_o} v_- \\ \\ I_s = \dfrac{v_s - v_-}{R_s + R_i} \end{cases}$$

$$I_s = \frac{1}{R_s + R_i}\left(1 - \frac{R_1 \| (R_2 + R_L \| R_o)}{R_s + R_i + R_1 \| (R_2 + R_L \| R_o)}\right) v_s$$

$$I_s = \frac{1}{R_s + R_i + R_1 \| (R_2 + R_L \| R_o)} v_s$$

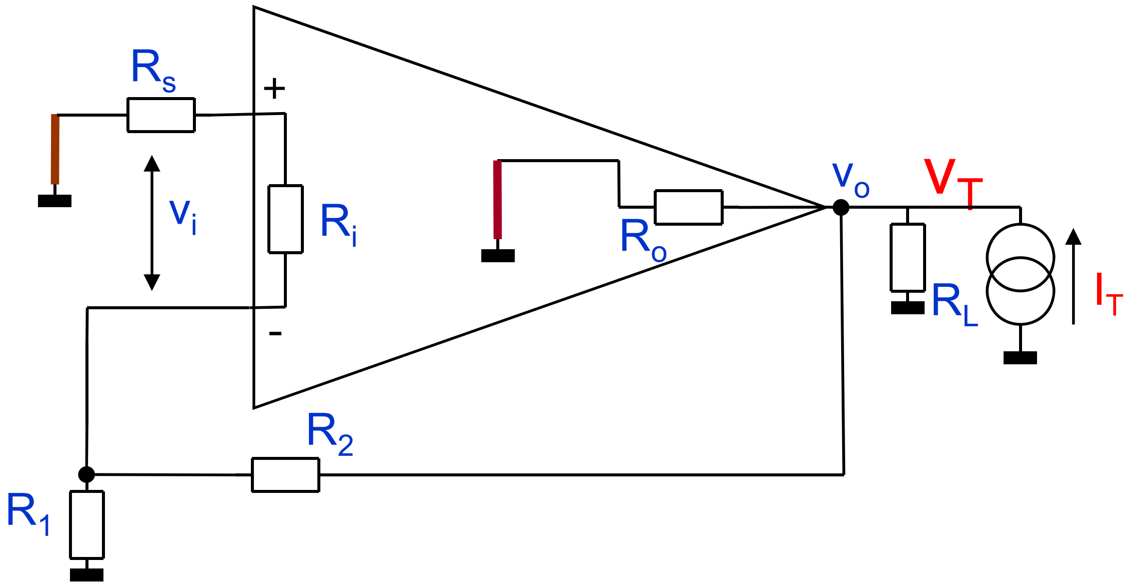
Therefore:

$$R_{ir} = \frac{v_s}{I_s} = R_s + R_i + R_1 \| (R_2 + R_L \| R_o) \approx 1.0012 \times 10^6\ \Omega$$

And::

$$R_{if} = [R_s + R_i + R_1 \| (R_2 + R_L \| R_o)]\,(1 - T)$$
$$\approx 1.0012 \times 10^6\ \Omega(1 - T) = 662.7 \times 10^6\ \Omega$$

The output impedance can be evaluate starting by nulling the input and the gain:



$$\begin{cases} i_T = \dfrac{v_T}{R_L} + \dfrac{v_T}{R_o} + \dfrac{v_T - v_-}{R_2} \\ \dfrac{v_T - v_-}{R_2} = \dfrac{v_-}{R_1} + \dfrac{v_-}{R_i + R_s} \end{cases}$$

$$\begin{cases} i_T = \dfrac{v_T}{R_L} + \dfrac{v_T}{R_o} + \dfrac{v_T - v_-}{R_2} \\ \dfrac{v_T}{R_2} = \dfrac{v_-}{R_2} + \dfrac{v_-}{R_{1'}} \end{cases}$$

$R_{1'} = R_1 || (R_i + R_s)$

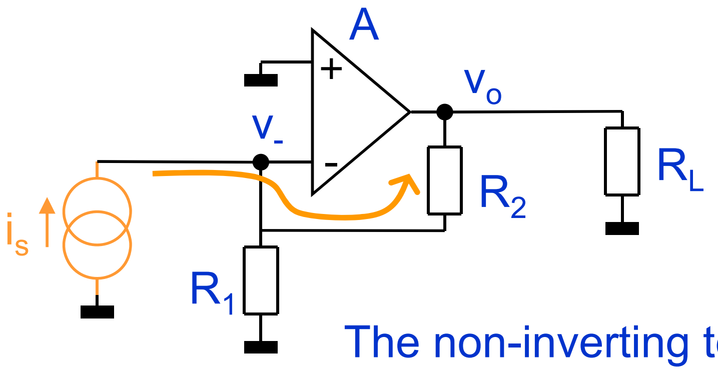
$$\begin{cases} i_T = \dfrac{v_T}{R_L} + \dfrac{v_T}{R_o} + \dfrac{v_T - v_-}{R_2} \\ v_- = \dfrac{R_{1'}}{R_{1'} + R_2} v_T \end{cases}$$

$$\begin{cases} i_T = \dfrac{v_T}{R_L} + \dfrac{v_T}{R_o} + \dfrac{v_T - v_-}{R_2} \\ v_- - v_T = -\dfrac{R_2}{R_{1'} + R_2} v_T \end{cases}$$

$$\begin{cases} i_T = \dfrac{v_T}{R_L} + \dfrac{v_T}{R_o} + \dfrac{v_T}{R_{1'} + R_2} \\ v_- - v_T = -\dfrac{R_2}{R_{1'} + R_2} v_T \end{cases} \Rightarrow R_{ool} = \frac{v_T}{i_T} = R_L \| R_o \| (R_2 + R_1') \approx 66.25\ \Omega$$

and: $R_{of} = \dfrac{R_{ool}}{1 - T} \approx \dfrac{66.25}{661.6} = 0.1\ \Omega$

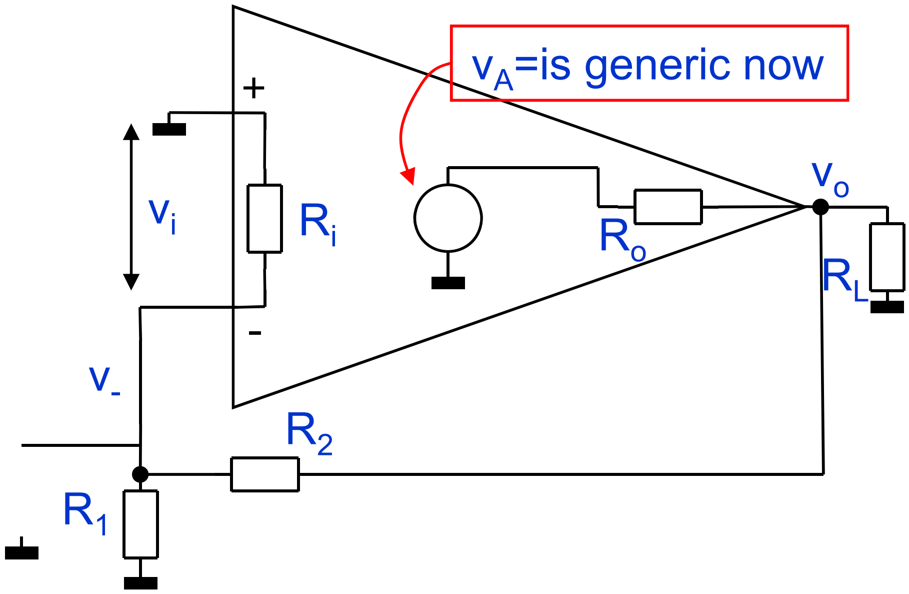
Appendix B (1)



In the circuit on the left we have: A=1000, R_o=100 Ω, R_1=1000 Ω, R_2=4000 Ω, R_L=200 Ω, R_i=10^5 Ω. Determine β, T, A_{OL}, R_{if} and R_{of}.

The non-inverting terminal is at constant potential and we know that the inverting terminal is at the same potential due to the feedback action: therefore we have a current input sensitivity.

Setting A=∞: v_-~0 and: $v_o = -R_2 i_s$ ➡ $\beta = -\dfrac{1}{R_2}$



Start:

$$\begin{cases} \dfrac{v_A - v_o}{R_o} = \dfrac{v_o}{R_L} + \dfrac{v_o - v_-}{R_2} \\ \dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1} + \dfrac{v_-}{R_i} \end{cases}$$

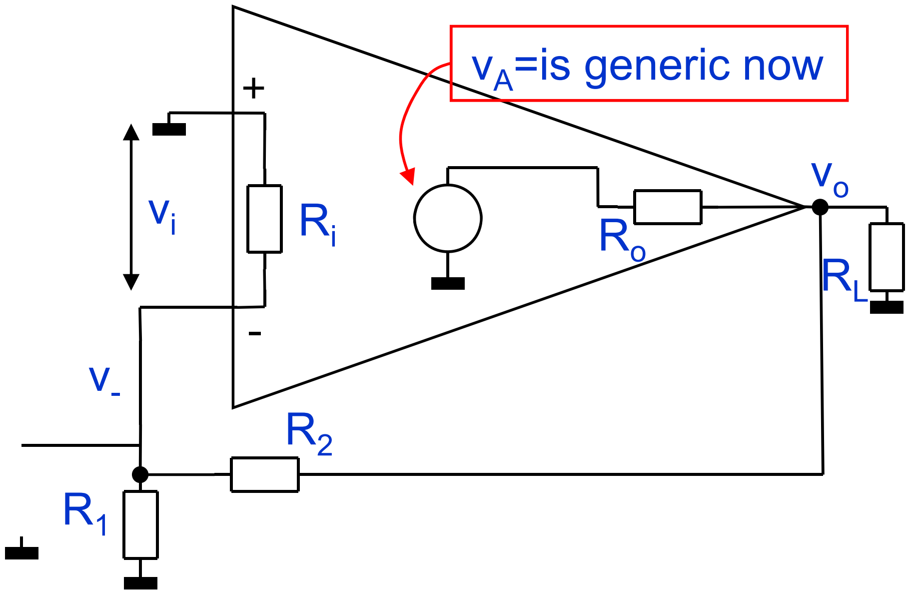
Now set:

$$R_1' = R_1 \| R_i = 990\ \Omega$$

➡

$$\begin{cases} \dfrac{v_A - v_o}{R_o} = \dfrac{v_o}{R_L} + \dfrac{v_o - v_-}{R_2} \\ \dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1'} \end{cases}$$

Appendix B (2)



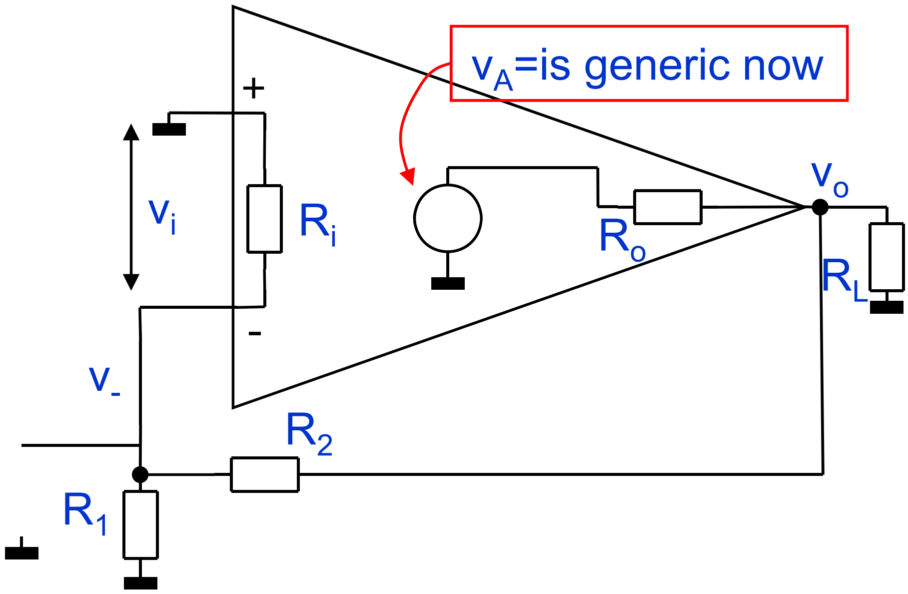
$$\begin{cases} \dfrac{v_A - v_o}{R_o} = \dfrac{v_o}{R_L} + \dfrac{v_o - v_-}{R_2} \\ \dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1'} \end{cases}$$

$$\begin{cases} \dfrac{v_A}{R_o} = \dfrac{v_o}{R_o} + \dfrac{v_o}{R_L} + \dfrac{v_-}{R_1'} \\ v_o = v_- + \dfrac{R_2}{R_1'} v_- \end{cases}$$

$$\begin{cases} \dfrac{v_A}{R_o} = \dfrac{R_L + R_o}{R_L R_o} \dfrac{R_1' + R_2}{R_1'} v_- + \dfrac{v_-}{R_1'} \\ v_o = \dfrac{R_1' + R_2}{R_1'} v_- \end{cases}$$

$$\begin{cases} v_A = \dfrac{(R_L + R_o)(R_1' + R_2) + R_L R_o}{R_L R_1'} v_- \\ v_o = \dfrac{R_1' + R_2}{R_1'} v_- \end{cases}$$

Appendix B (3)



$$\begin{cases} v_A = \dfrac{(R_L + R_o)(R_1' + R_2) + R_L R_o}{R_L R_1'} v_- \\ \\ v_o = \dfrac{R_1' + R_2}{R_1'} v_- \end{cases}$$

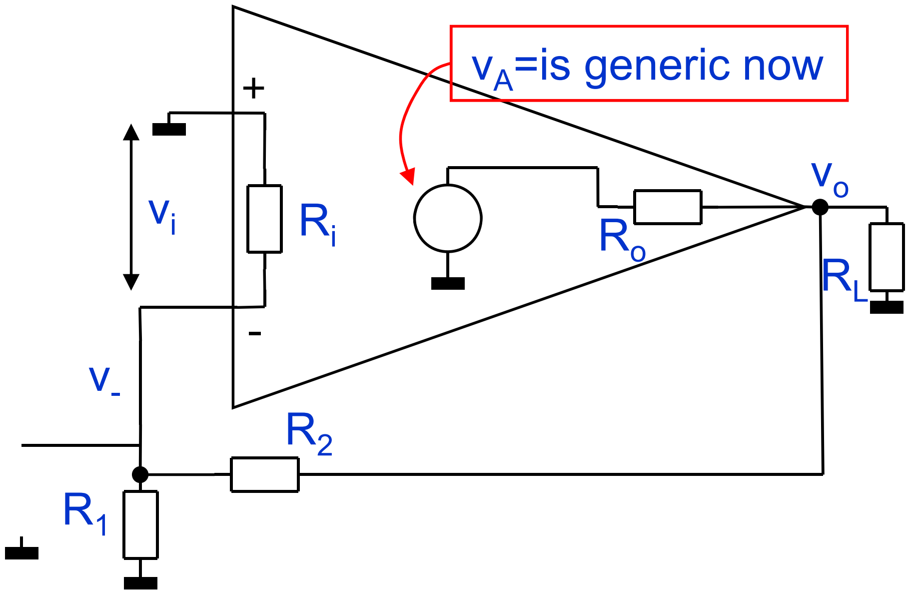
$$\begin{cases} v_A = \dfrac{R_L(R_1' + R_2) + (R_1' + R_2 + R_L) R_o}{R_L R_1'} v_- \\ \\ v_o = \dfrac{R_1' + R_2}{R_1'} v_- \end{cases}$$

With:

$$R_E = R_L \| (R_2 + R_1') = 192.3\ \Omega$$

$$\begin{cases} v_A = (R_1' + R_2) \dfrac{1 + R_o/R_E}{R_1'} v_- \\ \\ v_o = \dfrac{R_1' + R_2}{R_1'} v_- \end{cases}$$

Appendix B (4)



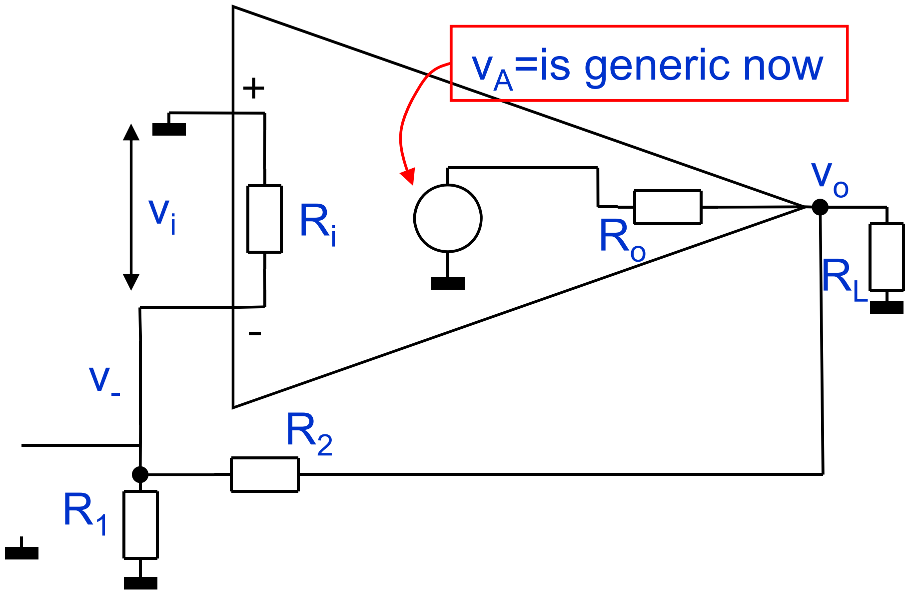
$$\begin{cases} v_A = (R_1' + R_2)\dfrac{1 + R_o/R_E}{R_1'} v_- \\ v_o = \dfrac{R_1' + R_2}{R_1'} v_- \end{cases}$$

$$\begin{cases} v_A = \dfrac{R_1' + R_2}{R_1'}\dfrac{R_E + R_o}{R_E} v_- \\ v_o = \dfrac{R_1' + R_2}{R_1'} v_- \end{cases}$$

Or:

$$v_- = \frac{R_1'}{R_1' + R_2}\frac{R_E}{R_E + R_o} v_A$$

Appendix B (5)



$$v_- = \frac{R_1'}{R_1' + R_2}\frac{R_E}{R_E + R_o} v_A$$

Then:

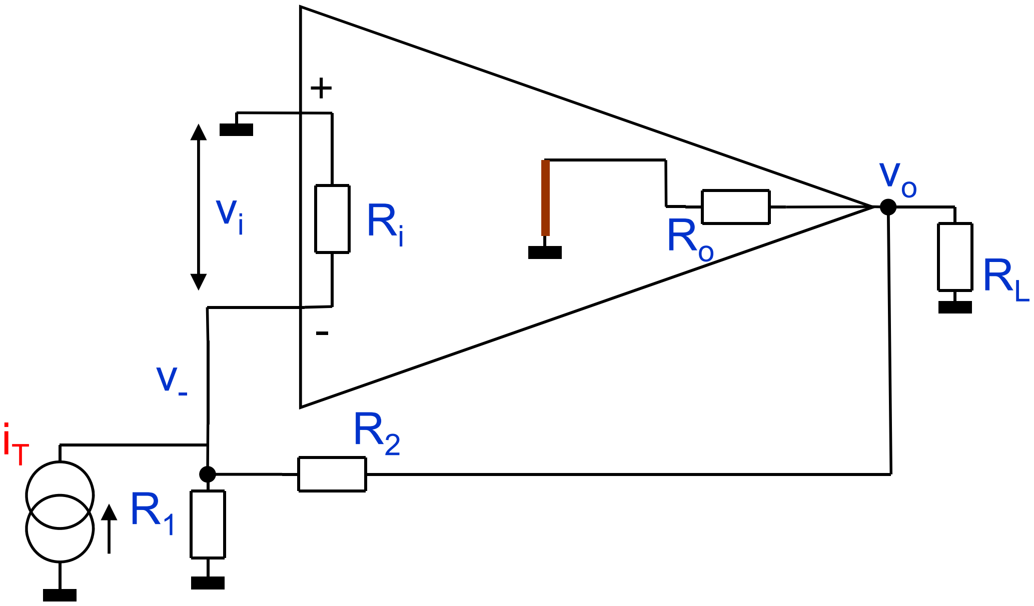
$$T = -A\frac{R_1'}{R_1' + R_2}\ \frac{R_E}{R_E + Ro} = -130.53$$

The closed loop gain, or 1/β, does not depends on R_1, but R_1 takes part in the loop gain T, lowering its absolute value.

In this configuration R_1 is not necessary and worse the loop gain and should be avoided. Putting $R_1=\infty$ we obtain:

$$T = -A\frac{R_i}{R_i + R_2}\ \frac{R_E}{R_E + Ro} = -640.6$$

Appendix B (6)



$$\begin{cases} i_T = \frac{v_-}{R_i} + \frac{v_-}{R_1} + \frac{v_- - v_o}{R_2} \\ \frac{v_- - v_o}{R_2} = \frac{v_o}{R_L} + \frac{v_o}{R_o} \\ R_{iol} = \frac{v_-}{i_T} \end{cases}$$

$$\begin{cases} i_T = \frac{v_-}{R_i} + \frac{v_-}{R_1} + \frac{v_- - v_o}{R_2} \\ \frac{v_-}{R_2} = \frac{v_o}{R_2} + \frac{v_o}{R_L \| R_o} \\ R_{iol} = \frac{v_-}{i_T} \end{cases}$$

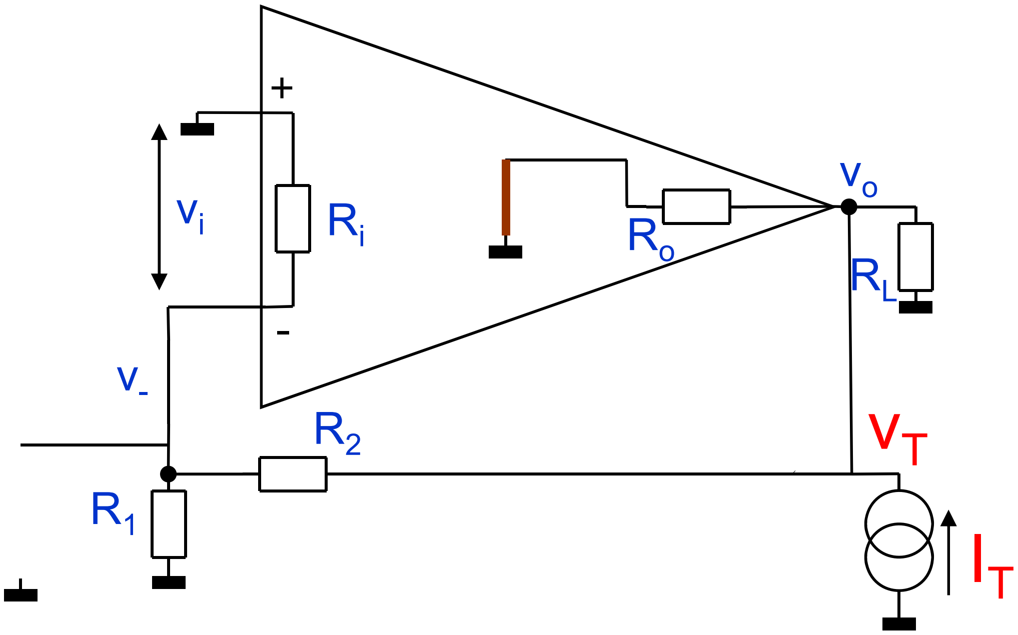
Solving:

$$R_{iol} = R_1 \| R_i \| (R_2 + R_L \| R_o) = 796.24\ \Omega$$

and:

$$R_{if} = \frac{R_{iol}}{1 - T} = 6.1\ \Omega$$

Appendix B (7)



$$\begin{cases} i_T = \dfrac{v_T}{R_L} + \dfrac{v_T}{R_o} + \dfrac{v_T - v_-}{R_2} \\ \dfrac{v_T - v_-}{R_2} = \dfrac{v_-}{R_1} + \dfrac{v_-}{R_i} \\ R_{ool} = \dfrac{v_T}{i_T} \end{cases}$$

$$\begin{cases} i_T = \dfrac{v_T}{R_L} + \dfrac{v_T}{R_o} + \dfrac{v_T - v_-}{R_2} \\ \dfrac{v_T}{R_2} = \dfrac{v_-}{R_2} + \dfrac{v_-}{R_1 \| R_i} \\ R_{ool} = \dfrac{v_T}{i_T} \end{cases}$$

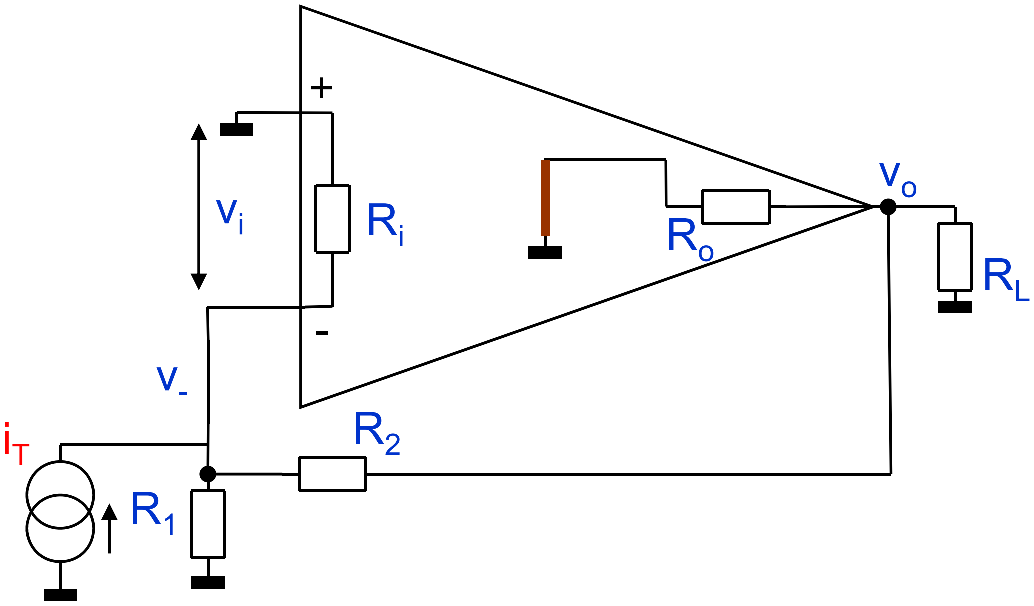
Solving:

$$R_{ool} = R_L \| R_o \| (R_2 + R_1 \| R_i) = 65.8\ \Omega$$

and:

$$R_{of} = \frac{R_{ool}}{1 - T} = 0.5\ \Omega$$

Appendix B (8)



Direct transmission:

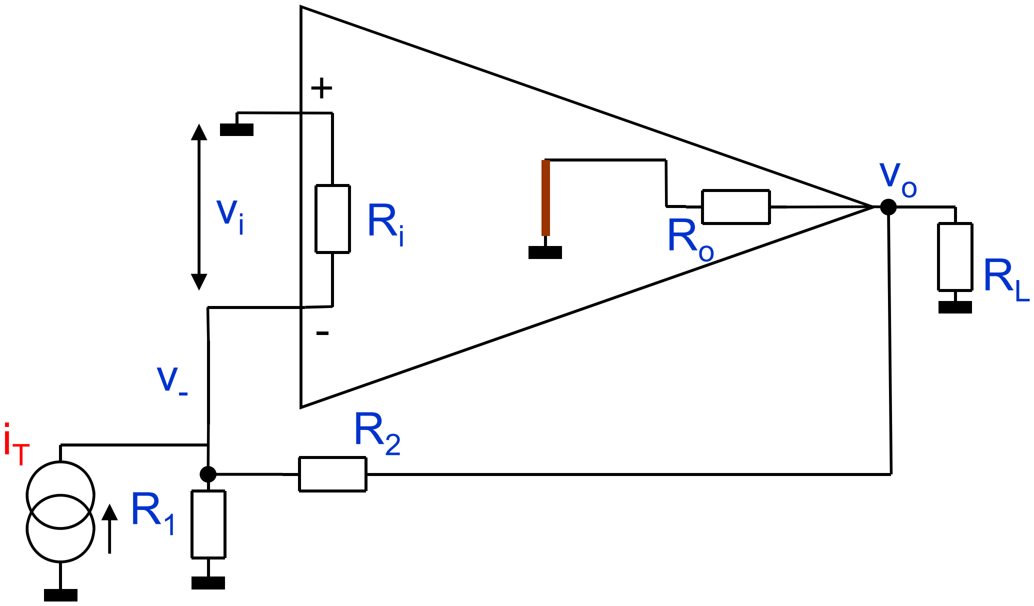
$$\begin{cases} i_T = \dfrac{v_-}{R_i} + \dfrac{v_-}{R_1} + \dfrac{v_- - v_o}{R_2} \\ \dfrac{v_- - v_o}{R_2} = \dfrac{v_o}{R_L} + \dfrac{v_o}{R_o} \end{cases}$$

$$\begin{cases} i_T = \dfrac{v_-}{R_i} + \dfrac{v_-}{R_1} + \dfrac{v_- - v_o}{R_2} \\ \dfrac{v_-}{R_2} = \dfrac{v_o}{R_2} + \dfrac{v_o}{R_L\|R_o} \end{cases}$$

$$\begin{cases} i_T = \dfrac{v_-}{R_1'} + \dfrac{v_- - v_o}{R_2} \\ v_- = \dfrac{R_L\|R_o + R_2}{R_L\|R_o} v_o \end{cases}$$

$$\begin{cases} i_T = \dfrac{R_1' + R_2}{R_2 R_1'} \dfrac{R_L\|R_o + R_2}{R_L\|R_o} v_o - \dfrac{v_o}{R_2} \\ v_- = \dfrac{R_L\|R_o + R_2}{R_L\|R_o} v_o \end{cases}$$

$$\begin{cases} i_T = \dfrac{(R_1' + R_2)(R_L\|R_o + R_2) - R_1' R_L\|R_o}{R_1' R_2 R_L\|R_o} v_o \\ v_- = \dfrac{R_L\|R_o + R_2}{R_L\|R_o} v_o \end{cases}$$

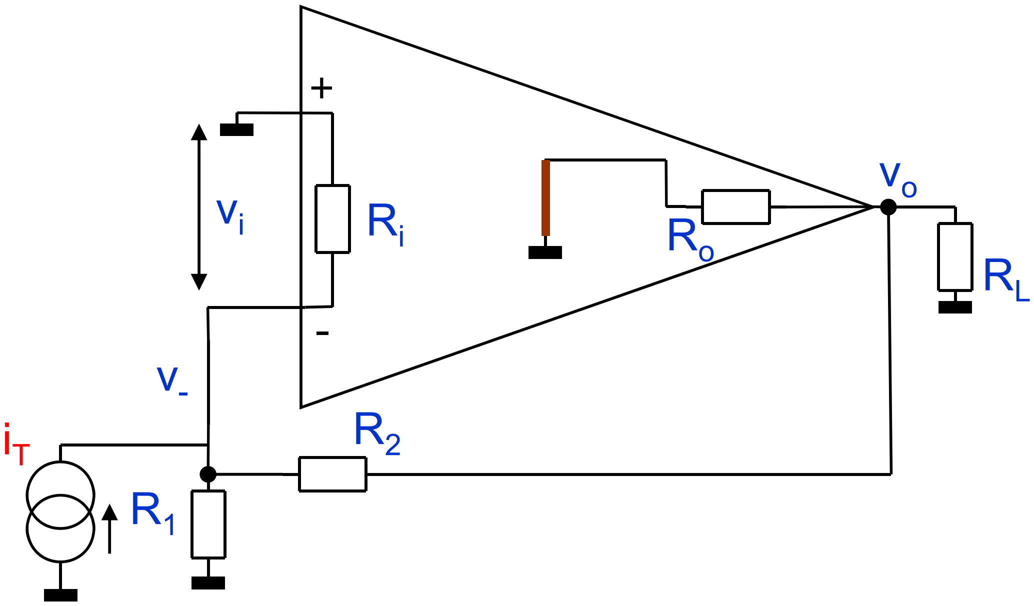


Direct transmission:

$$\begin{cases} i_T = \dfrac{(R_1' + R_2)(R_L \| R_o + R_2) - R_1' R_L \| R_o}{R_1' R_2 R_L \| R_o} v_o \\ v_- = \dfrac{R_L \| R_o + R_2}{R_L \| R_o} v_o \end{cases}$$

$$\begin{cases} i_T = \dfrac{R_1' R_2 + R_2(R_L \| R_o + R_2)}{R_1' R_2 R_L \| R_o} v_o \\ v_- = \dfrac{R_L \| R_o + R_2}{R_L \| R_o} v_o \end{cases}$$

$$\begin{cases} i_T = \dfrac{R_1' + R_L \| R_o + R_2}{R_1' R_L \| R_o} v_o \\ v_- = \dfrac{R_L \| R_o + R_2}{R_L \| R_o} v_o \end{cases}$$



Direct transmission:

$$\begin{cases} i_T = \dfrac{R_1' + R_L \| R_o + R_2}{R_1' R_L \| R_o} v_o \\ \\ v_- = \dfrac{R_L \| R_o + R_2}{R_L \| R_o} v_o \end{cases}$$

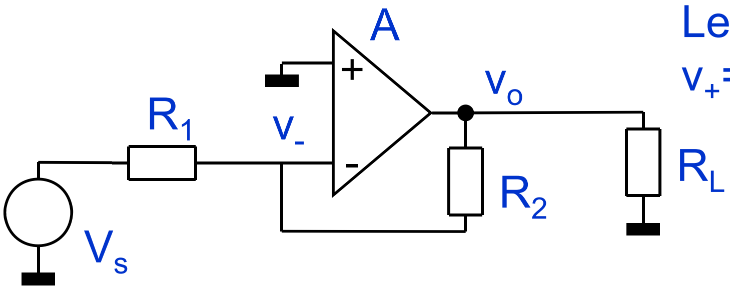
Finally:

$$v_o = \frac{R_1' R_L \| R_o}{R_1' + R_L \| R_o + R_2} i_T$$

$$v_o = \frac{R_1' (R_L \| R_o + R_2)}{R_1' + R_L \| R_o + R_2} \frac{R_L \| R_o}{R_L \| R_o + R_2} i_T$$

$$v_o = R_1' \| (R_L \| R_o + R_2) \frac{R_L \| R_o}{R_L \| R_o + R_2} i_T = A_R i_T$$

Appendix C (1)



Let's assume A=∞. Then, v_-~0, since v_+=0 and:

$$\frac{v_s}{R_1} = -\frac{v_o}{R_2} \Rightarrow v_o = -\frac{R_2}{R_1} v_s$$

This is called inverting configuration and, if R_1=R_2, it results: v_O=-v_s. Note that the gain depends on the impedance in series to the source, so the feedback is sensitive to the input current.

The loop gain is the same as that of the previous example of Appendix B:

$$T = -A \frac{R_1'}{R_1' + R_2} \frac{R_E}{R_E + Ro}$$

$$\left(\begin{aligned} R_1' &= R_1 \| R_i \\ R_E &= R_L \| (R_2 + R_1') \end{aligned} \right)$$

Output impedance is the same as that of Appendix B, as the output networks are similar.

As we saw, the impedance seen from the voltage source satisfies:

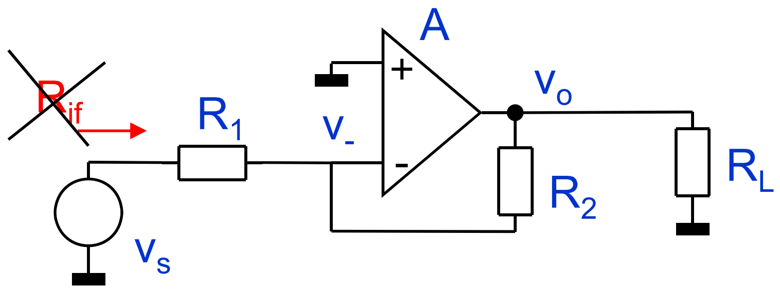
$$R_{ifs} = R_1 + R_{ifp\infty}$$

Being:

$$R_{if\infty} = \lim_{R_1 \to \infty} R_{ifp}$$

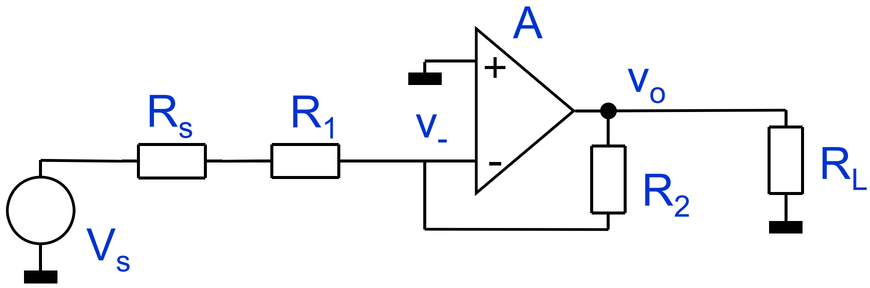
Appendix C (2)

A once-again remark: why it is not possible to evaluate the input impedance starting from the voltage source and then to divide it y (1-T)?.



The node connected to the source v_s is free and not read from the feedback; therefore it can be a whatever value and the current through R_s is dependent on v_s.

Appendix C (3)



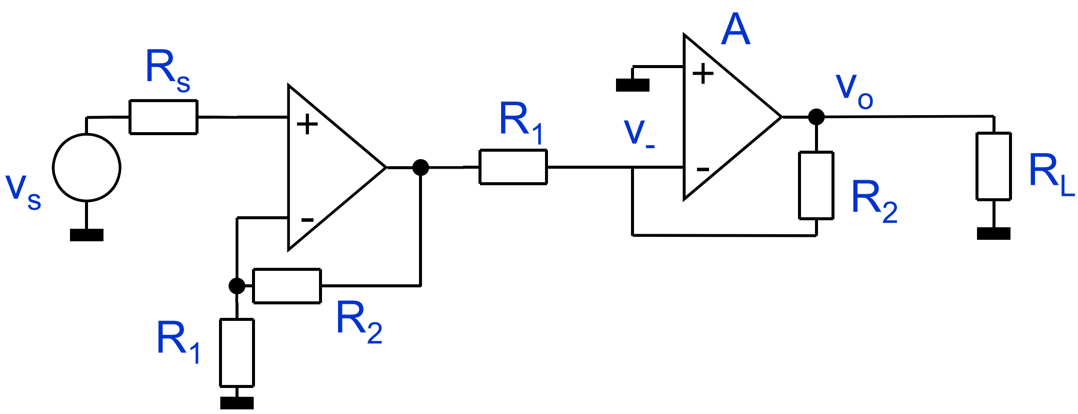
It has to be remarked that with this kind of feedback the connection at the input of a voltage source is unnatural and must be considered with care.

As it can be seen above if the source has a non negligible series impedance R_s its value enters in the gain since it is in series with R_1:

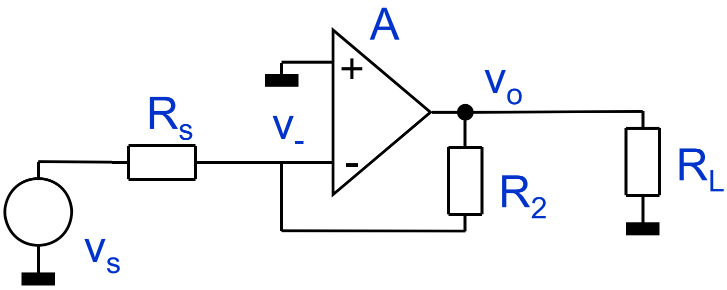
$$v_o = -\frac{R_2}{R_s + R_1} v_s$$

This means that the source impedance must be known with precision or must be negligible.

Normally this setting is used when the stage is driven by a voltage output fed backed stage, with known negligible impedance, as in this example:



Appendix D (1)



Side effect:

Let's suppose that R_1 is the source impedance R_s, namely a resistor that assumes small values, having a zero value in the ideal case.

In this situation the gain is subjected to 2 effects from both β and T:

$$\beta = -\frac{R_s}{R_2} \xrightarrow{R_s \to 0} 0, \quad \text{hence} \quad \frac{1}{\beta} \to \infty$$

$$T = -A \frac{R'_s}{R'_s + R_2} \frac{R_E}{R_E + Ro} \xrightarrow{R_1 \to 0} 0 \quad \left(R'_s = R_s \| R_i\right)$$

So:

$$v_o = -\frac{R_2}{R_s} \frac{-T}{1 - T} v_s \xrightarrow{R_s \to 0} -\frac{R_2}{[0]} \frac{[-0]}{1} v_s$$

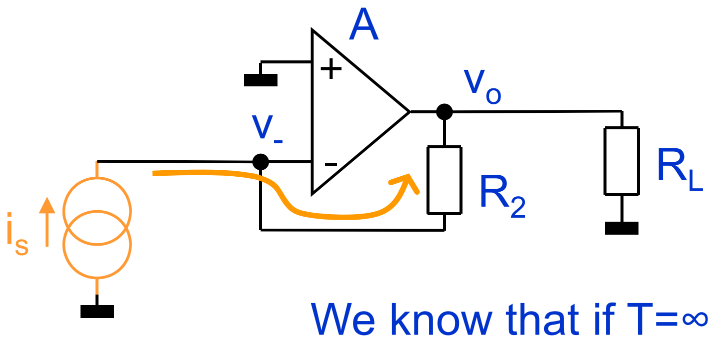
But we know that T=-Af(β)β:

$$v_o = \frac{1}{\beta} \frac{-Af(\beta)\beta}{1 + Af(\beta)\beta} v_s = \frac{-Af(\beta)}{1 + Af(\beta)\beta} v_s \xrightarrow{\beta \to 0} -Af(\beta) v_s$$

$$v_o = -A \frac{R'_s}{R'_s + R_2} \frac{R_E}{R_E + Ro} \frac{R_2}{R_s} v_s \xrightarrow{R_s \to 0} -A \frac{R_E}{R_E + Ro} v_s$$

Therefore, if β→0 the closed loop gain can not increase indefinitely since, at the same time, T →0: the resulting gain is limited by the OA gain and it results as the network is not closed loop.

Appendix D (2)



Another side effect:

Let's suppose to increase the closed loop current gain by increasing and increasing R_2.

We know that if T=∞ we get: $v_o = -R_2 i_s$

Coming back to the case T is not very very large:

$$\beta = -\frac{1}{R_2} \xrightarrow{R_2\to\infty} 0$$

$$T = -A\frac{R_1'}{R_1' + R_2}\frac{R_E}{R_E + Ro} \xrightarrow{R_2\to\infty} 0$$

But:

$$\frac{1}{\beta}\frac{-T}{1-T} \approx \frac{-T}{\beta} = Af(\beta) = A\frac{R_1'}{R_1' + R_2}\frac{R_E}{R_E + Ro}R_2 \xrightarrow{R_2\to\infty} A\frac{R_E}{R_E + Ro}$$

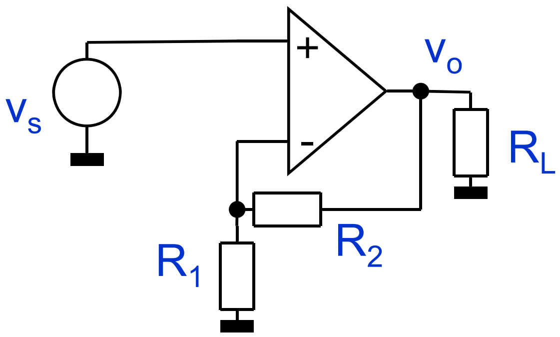
Therefore:

$$v_o = \frac{1}{\beta}\frac{-T}{1-T}i_s = \frac{-T/\beta}{1-T}i_s = \frac{Af(\beta)}{1-T}i_s \xrightarrow{R_2\to\infty} Af(\beta)i_s = -A\frac{R_E}{R_E + Ro}i_s$$

Again the network behaves as it were operating open loop.

To conclude: every time T→0 the circuit behaves as it is in open loop, as it is expected since T is the measure of the loop operation.

Appendix D (3)



A last side effect:

Again, if T=∞ we have:

$$v_o = \frac{R_1 + R_2}{R_1} V_s$$

If now $R_1 \rightarrow 0$ the close loop gain $\rightarrow \infty$, but:

$$\beta = \frac{R_1}{R_1 + R_2} \xrightarrow{R_1 \to 0} 0$$

$$T = -A \frac{R_1'}{R_1' + R_2} \frac{R_E}{R_E + R_o} \xrightarrow{R_1 \to 0} 0$$

But:

$$\frac{T}{\beta} = -Af(\beta) = -A \frac{R_1'}{R_1' + R_2} \frac{R_E}{R_E + Ro} \frac{R_1 + R_2}{R_1} \xrightarrow{R_1 \to 0} -A \frac{R_E}{R_E + Ro}$$

And so:

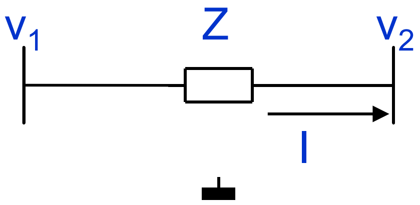
$$v_o = \frac{1}{\beta} \frac{-T}{1 - T} V_s = \frac{-T/\beta}{1 - T} V_s = \frac{Af(\beta)}{1 - T} \xrightarrow{R_1 \to 0} Af(\beta) = A \frac{R_E}{R_E + Ro}$$

Once again the network behaves as it is open loop.

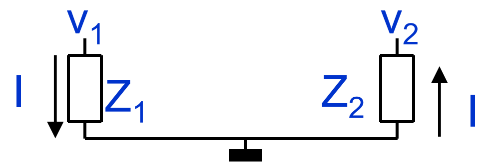
To conclude: every time $T \rightarrow 0$ the circuit behaves as it is in open loop, as it is expected since T is the measure of the loop operation.

Appendix E 1

The Miller effect



We want to show that the simple scheme on this left is equivalent to this:



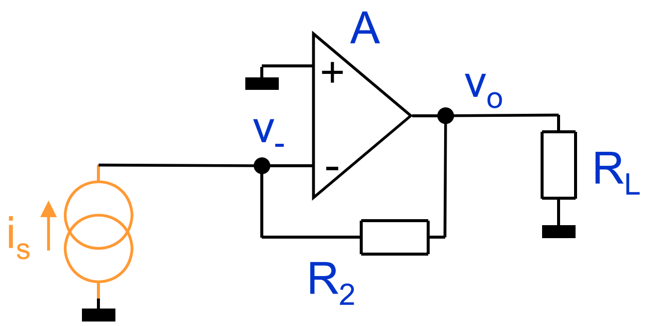
The current I must be equal in both branches: $I = \frac{v_1}{Z_1} = -\frac{v_2}{Z_2} = \frac{v_1 - v_2}{Z}$

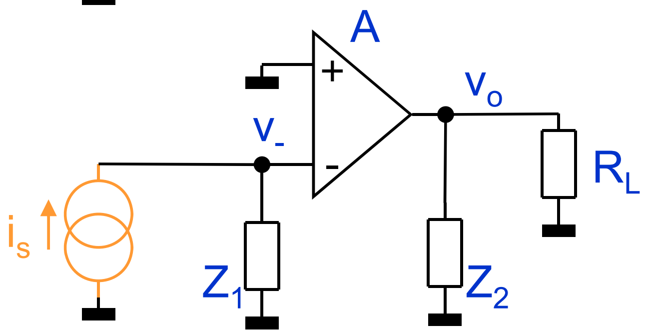
From which:

$$\frac{1}{Z_1} = \frac{1}{Z}\left(1 - \frac{v_2}{v_1}\right), \qquad Z_1 = \frac{Z}{1 - \frac{v_2}{v_1}}$$

$$\frac{1}{Z_2} = \frac{1}{Z}\left(1 - \frac{v_1}{v_2}\right), \qquad Z_2 = \frac{Z}{1 - \frac{v_1}{v_2}}$$

Let's apply the Miller effect to resistor R_2 of the example of Appendix B:





By assuming for a while negligible the output impedance of the OA:

$$\frac{v_O}{v_-} \approx -A$$

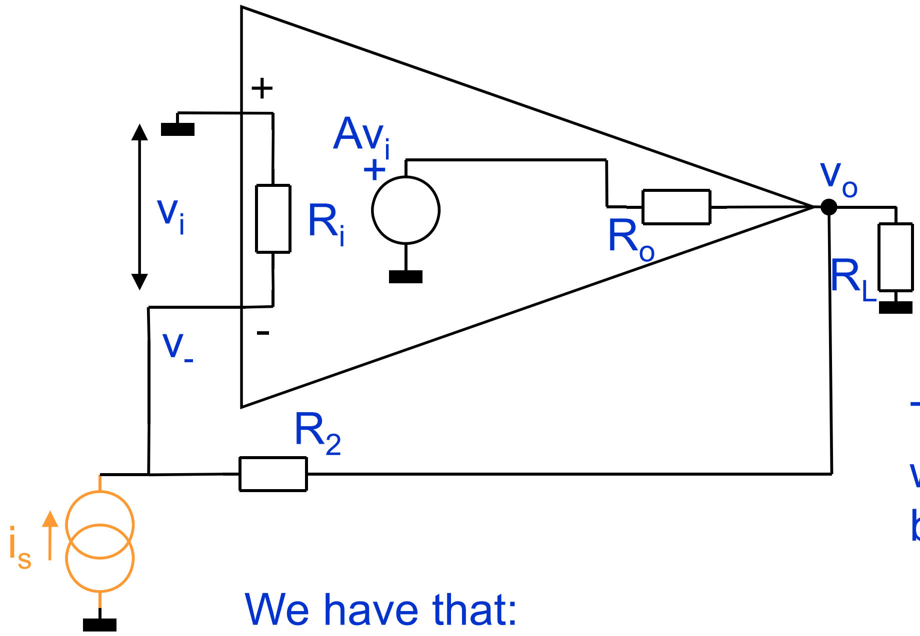
Hence:

$$Z_1 = \frac{R_2}{1 - \frac{v_O}{v_-}} \approx \frac{R_2}{1 + A} = 4\ \Omega$$

And we get the order of magnitude of the expected results

Appendix E 2

The result of the previous page has been obtained with some approximation and we can try now to improve the precision:



Let's assume that v_- is about zero::

$$v_O = -i_S R_2$$

To evaluate the Miller effect we must know the ratio between V_O and V_-.

We have that:

$$\frac{-Av_- - v_O}{R_O} \approx \frac{v_O}{R_2} + \frac{v_O}{R_L}$$

$$\frac{-Av_-}{R_O} = v_O\left(\frac{1}{R_2} + \frac{1}{R_L} + \frac{1}{R_O}\right) = v_O\left(\frac{1}{R_O} + \frac{1}{R_2 \| R_L}\right)$$

$$\frac{-Av_-}{R_O} = v_O\left(\frac{R_2 \| R_L + R_O}{R_O(R_2 \| R_L)}\right) \quad \Rightarrow \quad \frac{v_O}{v_-} = -A\frac{R_2 \| R_L}{R_2 \| R_L + R_O}$$

therefore:

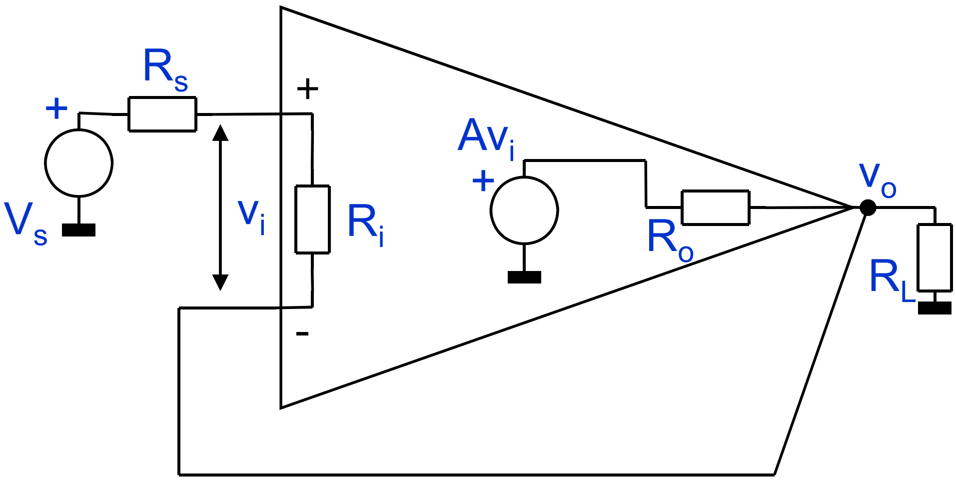
$$Z_1 = \frac{R_2}{1 - \frac{v_O}{v_-}} \approx \frac{R_2}{1 + A\frac{R_L \| R_2}{R_o + R_L \| R_2}} = \frac{4000}{1 + 1000 \times 0.656} = 6.09\ \Omega$$

With a greater precision.

Appendix F (1)

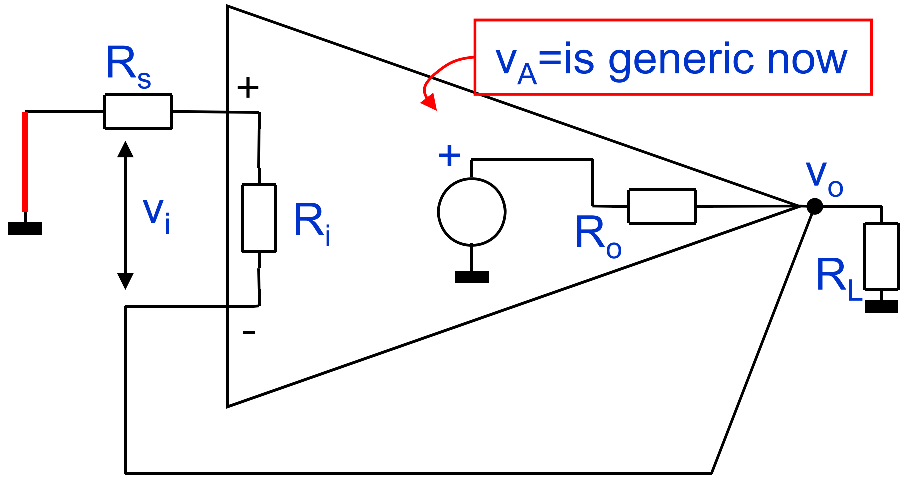
Let's determine now the input and output impedance of the unity gain buffer amplifier:

$A=10^4$, $R_i=10^6$, $R_O=100\ \Omega$, $R_L=200\ \Omega$, $R_s=50\ \Omega$.



By assuming for a while that $A=\infty$ we know that $v_+ \approx v_-$ and:

$v_o=v_s$, namely $\beta=1$.



Let's start with T:

$$V_o = \frac{R'_L}{R'_L + R_o} V_A$$

$$R'_L = R_L || (R_i + R_s)$$

$$= 199{,}96\ \Omega$$

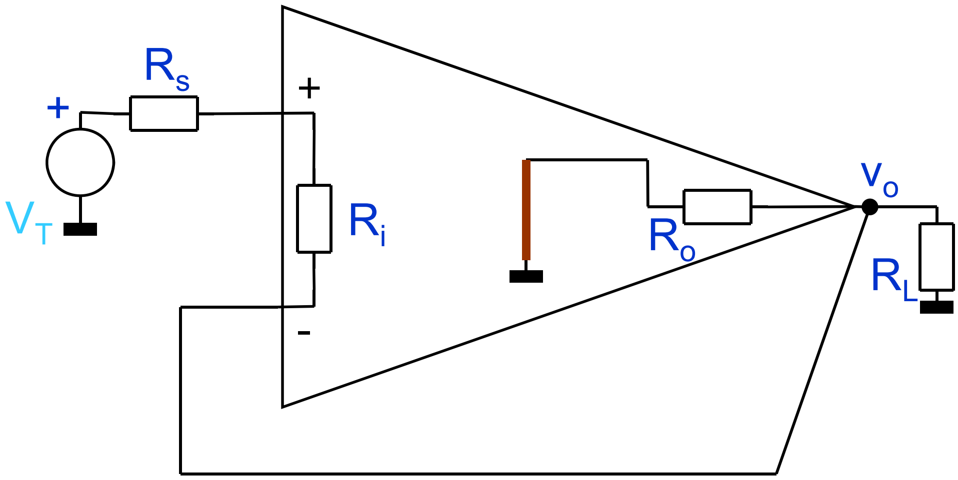
$$V_+ - V_- = -\frac{R_i}{R_s + R_i} V_o$$

Therefore:

$$T = \frac{A(V_+ - V_-)}{V_T} = -\frac{R'_L}{R'_L + R_o}\frac{R_i}{R_s + R_i} A \approx -6666$$

Appendix F (2)

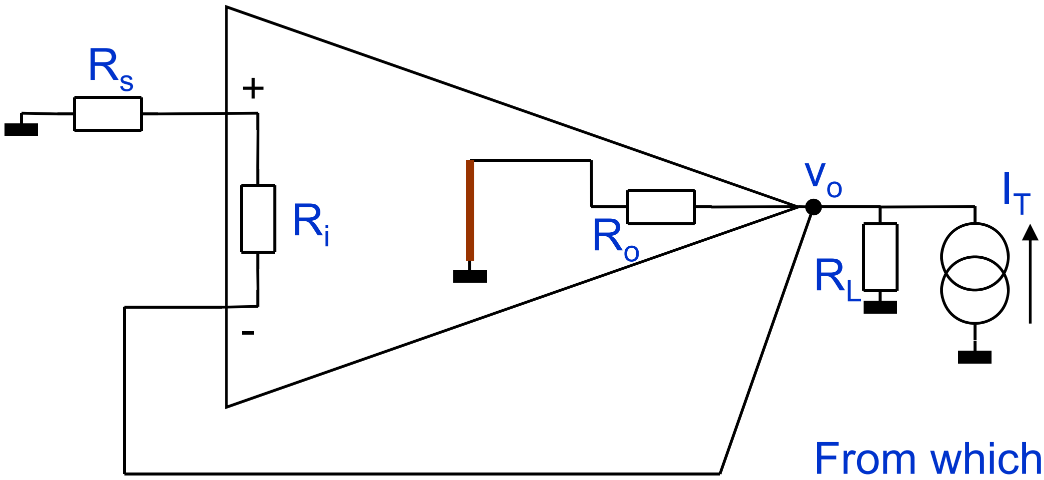
Finally the direct transmission:



From which:

$$R_{if} = R_{ir}(1 - T) = [R_s + R_i + R'_o]\,(1 - T) = 6{,}67\ \mathrm{G\Omega}$$

Finally the output impedance:

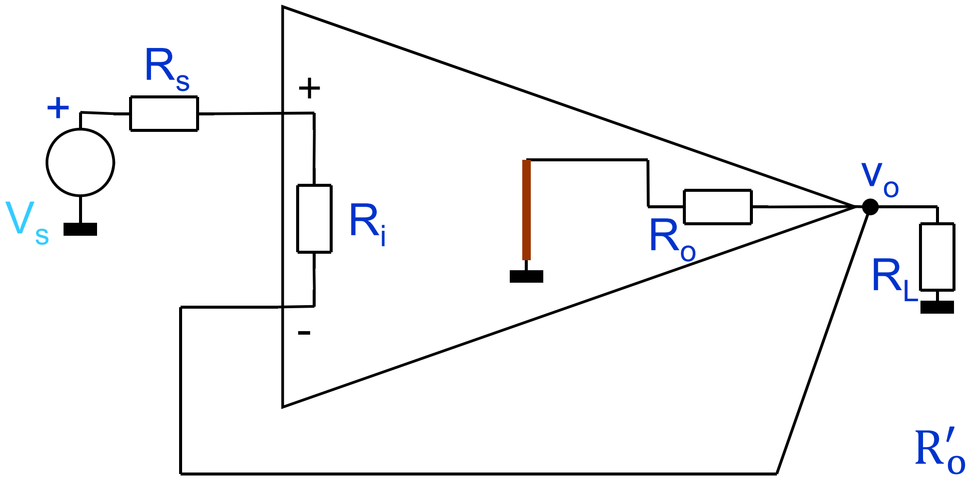


From which:

$$R_{or} = R_L \| R_o \| (R_s + R_i) = 66.66\ \Omega$$

$$R_{of} = \frac{R_L \| R_o \| (R_s + R_i)}{1 - T} \approx 10\ \mathrm{m\Omega}$$

Now the input impedance in open loop condition:



$$R'_o = R_o \| R_L = 66.66\ \Omega$$

It results:

$$v_o = \frac{R'_o}{R'_o + R_i + R_s} v_s = A_R V_s$$

$$v_o \approx \frac{R'_o}{R_i} v_s = 70 \times 10^{-6}\ V_s$$

And, considering the loop closed:

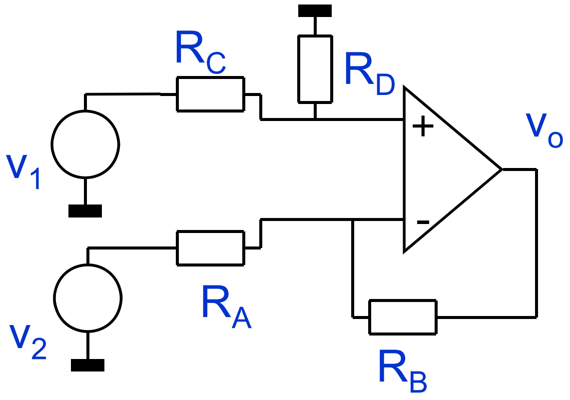
$$v_o = \frac{-T}{1-T} v_s + \frac{A_R}{1-T} v_s$$

$$v_o = \frac{6666}{1+6666} v_s + \frac{70 \times 10^{-6}}{1+6666} v_s$$

$$v_o = 0{,}99985\, v_s + 10{,}5 \times 10^{-9} v_s$$

Appendix G (1)

THE DIFFERENTIAL AMPLIFIER



Let's consider A=∞, from which we know it is valid that $v_+ \approx v_-$, and $i_+ \approx i_- \approx 0$, we get the following eq:

$$\begin{cases} v_+ = \dfrac{R_D}{R_D + R_C} v_1 \\ \dfrac{v_2 - v}{R_A} = \dfrac{v_- - v_o}{R_B} \end{cases}$$

Solving:

$$v_o = \frac{R_A + R_B}{R_A} \frac{R_D}{R_C + R_D} v_1 - \frac{R_B}{R_A} v_2$$

We must arrange the coefficients multiplying v_1 and v_2 to be equal for obtaining a true differential amplification., namely:

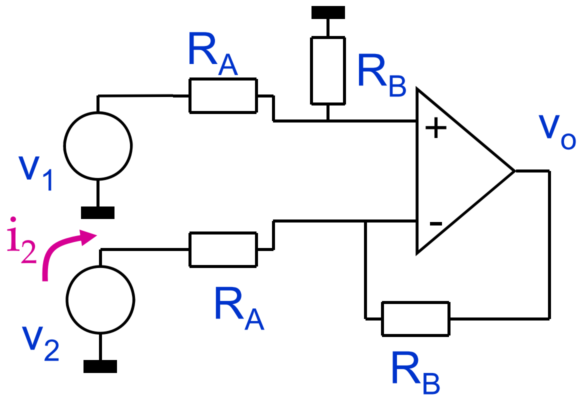
$$\begin{cases} R_D = R_B \\ R_A = R_C \end{cases}$$

From which now it is valid that:

$$v_o = \frac{R_B}{R_A}(v_1 - v_2)$$

Appendix G (2)

THE DIFFERENTIAL AMPLIFIER



The impedances seen from the 2 sources.

The impedance that the source v_1 sees is always R_A+R_B, whatever is v_2, in the approximation that $R_i >> R_A$, R_B.

Source v_2 has a different perspective and:

$$\begin{aligned} v_2 = v_1 &\quad : \quad R_{i2} = R_A + R_B, \\ v_2 = -v_1 &\quad : \quad R_{i2} = R_A \frac{R_A + R_B}{R_A + 2R_B} \\ v_1 = 0 &\quad : \quad R_{i2} = R_A \end{aligned}$$

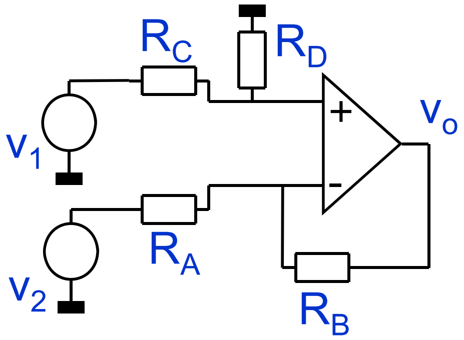
This means that if v_1 and v_2 had different source impedances, namely R_A is slightly different for the 2 sources, the differential gain would not be symmetrical.

If v_1=0 then v_-=0 and: $$i_2 = \frac{v_2}{R_A}$$

If $v_1=v_2$ then $v_- = \frac{R_B}{R_B+R_A} v_2$ and= $$i_2 = \frac{1}{R_A}\left(v_2 - \frac{R_B}{R_B + R_A} v_2\right)$$

If $v_1=-v_2$ then $v_- = -\frac{R_B}{R_B+R_A} v_2$ and: $$i_2 = \frac{1}{R_A}\left(v_2 + \frac{R_B}{R_B + R_A} v_2\right)$$

THE DIFFERENTIAL AMPLIFIER



Let's suppose now that the 4 resistors have not a very precise value. To simplify let's suppose that only inaccurate one is R_B $R_B=R_B(1-\varepsilon)$.

From the general result of the previous page:

$$v_o = \frac{R_A + R_B(1-\varepsilon)}{R_A}\,\frac{R_D}{R_C+R_D}\,v_1 - \frac{R_B(1-\varepsilon)}{R_A}\,v_2$$

$$\overset{R_B=R_D}{=} \frac{R_B}{R_A}\left\{\frac{(R_A+R_B)}{(R_C+R_B)}\,v_1 - \frac{\varepsilon R_B}{R_C+R_B}\,v_1 - (1-\varepsilon)v_2\right\}$$

$$\overset{R_A=R_C}{=} \frac{R_B}{R_A}\left\{v_1 - \frac{\varepsilon R_B}{R_A+R_B}\,v_1 - v_2 + \varepsilon v_2\right\}$$

$$= \frac{R_B}{R_A}\left\{v_1 - v_2 + \varepsilon\left(v_2 - \frac{R_B}{R_A+R_B}\,v_1\right)\right\}$$

If now $v_1=v_2=v_C$:

$$v_o = \varepsilon\frac{R_B}{R_A+R_B}\,v_c$$
$$= A_{cf}v_c$$

In the ideal case of perfect matching:

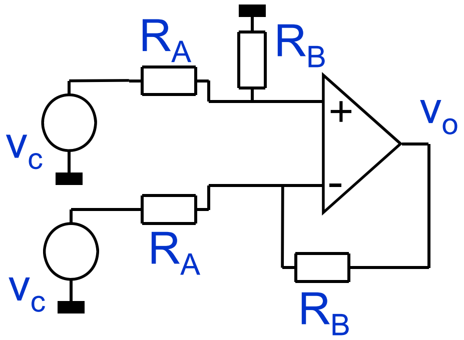
$$v_o = \frac{R_B}{R_A}(v_1 - v_2) = A_{df}(v_1 - v_2)$$

Therefore we can say that the Common Mode Rejection Ratio, CMRR is:

$$CMRR = 20\log_{10}\left(\frac{A_{df}}{A_{cf}}\right)$$

$$= 20\log_{10}\left(\frac{R_B}{R_A}\,\frac{1+R_A/R_B}{\varepsilon}\right) \overset{\substack{A_{df}=10\\ \varepsilon=0.1\%}}{\approx} 81\ \mathrm{dB}$$

THE DIFFERENTIAL AMPLIFIER



Another source of uncertainty is given from the OA itself, since it is not a perfect differential OA. In general it can be written:

$$v_o = A_d(v_+ - v_-) + A_c\left(\frac{v_+ + v_-}{2}\right)$$

Let's not neglect the common mode gain now:

$$\begin{cases} v_+ = \dfrac{R_B}{R_B + R_A} v_c \\[2ex] \dfrac{v_c - v_-}{R_A} = \dfrac{v_- - v_o}{R_B} \quad \Rightarrow \quad v_- = \dfrac{R_B}{R_B + R_A} v_c + \dfrac{R_A}{R_B + R_A} v_o \\[2ex] v_o = \left(A_d + \dfrac{A_c}{2}\right) v_+ - \left(A_d - \dfrac{A_c}{2}\right) v_- \end{cases}$$

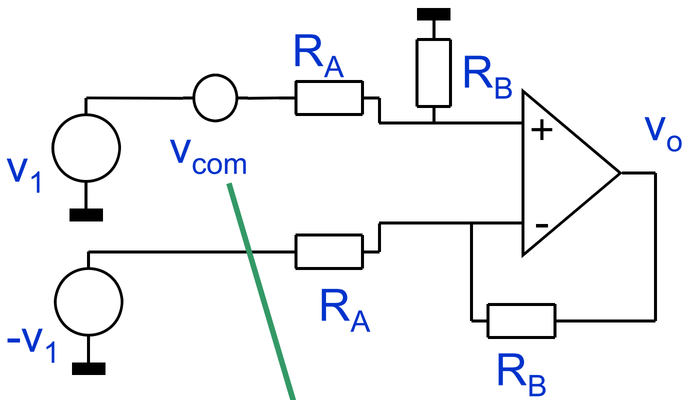
Replacing v_+ and v_- in the last eq:

$$v_o = A_c \frac{R_B}{R_B + R_A} \frac{1}{1 + \left(A_d - \frac{A_c}{2}\right)\frac{R_A}{R_B + R_A}} v_c$$

$$\approx \left(\frac{A_c}{A_d}\right)\left(\frac{R_B}{R_A}\right) v_c$$

Common mode signal is reflected at the input as it is a differential signal in open loop condition..

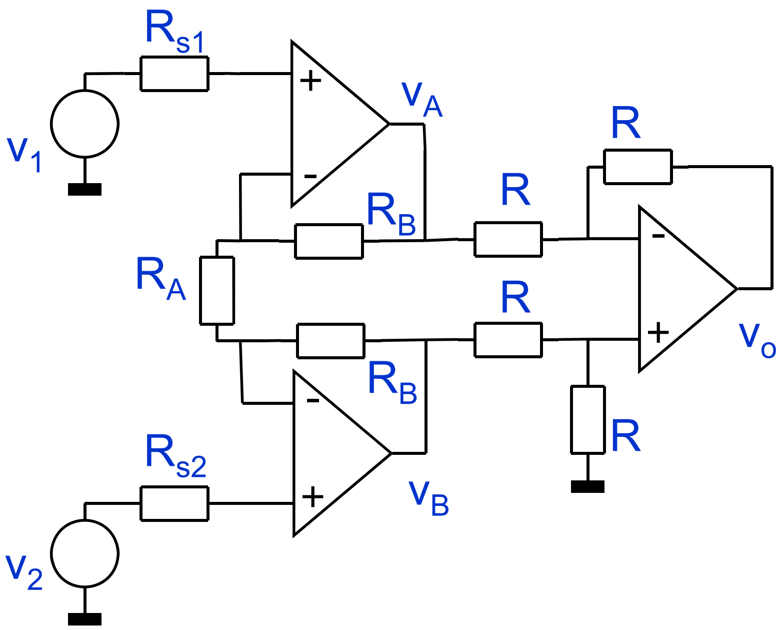
Closed loop gain



$$v_{com} = \frac{A_c}{A_d} v_c$$

Appendix G (5)

It is possible to improve the performance of the differential amplifier, at expense of a greater complexity.



Now the 2 sources see each one a very large impedance, due to the feedback.

Supposing that for the 3 OA is A=∞:

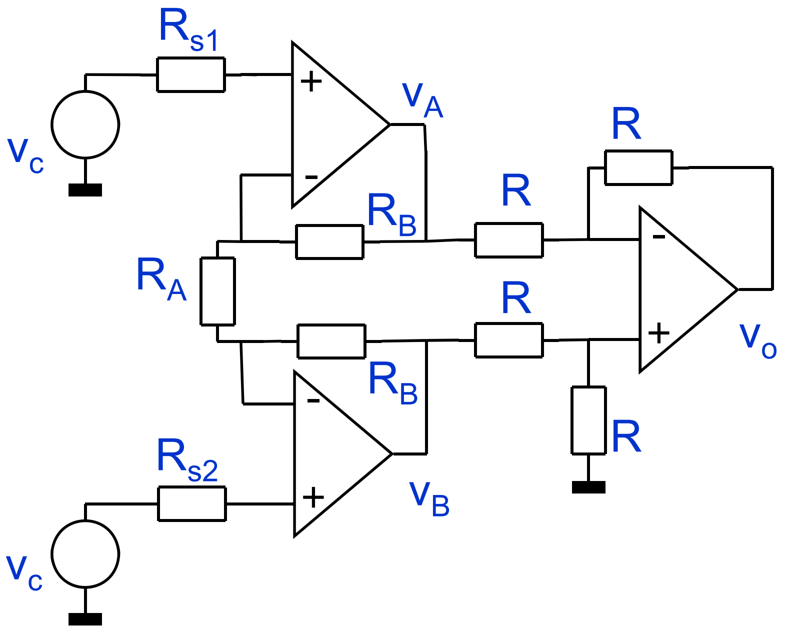
$$\begin{cases} \dfrac{v_A - v_1}{R_B} = \dfrac{v_1 - v_2}{R_A} \\ \dfrac{v_1 - v_2}{R_A} = \dfrac{v_2 - v_B}{R_B} \end{cases} \qquad \begin{cases} v_A = \dfrac{R_A + R_B}{R_A} v_1 - \dfrac{R_B}{R_A} v_2 \\ v_B = \dfrac{R_A + R_B}{R_A} v_2 - \dfrac{R_B}{R_A} v_1 \end{cases}$$

The output stage is simply a unity gain differential amplifier:

$$v_o = v_B - v_A = \frac{R_A + 2R_B}{R_A}(v_2 - v_1)$$

Appendix G (6)

Now it is possible to appreciate what happens to the common mode signal: $v_1=v_2=v_c$.



If the 2 inputs are equal the current through R_A is zero; as a consequence the current will be zero also in the 2 resistors R_B's. Therefore $v_A=v_B$ whatever is the tolerance of R_A and R_B's and the value of the R_s's.

If ε is the tolerance of the resistors R of the output OA, their CMRR is:

$$CMRR_R = 20\log_{10}\left(\frac{2R_B + R_A}{R_A}\ \frac{1}{\varepsilon}\right)$$

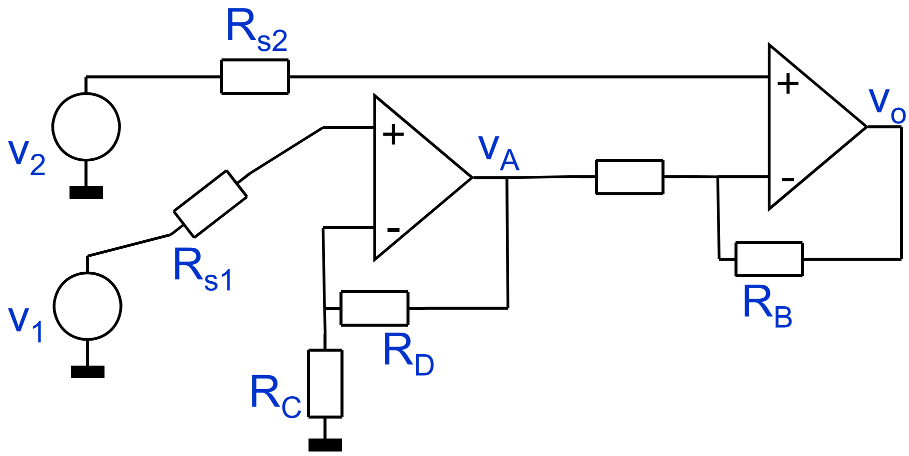
If the inputs OAs are very similar, that is the case when they share the same package, then the common mode gain A_c is similar, giving similar contribution to the nodes v_A and v_B. This way the CMRR is limited in accuracy by the output OA and:

$$CMRR_{AMPLI} = 20\log_{10}\left(\frac{2R_B + R_A}{R_A}\ \frac{A_d}{A_c}\right)$$

That is to say that the CMRR is larger and larger if the gain of the input stage is larger and larger.

Appendix G (7)

Here there is another differential configuration having a large input impedance:



Let's apply the superimposition principle and set v_1=0 first, that implies that v_A=0 V and:

$$v_{O2} = \frac{R_A + R_B}{R_A} v_2 = \left(1 + \frac{R_B}{R_A}\right) v_2$$

Now suppose that v_2=0 V, which sets to 0 V the non-inverting terminal of the output OA:

$$v_{O1} = -\frac{R_B}{R_A}\frac{R_C + R_D}{R_C} v_1 = -\frac{R_B}{R_A}\left(1 + \frac{R_D}{R_C}\right) v_1$$

Therefore:

$$v_O = v_{O1} + v_{O2} = \left(1 + \frac{R_B}{R_A}\right) v_2 - \frac{R_B}{R_A}\left(1 + \frac{R_D}{R_C}\right) v_1$$
$$= \left(1 + \frac{R_B}{R_A}\right) v_2 - \left(\frac{R_B}{R_A}\frac{R_D}{R_C} + \frac{R_B}{R_A}\right) v_1$$

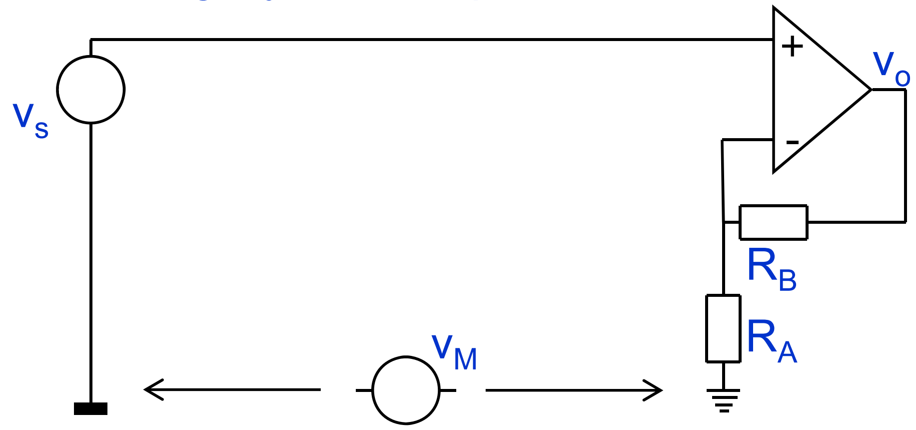
Finally, by setting: $$\frac{R_B}{R_A} = \frac{R_C}{R_D}$$

we get: $$v_O = \left(1 + \frac{R_B}{R_A}\right)(v_2 - v_1)$$

Appendix G (8)

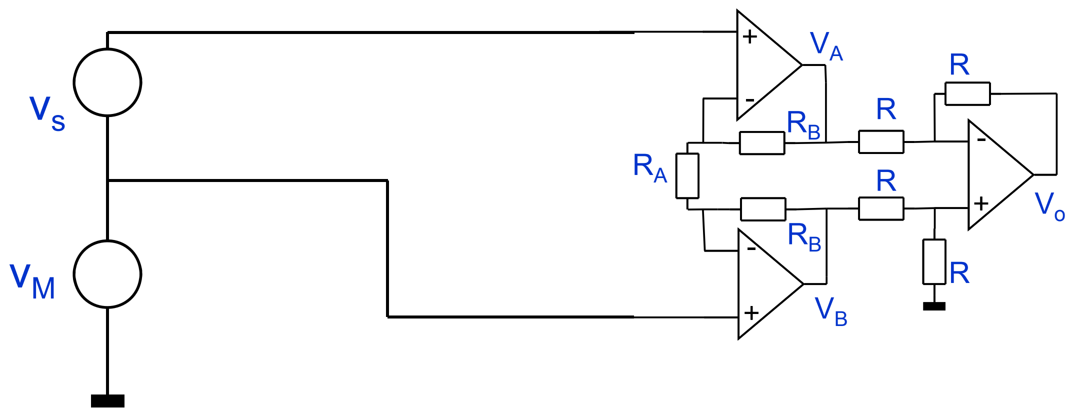
Application of the differential configuration.

Errors rise when the source and the amplifier are connected ground references far away: since currents flow around in the environment the 2 references could have slightly different potential.



This can be modelled this way:

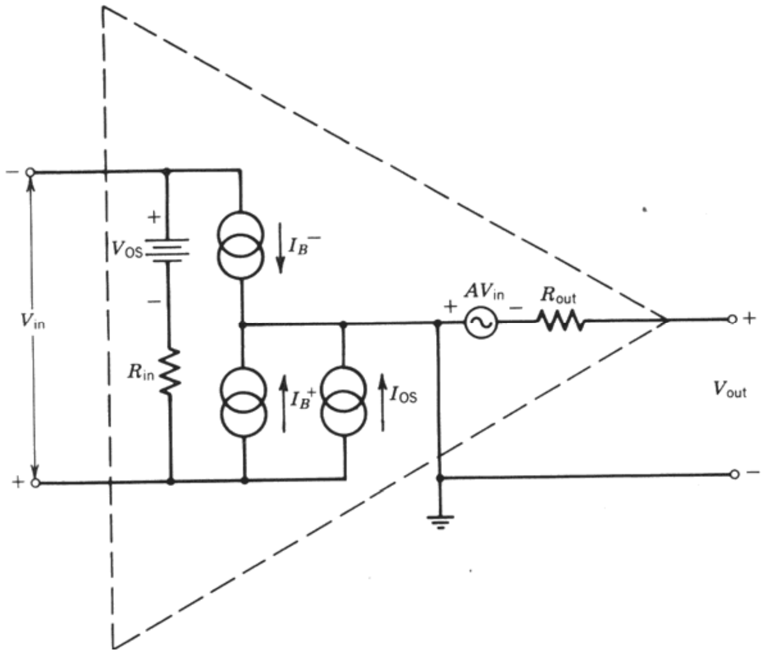
The amplified signal has superimposed the disturbance v_M to the signal v_s. If now we introduce our differential amplifier we have 2 reading terminals that can exploited to read the signal across the source only:



This way the disturbance is rejected since the amplifier behaves as it is virtually floating: v_o=A($(V_s+v_M) - v_M$)=Av_s.

Appendix H (1)

An actual OA is not ideal and shows several characteristics that can have affect in the application. Its datasheet list them and one can select the best suited off-the-shelf OA for the own application.



The characteristics are listed in the datasheet and, for a good OA, several are listed for a good detail.

Generally one focalizes the parameters important for the own application and do not consider the others.

Here some examples:

Input bias current The input bias current is one-half the sum of the separate currents entering the two input terminals of a balanced amplifier, as shown in Fig. 15-22. Since the input stage is that shown in Fig. 15-8, the input bias current is $I_B \equiv (I_{B1} + I_{B2})/2$ when $V_o = 0$.

Input offset current The input offset current I_{io} is the difference between the separate currents entering the input terminals of a balanced amplifier. As shown in Fig. 15-22, we have $I_{io} \equiv I_{B1} - I_{B2}$ when $V_o = 0$.

Input offset current drift The input offset current drift $\Delta I_{io}/\Delta T$ is the ratio of the change of input offset current to the change of temperature.

Appendix H (2)

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Input offset voltage The input offset voltage V_{io} is that voltage which must be applied between the input terminals to balance the amplifier, as shown in Fig. 15-13*a*.

Input offset voltage drift The input offset voltage drift $\Delta V_{io}/\Delta T$ is the ratio of the change of input offset voltage to the change in temperature.

Output offset voltage The output offset voltage is the difference between the dc voltages present at the two output terminals (or at the output terminal and ground for an amplifier with one output) when the two input terminals are grounded.

Input common-mode range The common-mode input-signal range for which a differential amplifier remains linear.

Input differential range The maximum difference signal that can be applied safely to the OP AMP input terminals.

Output voltage range The maximum output swing that can be obtained without significant distortion (at a given load resistance).

Full-power bandwidth The maximum frequency at which a sinusoid whose size is the output voltage range is obtained.

Power supply rejection ratio The power supply rejection ratio (PSRR) is the ratio of the change in input offset voltage to the corresponding change in one power supply voltage, with all remaining power supply voltages held constant.

Slew rate The slew rate is the time rate of change of the closed-loop amplifier output voltage under large-signal conditions.

- ✓ Other important parameters are the noise;
- ✓ The frequency response and the ability / inability to drive low value impedances and capacitive impedances;
- ✓ The value of the output impedance;
- ✓ The upper and lower limits of the output signals with respect to the supply voltages;
- ✓ Power dissipation;
- ✓ The maximum output current and short circuit current;
- ✓ Etc.