On the additivity of generation-recombination spectra
Part 3: The McWhorter model for 1/f noise in MOSFETs

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Abstract

This is a critical study on the McWhorter model, taking into account the conditions for additivity of GR spectra. A major finding is that the model does indeed give 1/f spectra. The calculated $\alpha$-values agree with experimental values of MOSFETs in strong inversion.

A simple comparison of experimental $\alpha$-values with values calculated from a $\Delta\mu$-model or a McWhorter $\Delta n$-model does not allow us to decide which the correct noise source is. However, the dependence of $\alpha$ on the gate voltage has often been used as a criterion for the correct source. Nevertheless, the $V_{GT}$-dependence does not lead to unambiguous conclusions either.

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1. Introduction

1.1. The notation chosen in this paper

This is the last of three papers, devoted to the problem of the summation of generation-recombination spectra. [1,2] At the end of part 2, the McWhorter model for 1/f noise was already treated as a special case of a general, many-level model. Here, we restrict ourselves to the McWhorter model. Because of the homogeneous trap distribution this model is much simpler than the general case. Therefore, the mathematics is less complicated, and we have no need for the rigorous approximations of part 2. We can easily describe the kinetics of the transitions between band and traps, so that we can make a clear distinction between the cases of mixing and addition of G-R spectra.

This paper can be read on its own. For those whose only interest is the McWhorter

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model, there is no need to consult the earlier parts. But for those who would like to compare the present and earlier results, we use the same notation as in parts 1 and 2. The notation deviates from semiconductor conventions in a few instances.

\[ n \]  
the total number of free electrons in the sample or the volume element, not the electron density

\[ N \]  
the number of electrons states in the sample or the volume element

\[ \frac{dn}{dx} \]  
electron density in a one-dimensional case (normal convention: \( n \) in cm\(^{-1} \))

\[ A \]  
number of traps in the sample or the volume element

\[ a \]  
number of electrons, trapped in \( A \)

\[ A-a \]  
number of empty traps

\[ a^* \]  
quantity, introduced to simplify the calculations

\[ \frac{1}{a^*} = \frac{1}{a} + \frac{1}{A-a} \quad \text{see (15)} \]

Thus, \( a^* \) is the smaller number of \( a \) and \((A-a)\).

\[ WL = 1 \text{ cm}^2 \]  
for the comparison of experimental results, all calculations are made for a MOSFET with a unit area of 1 cm\(^2 \). The area of a modern MOSFET is of the order of 1 \( \mu \text{m}^2 \), but by normalizing to 1 cm\(^2 \), we avoid width \( W \) and length \( L \) in the algebraic expressions

\[ C_{ox}, t_{ox} \]  
with this unit area, 1 cm\(^2 \), we follow the convention for MOSFETs where the capacitor, consisting of semiconductor-oxide-gate, is per unit area. The number of free electrons, induced by the effective gate voltage \( V_{GT} \), then is described by \( nq = Q = C_{ox} V_{GT} \), where \( C_{ox} = \frac{\varepsilon_0 \varepsilon_r}{t_{ox}} \) see (49)

\[ t_{ox} \]  
the thickness of the oxide layer between silicon and gate

\[ V_{GT} \]  
we consider a MOSFET with an \( n \)-channel in p-type material. \( V_G \) is the gate voltage, \( V_T \) is the so-called threshold voltage. \( V_{GT} = V_G - V_T \). The threshold voltage \( V_T \) is required for creating inversion. Then there are more free electrons near the interface than free holes beyond the depletion layer. The effective gate voltage \( V_{GT} \) is needed to induce more free electrons in the conduction band near the interface than holes in the valence band in the bulk of the semiconductor:

\[ n = C_{ox} V_{GT} / q \]

1.2. \( S \propto 1/f \)

A model for 1/f noise must explain two things:

(i) The shape of the spectrum. The power spectral density \( S \) is inversely proportional to the frequency \( f \) over a wide range of frequencies.

(ii) The magnitude of the noise (to be discussed in 1.3).

Most 1/f spectra have been measured with simple, standard equipment in a frequency range of \( 10^{-1} \text{ to } 10^5 \text{ Hz} \). By using special advanced techniques, 1/f noise has been observed at low frequencies, down to \( 10^{-5} \text{ Hz } \approx \text{ 1 day}^{-1} \) \([3,4]\) and at high frequencies, up to \( 10^7 \text{ Hz} \) \([5]\).

Nowadays the problem of the 1/f shape of the spectrum is often ignored, because at a very early stage, McWhorter \([6]\) presented a simple model for the 1/f noise, due to an oxide layer on a semiconductor crystal.

The model looks so simple that many people accept it blindly. McWhorter used a mathematical result of Surdin \([7]\), who showed that a wide range of Lorentzian spectra \((1)\), with a statistical weight proportional to \( 1/\tau \) \((2)\), produces a 1/f spectrum \((3)\):

\[ S(\tau) \propto \frac{\tau}{1 + \omega^2 \tau^2}, \quad (1) \]

\[ g(\tau) \, d\tau \propto (1/\tau) \, d\tau, \quad (2) \]
\[
S = \int_0^\infty g(\tau)S(\tau)\,d\tau \propto \int_0^\infty \frac{1}{\tau} \cdot \frac{\tau}{1 + \omega^2 \tau^2} \,d\tau \\
\propto \frac{1}{\omega} \text{arctan}\left[\frac{1}{\omega} \right] \propto \frac{1}{\sqrt{f}}.
\]

McWhorter proposed a physical model of the statistical weight (2), required in Surdin’s mathematical model. In McWhorter’s model, the semiconductor crystal is covered by an oxide layer, in which traps, \(x\), are homogeneously distributed along the distance \(s\) from the interface: \((dx/ds\) is constant). The free electrons in the conducting channel reach the traps by tunneling, which gives a trapping probability that exponentially depends on the trap distance \(s\).

\[
\tau(s) = \tau(0)e^{s/l},
\]

\[
g(\tau) = \frac{dx}{ds} = \frac{dx}{d\tau} = C \frac{1}{\tau}.
\]

The generation-recombination noise—GR noise—from each trap is a Lorentzian with \(\tau(s)\). The summation of the GR spectra results in a \(1/f\) spectrum, because of Surdin’s integral (3).

Two remarks can be made here with regard to the above:

1. **GR noise** is a fluctuation in the number of electrons. McWhorter’s model is a \(\Delta n\) model.

2. We have tacitly assumed that GR spectra can simply be added. The problem of additivity has been discussed in two publications [1,2].

### 1.3. **Magnitude of the noise**

The \(1/f\) noise in the conductance \(G\) of homogeneous samples of metals or semiconductors can be expressed by an empirical relation:

\[
\frac{S_G}{G^2} = \frac{\alpha_G}{fn},
\]

where \(n\) is the total number of free carriers in the sample (not the density!) and \(\alpha_G\) is a numerical parameter, usually with a value between \(10^{-6}\) and \(10^{-4}\). Until very recently there was no theoretical model behind this relation. The factor \(n\) is trivial. It says that whatever the electrons do, they do it independently.

Mechanical damage or implantation damage of the crystal lattice results in increasing \(\alpha_G\), by orders of magnitude, whereas the mobility only decreases by about 10% [8].

With inhomogeneous samples or devices (6) is applied to a homogeneous volume element, and \(S_G\) is integrated over the whole device [9].

### 2. **Fluctuations in number or in mobility?**

Everybody agrees that \(1/f\) noise is a fluctuation in the conductance. But there is no agreement on another key question: is the conductance fluctuation caused by fluctuations in \(n\) or in \(\mu\)?

First, the \(1/f\) noise in homogeneous samples is discussed.

\(\Delta n\): Conductance fluctuations caused by fluctuations in number. The empirical relation and the definition of \(\alpha\) becomes

\[
\frac{S_G}{G^2} = \frac{S_n}{n^2} = \frac{\alpha_G}{fn},
\]

\[
\alpha_n = fn \frac{S_n}{n^2} \propto \frac{\Delta n^2}{n}.
\]

Later, in Section 3.1, we will show that the variance of \(n\) equals \(n\) or \(m^*\), where \(m^*\) is the effective number of traps.

if \(n < m^*\), \(\Delta n^2 = \bar{n}\),

resulting in \(\alpha_n\) is constant,

\[
\text{if } n < m^*, \quad \Delta n^2 = m^*,
\]

resulting in \(\alpha_n \propto \frac{m^*}{n} \propto n^{-1}\). (9)

If samples were available with the same \(m^*\) and varying \(n\) \((n > m^*)\), we would find \(\alpha \propto n^{-1}\). Such experiments are easily performed on a MOSFET, where \(n\) is varied by the gate voltage \(V_{GT}\), while \(m^*\) is constant.

\(\Delta \mu\): Conductance fluctuations caused by fluctuations in mobility. The empirical relation reads as follows:

\[
\frac{S_G}{G^2} = \frac{S_\mu}{\mu^2} = \frac{\alpha_\mu}{fn}.
\]
There are \( n \) electrons in the sample. The mobility and its fluctuation of electron \( i \) are \( \mu_i \) and \( \Delta \mu_i \).

\[
G = c \sum_{i=1}^{n} \mu_i = cn\bar{\mu},
\]

\[
\Delta G = c \sum_{i=1}^{n} \Delta \mu_i \Rightarrow (\Delta G)^2 = c^2 \sum_{i=1}^{n} \sum_{j=1}^{n} \Delta \mu_i \cdot \Delta \mu_j
\]

\[
= c^2 \sum_{i=1}^{n} (\Delta \mu_i)^2 = c^2 n(\bar{\Delta \mu})^2,
\]

\[
\left( \frac{\Delta G}{G} \right)^2 = \frac{1}{n} \frac{(\Delta \mu_i)^2}{\mu_i^2} \Rightarrow \frac{S_G}{G^2} = \frac{1}{n} \frac{S_{\mu_i}}{\mu_i^2}
\]

\[
= \frac{1}{n} \frac{x_{\mu}}{\bar{\mu}}, \quad \text{with} \quad x_{\mu} \text{ constant.} \tag{11}
\]

\( x_{\mu} \) is proportional to the variance of the relative fluctuation per electron, which is independent of the number of electrons.

In cases, \( \Delta n \) and \( \Delta \mu \), the empirical relation is \( \frac{S_G}{G^2} = \frac{x_{\mu}}{fn} \). The fact that the number \( n \) appears in the denominator does not say anything about the nature of the noise, \( \Delta n \) or \( \Delta \mu \).

A measured conductance noise could be caused either by \( \Delta \mu \) or by \( \Delta n \). From conductance measurements alone, one cannot decide which source causes the conductance fluctuations. This is because \( \mu \) and \( n \) appear as the product \( \mu \times n \). A determination of the source requires noise measurements on a phenomenon where \( n \) and \( \mu \) do not appear as a product, but as a different function \( F \), like \( F = \mu \ln n \). Then in the case of

\[
\mu \text{ fluctuations : } \frac{S_F}{F^2} = \frac{S_{\mu}}{\mu^2} = \frac{x_{\mu}}{fn} = \frac{S_G}{G^2}, \tag{12}
\]

\[
n \text{ fluctuations : } \frac{S_F}{F^2} = \frac{S_{\ln n}}{(\ln n)^2} = \frac{(1/n^2)S_n}{(\ln n)^2}
\]

\[
= \frac{1}{(\ln n)^2} \cdot \frac{S_G}{G^2} < \frac{S_G}{G^2}. \tag{13}
\]

The fact that the \( 1/f \) noise in homogeneous layers is mobility noise is proved by comparison in the same sample of the conductance noise with the noise in thermo e.m.f., Hall effect, or other phenomena not determined by \( \mu \times n \).

This mobility source is further demonstrated by the \( \mu \)-dependence of \( x \) in samples with two scattering mechanisms, such as lattice scattering and impurity scattering.

In metals, it is obvious that the observed \( 1/f \) conductance noise is mobility noise.

The empirical relation and the \( \Delta n - \Delta \mu \) problem were extensively discussed in 1981 in a survey paper by Hooge et al. [10]. Further experimental results were presented in 1994 [11], when no essential corrections were made to the ideas presented in 1981.

In sum, experimental studies prove that in homogeneous layers the \( 1/f \) noise is mobility noise. Only recently has the first theoretical model of mobility noise been presented by Musha and Tacano [12,13]. Therefore, the whole \( \Delta n - \Delta \mu \) discussion has developed without a theoretical \( \Delta \mu \) model. This has been a weak point for the acceptance of the idea of mobility fluctuations.

In MOSFETs, the \( \Delta n \) or \( \Delta \mu \) nature has not been established experimentally through measurements of Hall voltage noise or noise in the magnetoresistance effect. For half a century, the very simple McWhorter model has been widely used. The McWhorter model is a number fluctuations model. In principle, it is possible that the \( 1/f \) noise in a MOSFET is of a different nature than the \( 1/f \) noise in homogeneous layers. It seems more plausible, however, that in MOSFETs the \( 1/f \) noise is mobility noise too, and that the McWhorter model cannot be applied [14]. A definite way to settle the \( \Delta n - \Delta \mu \) problem is to measure the \( 1/f \) noise of a different function than the product \( \mu \times n \) on a MOSFET.

3. Noise in the McWhorter model

Since part 3 is written in such a way that it can be read on its own, we have to repeat some definitions and relations from the parts 1 and 2 [1,2]. We do this only where it is necessary.

3.1. One trap only

Before discussing the McWhorter model, we repeat part of the treatment of GR noise in a semiconductor with one kind of trap.
The definitions of transition rates and numbers of states and electrons are given in Fig. 1 for an N-type semiconductor with A traps only, the so-called “A only” model. The definitions of $m/C_{17}$ and $n/C_{17} = a/C_{3}$ and $n/C_{17} = a/C_{3}$ are important, and here $m$ and $n$ do not denote mobility and frequency.

In equilibrium, $G = R$. The shot noise $\Delta G$ and $\Delta R$ causes fluctuations in $n$ and $a$. If at $t = 0$ a fluctuation $\Delta n$ is created, it will decay according to

$$\frac{d\Delta n}{dt} = G - R = \Delta G - \Delta R$$

$$= \frac{\Delta a}{a} - \frac{R}{n^2} \left( \frac{\Delta n}{n} - \frac{\Delta a}{a} \right)$$

$$= \frac{R}{n^2} \left( \frac{1}{a^*} + \frac{1}{A - a} \right) \Delta a - \frac{1}{n} \Delta n$$

$$= \frac{R}{n^2} \left( \frac{1}{a^*} \Delta a - \frac{1}{n} \Delta n \right).$$

Here, we introduced the definition

$$\frac{1}{a^*} = \frac{1}{a} + \frac{1}{A - a}.$$  \hspace{1cm} (15)

Since $a = fA$, where $f$ is the Fermi factor, (15) yields

$$a^* = f(1 - f)A.$$  \hspace{1cm} (16)

In an A-only model, the variance equals [15]

$$\frac{1}{(\Delta a)^2} = \frac{1}{(\Delta n)^2} = \frac{1}{n \ a^*}.$$  \hspace{1cm} (17)

From now on the bars in $G$ and $R$ will be omitted.

Eq. (14) shows that $\Delta n$ decays because of a $\Delta a$ term and a $\Delta n$ term. The arrows in Figs. 1b and c represent these $\Delta a$ and $\Delta n$ transitions. They only represent $\Delta G$ and $\Delta R$ for Boltzmann statistics. The quantity $a^*$ contains $a$ and $(A-a)$, and therefore takes care of the Fermi statistics.

A relaxation time $\tau_A$ and its inverse $\theta_A$ is defined according to

$$\frac{d\Delta n}{dt} = - \frac{\Delta n}{\tau_A} = - \theta_A \Delta n.$$  \hspace{1cm} (18)

In the “A-only” model, because of conservation of charge

$$\Delta a + \Delta n = 0.$$  \hspace{1cm} (19)

Substitution of (18) and (19) in (14) yields

$$\frac{1}{R\tau_A} = \frac{\theta_A}{R} = \frac{1}{a^*} + \frac{1}{n}.$$  \hspace{1cm} (20)

If $\beta$ electrons are removed from the traps to the band by the shot noise in the generation and recombination processes, $\Delta G$ and $\Delta R$, we find $\Delta n = \beta$ and $\Delta a = -\beta$. The number $\beta$ may be positive or negative.

In Fig. 1b, an arrow going down represents a number (+ or −) of free electrons leaving the band; the arrow going up represents a number (+ or −) of trapped electrons leaving the traps. In Fig. 1c, a negative number going up is not the same as a positive number going down. A negative number going up in 1c is a very small negative correction to the upgoing process $G$ in 1a. A positive number going down in 1c is a correction to $R$ in 1a. But as far as $d\Delta n/dt$ is concerned, the results of the two processes are the same. This is shown in the next calculation, where we do not immediately use (19), $\Delta a + \Delta n = 0$, because we...
distinguish between the $\Delta n$- and $\Delta a$-transitions here.

Two approximations will be used in (20):

(a) if $n/a^* \equiv \mu \ll 1$, Eq. (20) yields $\vartheta_A/R = -\Delta a/\Delta n$. Then (14) becomes:

$$\frac{d\Delta n}{dt} = \frac{R}{a^*} \Delta a - \frac{R}{n} \Delta n$$

$$= \frac{\mu}{1 + \mu} \vartheta_A \Delta a - \frac{1}{1 + \mu} \vartheta_A \Delta n$$

$$\equiv \mu \vartheta_A \Delta a - \vartheta_A \Delta n,$$

$$\frac{d\beta}{dt} = \mu \vartheta_A (-\beta) - \vartheta_A \beta \equiv -\vartheta_A \beta.$$ 

(21)

(b) if $n/a^* \equiv \nu \ll 1$, Eq. (20) yields $\vartheta_A/R = 1/a^* + \nu/a^* = (1 + \nu)/a^*$. Eq. (1) becomes:

$$\frac{d\Delta n}{dt} = \frac{R}{a^*} \Delta a - \frac{R}{n} \Delta n$$

$$= \frac{1}{1 + \nu} \vartheta_A \Delta a - \frac{\nu}{1 + \nu} \vartheta_A \Delta n$$

$$\equiv \vartheta_A \Delta a - \nu \vartheta_A \Delta n.$$ 

$$\frac{d\beta}{dt} = \vartheta_A (-\beta) - \nu \vartheta_A \beta \equiv -\vartheta_A \beta.$$ 

(22)

In both approximations, (21) for small $n$ and (22) for large $n$, the decay of $\Delta n$ is a negative exponential with $\vartheta_A$. So in the “$A$-only” case, it is of no consequence whether $\Delta n$ decays because of $\Delta n$ leaving the band, or $\Delta a$ being excited into the band. But the $\Delta n$, $\Delta a$ difference is essential in the McWhorter model (as in all models with more traps), where it leads to the distinction between mixing and adding of spectra.

3.2. The McWhorter model

The McWhorter model is presented in Fig. 2. The McWhorter states are distributed in the oxide layer, homogeneously in energy, and homogeneously along the distance $s$. The model discussed here is that of Fig. 4b in Section 7.3 of part 2 [2]. The less realistic Fig. 4a is not considered here. The 1/f noise is caused by transitions between the silicon conduction band and the traps in the oxide. The electrons reach the traps by tunneling. Therefore, the transition constants to traps at the same energy level exponentially depend on $s$:

$$\vartheta_s = \vartheta_0 e^{-s/kT}$$

(23)

The traps far from the interface have very small $\vartheta$-values. Such extremely slow states have no influence, since no transitions occur during the measuring time. The measuring time determines an effective thickness, $z$. One usually assumes $z = 30\text{ A (see (53))}$. Because of the homogeneous distributions,

$$\frac{dM}{dE} = \text{const. and } \frac{dM}{dz} = M/z.$$ 

(24)

In the noise calculations, we use an effective number of trapping states, $M$. $M$ is the number of states in a volume of 1 cm$^2 \times z$ cm, in a narrow energy band around the Fermi level. In the next calculation, it will be shown that the band has a width $kT$.

![Fig. 2. The McWhorter model. (a) Model of a MOSFET. (b) Model used in the calculations.](image-url)
We define \( m^* \) analogously to \( a^* \) in (16). At a fixed value of \( E \),
\[
\frac{dm^*}{dE} = f(E)(1 - f(E)) \frac{dM}{dE},
\]
(25)
where \( f \) is the Fermi factor.

The total effective \( m^* \) is found by integration over all \( E \) values,
\[
m^* = \int_{-\infty}^{\infty} \frac{dm^*}{dE} \, dE = \frac{dM}{dE} \int_{-\infty}^{\infty} f(1 - f) \, dE
= \frac{dM}{dE} kT \int_{-\infty}^{\infty} \frac{e^x}{(1 + e^x)^2} \, dx = \frac{dM}{dE} kT = M^*,
\]
(26)
where \( x = (E - E_F)/kT \). The integral equals 1.

The fact that \( m^* = M \) (26) seems to contradict the analogy with (16). Fig. 3 gives the answer to this problem. At any \( E \), holds \( dm^*/dE < dM/dE \), because of (25) in analogy to (16). But \( m^* \) follows from integration over all values of \( E \); whereas, by definition, \( M \) follows from integration over the narrow band \( \Delta E = kT \). The area under the curve \( dm^*/dE \) equals the area of the rectangle ABCD (in a linear plot). Both are \( M \).

The use of \( m^* = M \) gives an unimportant correction to Section 7 and Fig. 4b of part 2, where \( m^* = M/4 \) was used.

In the noise calculations, we need the variances \( \langle (\Delta m)^2 \rangle \) and \( \langle (\Delta n)^2 \rangle \). Since variances are equilibrium quantities, they do not depend on kinetic parameters [15]. Fig. 2b corresponds to Fig. 1a. Therefore, in analogy to (17)
\[
\frac{1}{\langle (\Delta m)^2 \rangle} = \frac{1}{\langle (\Delta n)^2 \rangle} = \frac{1}{n} + \frac{1}{m^*}.
\]
(27)

The oxide layer is divided in \( z/ds \) slices with thicknesses \( ds \). Consider the slice at \( s_K \) (i.e. between \( s_K \) and \( s_K + ds \)). The average number of trapped electrons is \( X_K = (dX/ds)ds \). All traps are at the Fermi level; their concentration does not depend on \( s \):
\[
\sum X_K = \int_0^z \frac{dX}{ds} \, ds = \int_0^z \frac{M}{z} \, ds = M,
\]
(28)
\[
\sum x_K = \int_0^z \frac{dx}{ds} \, ds = \int_0^z \frac{m}{z} \, ds = m,
\]
(29)
\[
\sum x_K^* = \int_0^z \frac{dx^*}{ds} \, ds = \int_0^z \frac{m^*}{z} \, ds = m^*,
\]
(30)
\[
\sum (\Delta x)^2 = \int_0^z \frac{d(\Delta x)^2}{ds} \, ds = \int_0^z \frac{d(\Delta m)^2}{z^2} \, ds
= \frac{\langle (\Delta m)^2 \rangle}{\langle (\Delta n)^2 \rangle} = \frac{n m^*}{n + m^*}.
\]
(31)
4. Kinetics of the decay

4.1. $\Delta n_K$ and $\Delta x_K$ transitions

The $M$ traps in the oxide layer are all in the same situation:

(i) Homogeneously distributed along $s$, $0 < s < z$.

(ii) In a band with width $kT$ around the Fermi level.

The $M$ states are divided into groups $d X_i$ with widths $d s$. According to (28) $d X_i = M \, ds/z$, where $i$ runs from $A$ to $Z$. The average number of electrons trapped in $X_i$ is $d x_i = m \, ds/z$ (29).

Consider $\Delta n$ free electrons that recombine in the traps. If the transition probabilities do not depend on $s$, then each trap receives the same fraction of $\Delta n$. The average number going into the traps $i$ then is

$$d \Delta n_i = \Delta n \, ds/z.$$  \hspace{1cm} (32)

If the transition probabilities are $s$-dependent, the fast traps $A$, $B$, etc. receive more electrons in unit time than the slow traps $Y$, $Z$. But we cannot distinguish between free electrons in the band. There is no difference, whatsoever, between the $n_A$ and $n_B$ electrons and the $n_Y$ and $n_Z$ electrons. The fluctuation $\Delta n_A$ does not decay faster than the fluctuation $\Delta n_Z$ although the $n_A$ electrons have larger transition probabilities $\beta_A$ than the $n_Z$ electrons with small $\beta_Z$. The electrons in the fraction $\Delta n_A$ with $\beta_A$ are replenished by electrons from the fluctuations near $z$. In this way, (32) holds good at any time and also with $s$-dependent $\beta$-values

$$\sum_i \Delta n_i(t) = \int_0^z \Delta n(t) \, \frac{ds}{z} = \Delta n(t). \hspace{1cm} (33)$$

Now we consider the $R_K$ and $G_K$ processes to and from the traps $K$ at $S_K$. The number of free electrons directly involved in $R_K$ and $G_K$ is $d n_K$. This $R_K$, $G_K$ subsystem is an “$A$-only” model, treated in Section 3.1.

For the averaged values, we find the following:

Band: $d N_K = N \, ds/z$ states, $d n_K = n \, ds/z$ electrons,

Trap $K$: $d X_K = M \, ds/z$ states, $d x_K = m \, ds/z$ electrons.

For the fluctuations, we cannot use $-\Delta n_K = \Delta x_K$ in a McWhorter model, since (19) exclusively holds in a real “$A$-only” model. We write $\Delta n$ and $\Delta x$ transitions as separate terms, as we did with the Eqs. (21) and (22).
For the \( R_K \), \( G_K \) subsystem we define \( \theta_K \) in analogy to (20):

\[
\theta_K = R_K \left( \frac{1}{x_{K}^* \theta^*_{K} + \frac{1}{n_K}} \right).
\tag{34}
\]

Eq. (34) will be used in two approximations:

1. If \( \frac{d\theta_K}{dx_K^*} = \frac{n}{m^*} \equiv \mu \ll 1 \),
   \[
   \theta_K \approx R_K/n_K,
   \]
   \[
   \frac{d\theta_K}{dx_K} = \frac{\mu}{1 + \mu} \frac{\theta_K \Delta x_K}{1 + \theta_K \Delta n_K} \approx \mu \theta_K \Delta x_K - \frac{1}{1 + \mu} \theta_K \Delta n_K.
   \tag{35}
   \]

2. If \( \frac{d\theta_K}{dx_K^*} = \frac{m^*}{n} \equiv \nu \ll 1 \),
   \[
   \theta_K \approx R_K/x_{K}^* \theta^*_{K},
   \]
   \[
   \frac{d\theta_K}{dx_K} = \frac{\theta_K \Delta x_K}{1 + \nu} - \frac{1}{1 + \nu} \theta_K \Delta n_K \approx \theta_K \Delta x_K - \nu \theta_K \Delta n_K.
   \tag{36}
   \]

4.2. The initial stage

The fluctuations \( \Delta n_K \) and \( \Delta x_K \) are generated by the shot noise in \( G_K \) and \( R_K \) (see Fig. 4). In a short time interval \( \delta t \), \( n\delta t \) electrons arrive in the band. The shot noise in the Poissonian process \( G_K \) generates a fluctuation \( \Delta n_{\delta t} \), where \((\Delta n_{\delta t})^2 = \pi_{\delta t} \). Suppose that in \( \delta t \), \( \beta \) electrons are removed from traps \( K \) and transported to the band by \( G_K \) and \( R_K \). \( \Delta n(t = 0) = \beta \) and \( \Delta x_K(t = 0) = -\beta \). The number \( \beta \) may be positive or negative. The fluctuation \( \Delta n \) equals \( \beta \), but \( \Delta n_K \) is a fraction of \( \beta \) according to (32).

The original fluctuations, \( \beta \) in the band and \( -\beta \) at the trap \( K \) partly recombine, and partly redistribute themselves over the other traps, from which they will later slowly decay. All these decaying fluctuations are correlated with \( \beta \).

We shall calculate correlation functions and spectra for the correlated fluctuations that originate “from \( K \)”. Then we integrate over all uncorrelated contributions “from \( A \), “from \( B \)”, etc.

1. The decay of \( \Delta n \) if \( n \ll m^* \).

If \( \theta_0 \) is independent of \( s \) \((\theta_s = \theta_0)\), then the fluctuation \( \beta \) is homogeneously distributed over all traps. The fraction going to \( K \) is \( \Delta n_K \) in analogy to (32). From (35) it follows that \( \Delta n_K \) decays with \( \theta_K \) (which for \( s \)-independent probabilities equals \( \theta_0 \)). With an \( s \)-dependent \( \theta_s \), (23), the fraction going to a trap at \( s \) decays proportionally to its \( \theta_s \).

\[
\frac{d\theta_s}{dt} = -\theta_s \beta \frac{ds}{z}.
\tag{37}
\]

To prevent that after some time \( \Delta n_A \) differs from \( \Delta n_K \); there is a continuous redistribution over \( \Delta n_A, \Delta n_B \ldots \Delta n_K \ldots \) relation (32) always holds. Because of (33) the total fluctuation \( \beta \) decays as

\[
\frac{d\beta}{dt} = \int d\Delta n = \int \theta \frac{\theta_0}{z} \frac{ds}{z} = \frac{\beta_0}{z} \int e^{-\nu \theta_0} ds = \frac{\lambda}{z} \theta_0.
\]

The decay of the fluctuation in the free electrons is described by one single constant \( \theta_1 \)

\[
\theta_1 = \frac{\lambda}{z} \theta_0 \quad \text{or} \quad \tau_1 = \frac{z}{\lambda} \tau_0.
\tag{38}
\]

The decay of \( \beta \) fills the traps \( A, B \), etc. much faster than \( X, Y \), and \( Z \). The generation processes \( G_A \), \( G_B \), etc. can be neglected compared with \( R_A \), \( R_B \), etc. as long as the concentrations in \( A, B \), etc. are low. As long as this approximation is allowed, we speak of the “initial stage”.

2. The decay of \( \Delta n \) if \( n \gg m^* \).

Instead of (35) we must now use (36). The total decay of \( \beta \) goes with \( \theta = \nu \theta_1 = \nu (\lambda/z) \theta_0 \).

3. The decay of \( \Delta x_K \)

The trapped electrons in \( K \), \( \Delta x_K = -\beta \), have no other option than to go to the band. Eq. (35) shows that \( \Delta x_K \) decays with \( \mu \theta_K \) if \( n \ll m^* \); Eq. (36) shows that \( \Delta x_K \) decays with \( \theta_K \) if \( n \gg m^* \).

4.3. Decay in the case of mixing

The following relations hold in this case: \( \mu = n/m^* \ll 1 \) (Fig. 4) and \( (\Delta m)^2 = (\Delta n)^2 = n \) (27).

We already discussed how a fluctuation \( \Delta n = \beta \) at \( t = 0 \) decays very fast with \( \theta_1 = \theta_0 \lambda z / (38) \). The recombination distributes the \( \beta \) electrons over the different traps, proportionally to \( \theta_s \). After time \( t \), \( \tau_1 < t < \tau_K \) the fluctuation \( \Delta n \) has almost disappeared. The \( \beta \) electrons are in traps with \( s < s_K \). Hardly any electron has fallen into \( K \), where we
still find $\Delta x_K = -\beta$. The faster traps are in “equilibrium” with the band. The equilibrium region extends from $s = 0$ to a distance we call $z_K$, $z_K < s_K$. Each trap in equilibrium contains $\Delta x_s = \beta/z_K$ electrons.

$\Delta n_s = (n_s/x_s)\Delta x_s = \mu\Delta x_s = \mu\beta/z_K ds$,

$\Delta n = \int_0^{z_K} \frac{\mu\beta}{z_K} ds = \mu\beta$.  \hspace{1cm} (39)

The exact value of $z_K$ is of no consequence, the important point being that there are $\beta$ electrons in the equilibrium states taken together (see Fig. 5).

This situation is rather stable since all transport has to go via the band, where only very few electrons are present, $\mu\beta$. There is a very slow decay of the $\beta$ trapped electrons, the $\mu\beta$ free electrons, and the $-\beta$ electrons still in $K$.

To distinguish the slow decay from the fast decay with $\tau_1$ we use the index 2. Since all fluctuations originate “from $K$”, the slow relaxation is characterized by $\tau_{2K}$.

At the $K$ level, we find

$$\frac{d(-\beta)}{dt} = -\beta ds/z - \mu\beta(-\beta) \simeq -\mu\beta(-\beta),$$

$\vartheta_{2K} = \mu\beta$. \hspace{1cm} (40)

(In part 2, we derived $\vartheta_{2K} = \mu\beta K/(r_K - 1)$ (45), where $r_K$ is the serial number of level $K$, $r_K \approx s_K/d_\tau$. In a McWhorter model, $r \to \infty$, which yields (40)).

Having determined $\vartheta_1 = (\lambda/z)\vartheta_0$ (38) and $\vartheta_{2K} = \mu\vartheta$ (40), we now calculate correlation functions $\phi$ and spectral densities. Since $n$ and $\Delta n$ are the only quantities that can be measured, we restrict ourselves to $\phi$ and $S$, without explicitly writing the index $n$.

From $K$:

$$K\Delta n(t) = K\Delta n_1(t) + K\Delta n_2(t)$$

$$= K\Delta n_1(0)e^{-\beta_1 t} + K\Delta n_2(0)e^{-\beta_2 t},$$

$$K\phi = K\Delta n(0) \cdot K\Delta n(t)$$

$$= \langle [K\Delta n_1(0) + K\Delta n_2(0)] \rangle$$

$$\times [K\Delta n_1(t) + K\Delta n_2(t)]$$

$$= \langle [\beta] \cdot [(1 - \mu)\beta e^{-\beta_1 t} + \mu\beta e^{-\beta_2 t}] \rangle$$

$$\simeq \beta^2 [e^{-\beta_1 t} + e^{-\beta_2 t}].$$ \hspace{1cm} (41)

Since $\Delta k = -\beta$, it follows that $\beta^2 = (\Delta k)^2 = (\Delta m)^2 ds/z = n ds/z$,

$$K\phi = n(e^{-\beta_1 t} + e^{-\beta_2 t}) ds/z = K\phi_1 + K\phi_2.$$  

$K\phi_2$ is the interesting term for the low-frequency spectra

$$K\phi_2 = \mu ne^{-\beta_2 t} ds/z,$$
\[ k S_2 = 4 \int_0^\infty k \phi_2 e^{\lambda t} \, dt = 4 \mu n \int_0^\infty \frac{e^{-t/\tau_{2K}}}{z} \, dt \]
\[ = 4 \mu n \frac{\tau_{2K}}{z} \frac{1}{1 + \omega^2 \tau_{2K}^2} \, dx. \]

From all traps:
\[ S_2 = 4 \mu n \int_0^Z \frac{\tau_{2K}}{1 + \omega^2 \tau_{2K}^2} \, ds \]
\[ = 4 \mu n \frac{\lambda}{z} \int_0^z \frac{1}{1 + \omega^2 \tau_{2K}^2} \, d\tau_{2K}. \]

In the last step, we used
\[ dx = \frac{\partial s}{\partial \tau_{2K}} d\tau_{2K} = \frac{\lambda}{\tau_{2K}} d\tau_{2K}. \]

The correlation function (41) for decay in the case of mixing has two exponential terms (see Fig. 5).

The first term in (41), \( \beta e^{-\beta t} \) with \( \beta = \beta_1 \), is the same for the fluctuations from every \( K \). Summing over all \( K \) values, we obtain the following for the white branch of the Lorentzian spectrum:
\[ S_1(white) = \sum \beta e^{-\beta t}, \]
\[ = 4 \mu n \frac{\tau_{2K}}{z} \frac{1}{1 + \omega^2 \tau_{2K}^2} \, dx. \]

The second term in (41), \( \mu \beta e^{-\beta t} \), gives 1/f noise, with \( S_2 \) being given by (42). Surprisingly enough, we find that an 1/f spectrum is produced in the case of mixing as well. The question is, however, whether it can be observed on top of the white \( S_1 \) spectrum. There is a corner frequency \( f_c \), below which the 1/f spectrum can be observed and above which it is invisible. The corner frequency follows from \( S_1 = S_2 \) (42, 44).
\[ 4(z/\lambda)\tau_0 n = \alpha \frac{1}{f_c}, \]
\[ f_c = \frac{\alpha}{4(z/\lambda)\tau_0}. \]

With the numerical values, to be discussed in Section 5, we find
\[ f_c = 10^4 \text{ Hz}, \quad \text{if} \quad \alpha = 10^{-4}, \]
\[ f_c = 100 \text{ Hz}, \quad \text{if} \quad \alpha = 10^{-6}. \]

So the white \( S_1 \) spectrum does not obscure the 1/f spectrum in the usual frequency range of the measurements.

4.4. Decay in the case of addition

The following relations hold in this case:
\[ v = m^*/n \leq 1 \quad (4) \quad \text{and} \quad (\Delta n)^2 = (\Delta m)^2 = m^* \quad (27). \]

The calculations are easy now, since the decay of \( \Delta n \) is exclusively determined by the uncorrelated \( \Delta K \)-transitions. The result is a straightforward McWhorter 1/f spectrum.

From \( K \): Since \( \Delta k = -\beta \), we obtain
\[ \overline{\beta^2} = (\Delta K)^2 = \overline{\Delta n} = \frac{m^*}{z} \frac{dx}{z}, \]
\[ k \Delta n = \beta e^{-t/\tau_K}. \]

\( \Delta n \) decays with \( \Delta k \). As long as \( \Delta n \) remains of the order of \( \beta \), some electrons will be trapped in the

![Fig. 6. Decay of a fluctuation from \( K \). addition \( n \gg m^* \).](attachment:fig6.png)
fast traps $A$, $B$, etc. Therefore, $\sum \Delta x_i$ (with $0 < s \ll s_K$) will be built up, and will slowly decay with $s_k$ (see Fig. 6).

$$K \phi = \Delta m(0) \cdot \Delta n(t) = \beta^2 e^{-t/s_k},$$

$$K S = \int \frac{\tau_K}{1 + \omega^2 \tau_K} \, ds, \quad \int \frac{\tau_K}{1 + \omega^2 \tau_K} \, ds.$$ 

All traps:

$$S = 4m^* \lambda z \int_0^\infty \frac{\tau_K}{1 + \omega^2 \tau_K} \, d\tau,$$

$$S = 4m^* \lambda z = \frac{1}{n} d\tau.$$

$$S_n = \frac{\lambda}{z} \frac{1}{n f} \equiv \alpha \frac{1}{n f}.$$

$$\alpha = \frac{\lambda}{z} = \frac{m^* \lambda}{n z}. \quad (46)$$

5. Numerical results

Fig. 7 shows $n$, $m^*$ and $\alpha_{\text{McW}}$ as functions of $n$. The overall picture agrees with Fig. 4b in part 2 [2]. The presentation in Fig. 7 differs slightly from that in part 2, Fig. 4b, where the Fermi level was used as the independent variable. The only use we made here of $E_F$ was in defining $m^*$ (Fig. 3, Eqs. (24)–(26)). Now we prefer a horizontal log $n$-axis since the discussion of experimental work requires $V_{GT}$ as the independent variable. Many authors have seen the experimentally found dependence $\alpha_{\text{exp}} \propto V_{GT}^{-1}$ as proof of the correctness of the number fluctuations as predicted by the McWhorter model. This dependence can easily be discussed using Fig. 7, since $n \propto V_{GT}$. Therefore, we draw a log $V_{GT}$-axis underneath the log $n$-axis.

Now numerical estimates of $N$, $M$, $n$ and $\alpha_{\text{McW}}$ will be given for a unit area of a MOSFET. The accuracy is not that important; it is the $n$- or $V_{GT}$-dependence that we are mainly interested in.

$N$. The total number of states in the potential well near the interface is $N = a N_c$. The effective width of the channel is $a$. We take $a = 10^{-6}$ cm [16–20]. The effective density of states in the conduction band, $N_c$ is for Si $2 \times 10^{19}$ cm$^{-3}$. Thus

$$N = 2 \times 10^{13}. \quad (47)$$

$M$. The density of McWhorter states varies in different MOSFETs, but a representative value is $2 \times 10^{17}$ cm$^{-3}$ eV$^{-1}$ [21] In Fig. 2 we defined $M$ as the number of traps in an area $kT \times z$. The depth $z = 30 \, \lambda$ is the maximum distance from the interface from where in the measuring time a
transition to the band may occur (see (53)):

$$M = 10^9.$$  \hfill (48)

$$n = \ldots V_{GT}.$$  For a MOSFET in strong inversion we assume that nearly all charge induced by the gate voltage is mobile, without much immobile charge in the interface. From \( Q = CV \), it then follows that

$$n = \frac{1}{q} \frac{\varepsilon_{ox} \varrho_0}{t_{ox}}, \quad V_{GT} = 2 \times 10^6 \frac{V_{GT}}{t_{ox}}. \hfill (49)$$

In modern MOSFETs, with thin oxide layers \( t_{ox} \approx 10^{-6} \text{ cm} \):

$$n = 2 \times 10^{12} V_{GT}. \hfill (50)$$

In older MOSFETs, the thickness can be a factor 10 higher. The value of \( t_{ox} \) shifts the \( V_{GT} \) axis parallel to the \( n \)-axis. All lines in the plot keep their positions relative to \( n \). So the somewhat arbitrarily chosen value of \( t_{ox} = 10^{-6} \text{ cm} \) is not too critical for the analysis.

\( \alpha_{\text{McW}} \). From (46) the \( \alpha \) value for strong inversion follows:

$$\alpha_{\text{McW}} = \frac{\lambda}{z} m^* n = 3 \times 10^7 n^{-1}, \hfill (51)$$

which by using (50) can be transformed into

$$\alpha_{\text{McW}} \approx 10^{-5} V_{GT}^{-1}, \quad \text{if} \quad t_{ox} = 10^{-6} \text{ cm}. \hfill (52)$$

\( \lambda, z \). The values \( \lambda = 1 \text{ Å} \) and \( z = 30 \text{ Å} \) are used here. There is general agreement on \( \lambda \), which is the characteristic length of the tunneling process into the oxide layer, given by (23): \( \varrho_s = \varrho_0 e^{-s/\lambda} \) or \( \tau_s = \tau_0 e^{s/\lambda} \).

The value of \( z \) is determined by the measuring time used. If in the measuring time, \( 1/f_{\text{low}} \), hardly any transition occurs to traps at \( s > z \), then the traps are not considered to contribute to the \( 1/f \) noise. These very slow traps beyond \( z \) determine the low-frequency limit of the \( 1/f \) spectrum. Using \( \tau_0 = 10^{-13} \text{ s} \) [16] and \( \tau_s = 10^3 \text{ s} = 10^{13} \tau_0 \) we estimate

$$z = 30 \text{ Å}. \hfill (53)$$

Other values down to 20 Å are acceptable, but \( z = 10 \text{ Å} \) definitively is too low. The numerical value of \( z \) is not very sensitive to the value chosen for \( z \), but the width of the frequency range of the \( 1/f \) spectrum requires \( z > 10 \text{ Å} \).

In Fig. 7, we distinguish two branches of \( \alpha(n) \). The border is at \( n = m^* \). In the first place, the ratio \( n/m^* \) determines the variance: \( (\Delta n)^2 = n \) if \( n < m^* \), and \( (\Delta n)^2 = m^* \) if \( n > m^* \). But the ratio \( n/m^* \) also determines whether the downward \( \Delta n \) transitions or the upward \( \Delta m \) transitions dominate (Fig. 4).

If the \( \Delta n \) transitions dominate, then the electrons are distributed over all traps: mixing. If the transitions from a trap \( K \) to the band dominate, then the electrons recombine with \( \tau_K \): addition. Thus, \( n = m^* \) is both the border between addition and mixing, and between \( (\Delta n)^2 = n \) and \( (\Delta n)^2 = m^* \) as well.

The McWhorter model gives \( \alpha_{\text{McW}} \propto n \) or \( \alpha_{\text{McW}} \propto n^{-1} \) if the correct addition-mixing conditions are taken into account and the correct variance is used. Naive McWhorter calculations give \( \alpha \propto n^{-1} \). This is the correct result for strong inversion, where \( \langle n^2 \rangle = m^* \) automatically.

The strong-inversion line \( \alpha \propto n^{-1} \) cannot be extrapolated into the \( (n < m^*) \) region. If in this region, the correct variance \( m^* \) and the incorrect addition formalism are used, one finds the dotted line \( \alpha_{\text{McW}} = \lambda/z = 1/30 \).

Fig. 7 shows that serious errors can be made if the trap density is determined from the \( 1/f \) noise under flat band conditions. Under this condition, \( n < m^* \) holds; hence mixing occurs and the \( 1/f \) spectrum follows then from (43). But in the determination of the trap densities, one usually assumed addition [22].

6. \( \alpha_z \), The noise parameter of a device [23]

6.1. Dependence on the gate voltage \( V_{GT} \)

The number of free electrons in the channel of a MOSFET is proportional to the gate voltage \( V_{GT} \). We can easily vary \( n \) by varying \( V_{GT} \), while keeping the number of traps constant. If \( (\Delta n)^2 = \pi \), the parameter \( \alpha \) of the channel material is constant, both for \( \Delta n \)-sources and \( \Delta \mu \)-sources. But if \( (\Delta n)^2 = m^* = \text{constant} \), we expect \( \alpha \propto V_{GT}^{-1} \) according to (52). This is also the result of the McWhorter model in the case of strong inversion.
When measurements on n-channel MOSFETs showed $\varphi_{\text{exp}} \propto V_{GT}^{-1}$, the conclusion often was that 1/$\varphi$ is due to number fluctuations of the McWhorter model. This reasoning assumed that the measurements were performed on what we will call a simple channel, i.e., a channel where $\varphi$ does not depend on $s$, the distance from the interface. A simple channel is homogeneous up to the interface; there is no surface layer. In reality, a surface layer with different properties may be present. Since the noise is measured outside the channel, there may also be an influence of series resistors. In this way, the measured $\varphi_{\text{exp}}$ is some average of $\varphi_1$, of the channel and $\varphi_2$ of the complications. Therefore, we have to calculate how $\varphi_S$ follows from $\varphi_1$ and $\varphi_2$ for the complete device with complications.

6.2. $\varphi_S$ of a simple channel

If the empirical relation (6) is applied directly to the simple channel, we write $S_G/G^2 = \varphi_G/fn_{\text{tot}}$.

This direct application to the inhomogeneous channel is not always allowed. In the conducting channel $dn/ds$, $\mu$ and $\varphi$ may depend on $s$, the distance from the interface. Then (6) must be applied to thin homogeneous layers, parallel to the interface. The thickness of the layer is $ds$, its area $WL$, its concentration $dn/ds$. The number of electrons in the layer is $dn$.

$$dn = \frac{dn}{ds} ds \rightarrow n = \int_0^\infty \frac{dn}{ds} ds,$$

$$dG = \sigma W L \frac{dn}{ds} \rightarrow \mu = \frac{q^2}{L^2} \int_0^\infty \frac{dn}{ds} ds,$$

$$S = \frac{\varphi}{f \frac{dn}{ds}} (\frac{dG}{L})^2 = \frac{q^2}{L^4} \int_0^\infty \frac{dn}{ds} ds \rightarrow S = \frac{q^2}{L^4} \frac{\mu^2}{ds} \frac{dn}{ds} ds$$

In the general case with $\varphi(s)$, $\mu(s)$ and $dn(s)$, we find

$$S_G = \int_0^\infty \frac{\mu^2}{ds} (\frac{dn}{ds}) ds = \frac{1}{f} \frac{\mu^2}{ds} \int_0^\infty (\frac{dn}{ds}) ds,$$

instead of $\varphi_G/fn_{\text{tot}}$.

In a simple channel, $\mu$ and $\varphi_G$ are constants and the concentration profile depends on $s$. Relation (54) then gives

$$S_G/G^2 = \int_0^\infty (\frac{dn}{ds}) ds = \frac{\varphi_G}{fn_{\text{tot}}}.$$

Thus, for a simple channel

$$\varphi_S = \varphi_G.$$

6.3. Influence of a surface layer [23]

The MOSFET with a surface layer at the Si–SiO$_2$ interface is treated as two parallel conductors: (i) the conducting channel with $G_1$, $\varphi_1$, $L_1$, $W_1$, $t_1$ and $n_1 V_{GT}$, (ii) the surface layer with $G_2$, $\varphi_2$, $t_2$, constant $n_2$ and the same $\mu$, $L_1$ and $W_1$. The crystal lattice of the surface layer is far from perfect: $\varphi_2 > \varphi_1$.

$$\varphi_1 = \frac{n_1}{L_1^2} = \frac{n_1}{n_2}. \quad \ldots (57)$$

For the calculation of $\varphi_S$, the conductances $G_1$ and $G_2$ are added, and so are $S_1$ and $S_2$, the spectral densities of the conductance fluctuations.

$$\frac{S_1}{G_1} = \frac{S_2}{G_2} = \frac{S_1 + S_2}{(G_1 + G_2)^2},$$

$$\frac{\varphi_S}{n_2} = \frac{(\varphi_1/n_1) n_1^2 + (\varphi_2/n_2) n_2^2}{(n_1 + n_2)^2},$$

$$\varphi_S = \left[ \varphi_1 n_1 + \varphi_2 n_2 \right] / (n_1 + n_2) \ldots (58)$$

Relation (58) is approximated by considering the dominant term only in each of the expressions between square brackets. The results are presented in Table 1. Three regions are found for increasing $n_1$, A : $n_1 < n_2$, B : $n_2 < n_1 < \frac{\varphi_1}{\varphi_2} n_2$ and C : $\frac{\varphi_1}{\varphi_2} n_2 < n_1$. In each region, an expression for $\varphi_S$ is given, followed by the values of $\gamma$ for two noise sources: bulk mobility noise, $\varphi_\mu$ and McWhorter fluctuations, $\varphi_{\text{McW}}$.

Only over a limited range of $V_{GT}$-values are measurements possible on MOSFETs in strong inversion. $V_{GT}$ can be varied by a factor 10–20 at best. Therefore, one can never scan the whole $n_1$-range presented in Table 1. In most cases, $\varphi_{\text{exp}}$
Dominant terms in (58) in (62)

<table>
<thead>
<tr>
<th>Dominant terms in (58) in (62)</th>
<th>Simple channel</th>
<th>Series resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>$x_S$</td>
<td>$x_1$</td>
<td>$x_2$</td>
</tr>
<tr>
<td>$\gamma$ if $x_1 = x_\mu$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma$ if $x_1 = x_{McW}$</td>
<td>$-1$</td>
<td>0</td>
</tr>
</tbody>
</table>

$S_2 = \frac{x_2}{fn_2}R_2^2 = \frac{1}{f\pi^2\mu^2} \cdot \frac{x_2a^4}{n_2^2}$.  

Because of the analogy to (59) and (60), a point contact can be treated as a series resistance with $n_2$, $L_2 = a$ and $x_2 = x_1$.

The general case with series resistors outside the channel, where $n_2$ is constant and $n_1 \propto V_{GT}$, is described by

$$S_\Sigma = \frac{x_S}{fn_\Sigma} = \frac{S_1 + S_2}{(R_1 + R_2)^2},$$

where

$$x_S = \frac{[n_1 + n_2][S_1 + S_2]}{[R_1 + R_2]^2}.$$

Table 1 shows the approximated results. Four regions are found for increasing $n_1$: K: $n_1 < n_2$, L: $n_2 < n_1 < \left(\frac{a}{L_2}\right)^{1/3} \left(\frac{L_2}{a}\right)^{1/4} n_2$, M: $\left(\frac{a}{L_2}\right)^{1/3} \left(\frac{L_2}{a}\right)^{1/4} n_2 < n_1 < \left(\frac{L_2}{a}\right)^2 n_2$ and N: $\left(\frac{L_2}{a}\right)^2 n_2 < n_1$. The $\gamma$-values for a constriction inside the conducting channel, with $n_2 \propto n_1$ and $x_2 \propto x_1$, become

$\gamma = 0$ if $x_1 = x_2 = x_\mu$,  

$\gamma = -1$, if $x_1 = x_2 = x_{McW}$.  

6.4. Influence of series resistances [24–26]

In the case of series resistances, a model with two resistors can be considered: (i) the channel with $R_1$, $x_1$, $L_1$, $A_1$ and $n_1 V_{GT}$, (ii) the series resistances that are represented as one resistor $R_2$ with $x_2$, $L_2$, $A_2$ and constant $n_2$.

If the $x$ values do not depend on $n_1$, we take $x_1 = x_2$, because $x_\mu$-values of metals and of semiconductors are of the same order of magnitude.

$$R_1 = \frac{1}{q\mu} \frac{L_1^2}{n_1},$$

$$S_1 = \frac{x_1}{fn_1} = \frac{1}{q\pi^2\mu^2 \rho} \cdot \frac{x_1L_1^4}{n_1^2}. \tag{59}$$

Series resistors are often formed by point contacts or constrictions, where crowding effects occur. The simplest model of a point contact between a metal wire and a semiconductor is a hemisphere in the semiconductor material with radius $a$ and the normal semiconductor properties, like electron density $c$, $\mu$ and $\rho$. The number of electrons in the hemisphere is

$$R_2 \approx \frac{\rho}{a^2} = \frac{1}{q\mu} \cdot \frac{a^2}{n_1^2}, \tag{60}$$

$$S_2 = \frac{x_2}{fn_2}R_2^2 = \frac{1}{f\pi^2\mu^2} \cdot \frac{x_2a^4}{n_2^2}.$$
7. Discussion and results

In the introduction, we stated that a theoretical model should explain the shape of the spectrum \( S \propto 1/f(1.2) \) and the magnitude of the noise, i.e. the value of \( \alpha \) (1.3).

Studying the literature on experimental work on MOSFETs, we found that the voltage dependence of \( \alpha \), \( \alpha \propto V_{GT} \), plays an important part in deciding what the correct noise source is in a MOSFET, being either the bulk \( \Delta \mu \)-source or the McWhorter \( \Delta n \)-source.

These three problems set the outline of Section 7.

7.1. \( S \propto 1/f \).

The success of the McWhorter model is largely due to the easy explanation it gives for the frequency dependence, \( 1/f \). The \( 1/f \) spectrum is the result of addition of GR spectra (46). We showed that this addition is allowed if \( n > m^* \). If, however, \( n < m^* \) then the GR spectra from the different traps result in a single Lorentzian with \( 1/t_L = \sum 1/t_i \) (38), because of mixing instead of addition of the original GR spectra. Unexpectedly, we also get the \( 1/f \) spectrum (42), which is not the sum of the original GR spectra.

We analyzed what happens to a group of free electrons excited at \( t = 0 \) from specific traps \( K \). Fast decay with \( t_1 \) of the free electrons distributes them over all traps. Then—via the "empty" band—the trapped electrons very slowly recombine with the holes in the traps \( K \), from which they originally came.

The slow recombination processes produce the \( 1/f \) spectrum (42). Thus, in both cases, \( n > m^* \) and \( n < m^* \), and \( 1/f \) spectrum is produced. Our complicated analysis is of no consequence for the frequency dependence, but it is for the numerical values of \( \alpha \) and for the \( V_{GT} \) dependence of \( \alpha \).

The experimentally found frequency range of the \( 1/f \) spectrum is very wide. This requires a wide range of recombination times in the McWhorter model. In (53), we showed that there is no problem if the oxide layer is at least 30 \( \AA \) thick. But, in any case, 10 \( \AA \) is much too thin. (\( e^{10} \approx 2 \times 10^9 \)). The fact that wide \( 1/f \) spectra have been observed with MOSFETs where the oxide layer is between 10 and 20 \( \AA \) [27,28] seems a strong argument against the McWhorter model. However, one should consider quantization effects, which shift the peak in the carrier concentration away from the interface/the average trap distance remains 20–30 \( \AA \). So that the tunneling distance still permits the wide range of \( \tau \)-values [29,30].

7.2. \( \alpha \)

There is ample experimental evidence that proves that the \( 1/f \) noise in homogeneous Si layers is mobility noise (Section 2). Mobility noise has \( \pi_\mu \) values between \( 10^{-6} \) and \( 10^{-4} \). Values of \( \pi_{\exp} \) obtained from constriction resistances on homogeneous Si wafers are confined in the very narrow range \( 1 \times 10^{-6} - 2 \times 10^{-6} \) [31–33]. These are the kinds of wafers in which the MOSFET structures are made.

B implanted resistors showed \( \pi_{\exp} \) of \( 3 \times 10^{-4} \), if poorly annealed at 450 \( ^\circ \)C. After the lattice damage had been restored by annealing at 900 \( ^\circ \)C, \( \pi_{\exp} \) was down to \( 10^{-6} \) [8].

Fig. 7 shows estimates for \( \pi_{\McW} \) that follow from the McWhorter model, if we use \( N = 2 \times 10^{13} \) (47), \( M = 10^9 \) (48) and \( z = 30 \) \( \AA \) (53). The calculated \( \pi_{\McW} \) values are in a very wide range from \( 10^{-6} \) to \( 3 \times 10^{-2} \), whereas most \( \pi_{\exp} \) values are found in the range \( 10^{-6} - 10^{-4} \) [21]. In part 2, we considered the wide range of calculated \( \pi_{\McW} \) values a serious argument against the McWhorter model. But, on second thought, we realized that \( \pi_{\exp} \) values are obtained from devices in strong inversion [21] (see the right-hand side of Fig. 7). For strong inversion, the McWhorter model gives the same range \( 10^{-6} - 10^{-4} \). The numerical values of \( \pi_{\exp} \), on their own are not fatal to the McWhorter model. They do not prove or disprove the mobility model or the McWhorter model.

But free electrons in the conduction band of the channel are scattered by the phonons of the Si lattice, and—as always—produce \( 1/f \) noise [12,13]. Comparison of \( \pi_{\exp} \) and \( \pi_\mu \) show that there is no need for another \( 1/f \) noise generating mechanism. If there is any McWhorter noise at all, it does not have an appreciable contribution.
In Section 7.2, it was shown that calculated $\alpha_{McW}$ values of MOSFETs in strong inversion compare reasonably well with $\alpha_{exp}$. However, $\alpha_{McW}$ depends on the chosen values of $N$, $M$ and $z$. In literature, one finds a spread in $N$ and $M$. So, the uncertainty for an individual device is rather high, amounting perhaps to a factor $10^2$. This is why the $V_{GT}$-dependence of $\alpha$ plays such an important role in the discussion. If we write $\alpha = an^\gamma = bV_{GT}^\gamma$, it is not so much the numerical values of $a$ and $b$ that are of interest, but the exponent $\gamma$. In strong inversion $V_{GT}$ can be varied by about a factor 10. An accurate value of $\gamma_{exp}$ can easily be obtained, whereas according to different models and sources, $\gamma = -1, 0, 1$ or another integer (Section 6). Many experimental results can be classified according to Table 2.

In Table 2, “simple” and “complications” mean that the source explains $\gamma_{exp}$ in a simple channel or in a channel with complications, like series resistors or surface layers.

A $\Delta\mu$-source explains $\gamma_{exp}$ of p-channels, but for n-channels one has to resort to complications. The McWhorter model gives the opposite results: n-channels are readily explained, whereas p-channels require complications. Studies of the $V_{GT}$ dependence do not give an unambiguous confirmation of the source, $\Delta\mu$ or McWhorter.

7.4. Conclusions

The $1/f$ noise of MOSFETs is measured as conductivity fluctuations of free electrons in the Si channel. The electrons are scattered by phonons in the Si lattice, producing the ubiquitous $1/f$ noise with $\alpha_{\mu}$. Values of $\alpha_{exp}$ of MOSFETs are in good agreement with $\alpha_{\mu}$ (Section 7.2). Model calculations give estimates for $\alpha$ and $\gamma$. Calculated values obtained from a $\mu$-model or from a McWhorter $\Delta\mu$-model do not rule out any of the models (Sections 7.2 and 7.3).

If one wishes to investigate whether the $1/f$ noise is produced by a McWhorter source, then one should focus on the essential feature of that model: it is a $\Delta\mu$-model. Measurements should be performed on a property other than the conductivity. In expressions for that property, $n$ and $\mu$ should not function as factors in the produced $n \times \mu$, as they do in the conductivity (Section 2).

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References