1/f NOISE

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A survey is given of the recent literature on 1/f noise. Proposals for mathematical models, for empirical relations and for physical models are discussed.

The present situation is evaluated and some unsolved problems are indicated.

1. Introduction

In the last two years some 50 papers have appeared on 1/f noise in semiconductors and metals. Furthermore, some 10 papers have been written on 1/f noise in neuromembranes or in membranes in ionic solutions [1-10]. The latter 10 papers will not be discussed here.

That the papers appear in such large quantities is already an indication that the problem of the physical origin of this noise is not yet solved, and that many people are trying to approach it from different angles.

Three main streams can be distinguished in the modern literature on 1/f noise. They are the following:

A. Mathematical models and studies of the mathematical properties of 1/f noise.
B. The empirical relation \((\Delta R/R)^2 = \alpha f/N f\).
C. Proposals for physical models.

The papers treated here are mainly from the last two years, with a few older publications that are the sources of the streams considered here.

2. Mathematical aspects

The first group of papers we would like to discuss deals with mathematical models for 1/f noise without bothering much about physical reality. In these papers models are proposed in which the spectrum follows from the shape of pulses, correlations between pulses, random walks with some drift, or with some kind of memory, etc. [11-37]. Here it does not matter much that sometimes unrealistic physical models are proposed as realisations of the mathematical model. For instance, Tunaley's proposal for the 1/f noise in metal films assumes a hopping mechanism between islands [11]. It has experimentally been found, however, that continuous metal films also show 1/f noise. The mathematical model could essentially be correct, but one must find another model for the physical realisation. Deviations from reality are much more serious when the mathematical model is based on mathematical properties that erroneously are assumed to be experimentally established. For instance, one publication starts with the statement "It is established that the noise signals are nonstationary in that the local means and variances change with time" [12]. Whether the variance changes with time is highly questionable. Some authors say so [13-18], others deny it [19]. The best we can say is that sometimes 1/f noise can have a varying variance, but there is also "quiet" 1/f noise with constant variance [20]. Thus a varying variance is not essential for 1/f noise. Because of arguments of this kind we need good experimental investigations on the mathematical properties of 1/f noise. In this class of highly needed experiments are the investigations of Strasilla and Strutt on the variance of 1/f noise [21], and the paper by Jones and Francis that will be presented in this session [22,23].

The simplest way of obtaining a 1/f spectrum is to add generation-recombination spectra with a wide range of relaxation times \(\tau\) and a statistical weight inversely proportional to \(\tau\). This has led to physical models that are variations of McWhorter's simple model in which a constant concentration of traps in an oxide film is assumed [24]. If the rate of penetration of electrons into the film is assumed to be proportional to \(\exp(-x/x_0)\) then the required weight
\( \alpha \tau^{-1} \) follows immediately, where \( \alpha \) is the distance from the interface into the oxide film. There is nothing wrong with the mathematical model but the physical elaboration in McWhorter's model can at most be correct in very special cases (MOS transistors?). It is not the correct description of the generally occurring 1/f noise, which certainly is no surface effect.

One main problem is whether there exists a frequency below which the spectrum flattens. Up till now there is no experimental evidence for such lower limit to the 1/f spectrum. Caloyannides carried out measurements down to \( 10^{-6} \) Hz without finding any indication of flattening [25]. Realising that the integration of an exact 1/f spectrum from \( f = 0 \) to \( f = \infty \) leads to infinite results at both limits, we can go two ways. Either we stick to the exact 1/f spectrum and see what happens near \( f = 0 \), which leads us to an infinite variance and to non-existence of the mean, or we avoid these particularities by assuming that the spectrum is 1/f only over a limited frequency range.

That no flattening has been observed at the lowest frequency now attainable does not say much, since the contribution to the total power of a frequency range from \( f_1 \) of \( f_2 \) is only proportional to \( \log (f_1 / f_2) \).

Since the analytical treatment even of relatively simple models is already difficult, serious problems arise when more complex models are treated. The use of Monte-Carlo simulations then seems attractive. However, here are other pitfalls. Success of a model is often claimed when a spectrum is obtained that is more or less 1/f in a certain frequency range. When this frequency range widens if more steps are taken, then it is too easily concluded that a good model has been found. Bird has shown that this can be a computer phantom [26]. See fig. 1, where \( N \) is the number of steps used. For higher \( N \), the region where a “1/f spectrum” is found widens proportionally to \( N \), but for \( N \to \infty \) there is no 1/f spectrum at all.

3. The empirical relation \( \langle (\Delta R/R)^2 \rangle = (\alpha \Delta f/Nf) \)

3.1. Homogeneous samples

A second group of papers deals with the empirical relation

\[
\langle (\Delta R/R)^2 \rangle = C \Delta f/f = \alpha \Delta f/Nf,
\]

where \( R \) is the resistance, \( f \) the frequency, \( \alpha \) a dimension less constant with the value \( 2 \times 10^{-3} \) and \( N \) the total number of free carriers [38]. Relation (1) holds for homogeneous samples of metals or semiconductors. For all materials investigated \( \alpha \) is about \( 2 \times 10^{-3} \). The inaccuracy of the measurements is such that at this stage it cannot be decided whether \( \alpha \) has exactly the same value for all materials or whether it is slightly material dependent. Measurements on gold have shown that if there is any temperature dependence at all, it is very weak.

A practical consequence of relation (1) is that if \( N < 10^{13} \) the 1/f noise will easily be measurable above the thermal noise in the frequency range 1 Hz to 10 kHz without the current overheating the sample. This means that semiconductor samples can have dimensions of 0.1 to 10 mm whereas metal samples must be very small. Measurements of 1/f noise on homogeneous metal samples, usually in the form of films, are difficult [39–41]. Therefore, the use of point contacts is the appropriate way to study 1/f noise of metals (see section 3.2). Clarke and Voss did measurements on metal films that agreed with relation (1) [42].

The validity of relation (1) for GaAs has been disputed [43], but recent measurements on homogeneous single crystals of GaAs showed that relation (1) holds also for this material [44]. During the last few years more results have been published that prove the correctness of relation (1) [45,46]. Most of Bilger's re-
sults from ion-implanted layers in Si can also be interpreted by this relation by assuming effective concentrations and thicknesses [47].

Having accepted relation (1) as an empirical relation one may proceed in two directions:
A. One may try to apply this relation to structures more complicated than homogeneous samples.
B. One may try to find a physical interpretation for this relation.

3.2. Application to more complicated structures

When treating complicated structures one might try the following procedure. The structure is divided into small volumes. Each volume is considered to be homogeneous, and relation (1) is applied to it. By integrating \((\Delta R)^2\) or \((\Delta G)^2\) one obtains an expression for the \(1/f\) noise of the structure. This procedure proved very successful for point contacts but it failed completely for MOS transistors.

For a point contact between two spheres one finds in first approximation [48,49]

\[ R = \rho / \pi a , \]  
(2)

\[ \left\langle \frac{(\Delta R)^2}{R} \right\rangle = \frac{\alpha n^2 R^3 \Delta f}{20np^3} , \]  
(3)

where \( \rho \) is the resistivity and \( n \) the concentration of the free charge carriers. Substitution of (2) in (3) shows that (3) is essentially the same as (1), the right-hand side being equal to \( \alpha \Delta f/20n^4a^3\). The relations (2) and (3) are derived by approximating the equipotential surfaces by hemispheres. Integrating the resistances of the shells between the hemispherical equipotential surfaces yields (2). Applying (1) to the shells and integrating \((\Delta R)^2\) yields (3).

This simple model describes the experimental results very well. See fig. 2. The straight lines in this figure have been calculated from (3). In these experiments two crossed cylindrical bars are pressed together. By varying the force on the bars the diameter of the contact area \( a \) is varied, and thereby \( R \) and \((\Delta R/R)^2\). According to the model, \((\Delta R/R)^2\) should be proportional to \( R^3 \). For metals this is true. Also for semiconductors one sometimes finds \( (\Delta R/R)^2 \) proportional to \( R^3 \). However, sometimes very large deviations are observed, the noise being a factor of \( 10^3 \) higher or

![Fig. 2. 1/f Noise of point contacts of 10 metals. The solid lines represent values expected on the basis of eq. (3). From F.N. Hooge, Physica 60 (1972) 130.](image-url)
lower than calculated from (3). The solid lines are calculated according to the model of Vandamme. From L.K.J. Vandamme, J. Appl. Phys. 45 (1974) 4563.

Vandamme has shown that such $\langle (\Delta R/R)^2 \rangle$ versus $R$ plots can be explained by taking into account an oxide film on the contact members [50]. By adding the noise contributions from the oxide film and from the constriction in the contact members, he obtained a rather complicated relation for the 1/f noise of such a contact. There are two extreme situations:

(i) Constriction dominated contacts, i.e. the influence of the film on the resistance and on the noise is negligible.

\[ a \propto F^{1/3}, \quad R \propto a^{-1} \propto F^{-1/3}, \]

\[ \langle (\Delta R/R)^2 \rangle \propto a^{-3} \propto R^3. \]

(ii) Film dominated contacts, i.e. the film determines the noise and the resistance

\[ a \propto F^{1/3}, \quad R \propto a^{-2} \propto F^{-2/3}, \]

\[ \langle (\Delta R/R)^2 \rangle \propto a^{-2} \propto R. \]

Here $a$ is the radius of the contact area and $F$ the force on the contact members. In this interpretation the film is no "deus ex machina" that has to explain the observed deviations, but the presence of the film is experimentally demonstrated in $R$ versus $F$ plots: (a) without film $R \propto F^{-1/3}$, (b) film dominated $R \propto F^{-2/3}$

The value of $\rho_{film}$ and $\mu_{film}$ that were estimated from noise measurements of film dominated contacts are of the right order of magnitude.

Another example where relation (1) has been applied is the multiple spot contact [51]. We assume that such a contact consists of $k$ parallel conducting spots each with a radius $a$. If the spots are relatively wide apart. The noise is given by

\[ R = \rho/kna, \]

\[ \langle (\Delta R/R)^2 \rangle = k^2 \frac{\alpha \pi^2 R^3 \Delta f}{20np^3 f}. \]

The relative noise of a multiple spot contact with $k$ spots is a factor $k^2$ higher than the noise of a single spot contact with the same resistance. By measuring the resistance and the 1/f noise of a contact, $k$ and $a$ can be determined. Here 1/f noise serves as an instrument for analysing contacts. In this way the formation of metal–semiconductor contacts by repeated discharges through the contact can be followed during the "fritting" procedure without destroying the contact. One can follow how $k$ and $a$ have grown after each discharge [52].

Much work has been done on the 1/f noise of MOS transistors [53–58]. See fig. 4. Here the 1/f noise is generally assumed to be generation–recombination noise from traps at the surface with a wide range of relaxation times. The models proposed are variations of the simple McWhorter model. There is good experimental evidence for the 1/f noise being proportional to the concentration of the surface states $C_{ss}$. It has been shown [56] that this proportionality remains true even for very low concentrations $C_{ss} = 10^9$ cm$^{-2}$ eV$^{-1}$. Thus the MOST seems clearly to illustrate that
1/f noise is a surface effect, whereas according to relation (1) it is a volume effect. Therefore, the MOST is a challenge to those who have faith in relation (1).

One may try to treat the noise of the MOST by dividing the device into thin slices perpendicular to the oxide layer. One then calculates the concentration \( n(x) \) as a function of \( (V_{\text{gate}} - V(x)) \). After applying relation (1), \( (\Delta R)^2 \) is found. Then by integration follows \( (\Delta R)^2_{\text{SD}} \). This procedure fails even for the very simple case where \( V_{\text{SD}} \ll V_{\text{gate}} \) so that the device is homogeneous in the SD direction. It should then give 1/f noise as if it were an ohmic resistor. The experimentally determined noise is \( 10^3 \) to \( 10^4 \) times as small as the noise calculated by this procedure.

One often finds such large discrepancies in inhomogeneous situations. For instance, the GaAs devices used by Mircea et al. gave 1/f noise that did not agree with relation (1) \([43]\). The ion-implanted samples of Bilger generally agree with that relation \([47]\). But by putting a voltage across the p–n junction separating the implanted layer from the underlying material, he changed the concentration in the depletion layer in such a way that \( R \) did not change much but \( (\Delta R/R)^2 \) changed a great deal.

Probably, relation (1) is not the fundamental relation for describing local 1/f noise. There must be some other relation that for homogeneous samples simplifies to relation (1). Honig \([59]\) tried to reformulate relation (1) for local noise, suggesting that (1) should be rewritten as

\[
\left\langle \frac{(\Delta V/V)^2}{\nu^4} \right\rangle = \int \frac{\alpha F^4}{np^2} \, d\tau \cdot \frac{R^2 \Delta f}{V^4 f},
\]

where \( F \) is the field strength.

An inhomogeneous current density such as in point contacts does not seem to jeopardise the application of relation (1), whereas an inhomogeneous concentration seems to forbid such application.

3.3. Towards an interpretation of the empirical relation \( (\Delta R/R)^2 = \alpha \Delta f/Nf \)

If we accept that the 1/f noise is a fluctuation in the conductivity, then we meet the problem of what is actually fluctuating, the concentration of the charge-carriers or their mobility? This problem was treated some years ago. If we believe that the 1/f noise of ionic solutions is physically the same as the 1/f noise of electrons and holes, then the analysis of the noise in the voltage of a concentration cell is relevant. See fig. 5. We studied concentration cells where a membrane with a small hole separates two compartments filled with solutions of the same salt but with different concentrations \( c_1 \) and \( c_2 \). Across the point contact formed by the hole, there was a diffusion voltage. Between two electrodes in the solutions the cell voltage and its fluctuations were measured.

The 1/f noise of concentration cells is closely connected with the noise properties of nerve cells where 1/f noise is found in the voltage across the membrane of the node of Ranvier. In fact the 1/f noise in concentration cells was discovered on the nodes of Ranvier \([60]\).

For the fluctuations in the cell voltage one experimentally finds \([61]\)

\[
(\Delta V/V)_{c_1, x_2}^2 = (\Delta R/R)^2_{c_2}.
\]

The interpretation \([38]\) starts with the equations for the currents of the positive and negative ions. One
takes the total current equal to zero, and finds an expression for the cell voltage. Then one substitutes \( n + \Delta n \) in the expressions for the currents and finds an expression for \( \Delta V_{\text{cell}} \). The relative noise in the cell voltage is found to be much smaller than in the concentration cell model. The model with \( 1/f \) fluctuations in the concentrations does not agree with the experimental result (7). If one substitutes \( \mu + \Delta \mu \) for \( \mu \) in the expressions for the currents, then the relative noise in the cell voltage equals the relative noise in the concentration cell model. Thus it has been proved that the \( 1/f \) noise stems from mobility fluctuations.

His analysis is much more detailed than the previous ones. The valence and conduction bands are divided into very narrow subbands with concentrations \( n(E) \) and relaxation times \( \tau(E) \). \( E \) is the kinetic energy of the charge carriers. He introduces \( \Delta n, \Delta n(E) \) and \( \Delta \mu(E) \) successively into the expressions for the currents.

In the first model the total number of electrons in the conduction band fluctuates. Fermi distribution is assumed for \( \Delta n \). This is the model that is implicitly used by McWhorter and others who consider \( 1/f \) noise to be a summation of generation–recombination spectra.

The second model with \( \Delta n(E) \) assumes a constant total number of free electrons. There is a continuous redistribution of electrons between the various subbands \( (\Delta n(E))^2 \propto n(E) \). Although \( \mu(E) \) does not fluctuate, the average mobility \( \bar{\mu} = \int_0^\infty n(E)\mu(E)dE \) does.

In the third model with \( \Delta \mu(E) \) the concentrations in the subbands \( n(E) \) are constant. It is assumed that there is no correlation between the mobility fluctuations in the various subbands. To each subband one applies relation (1), here in the form

\[
\frac{\langle (\Delta \mu(E))^2 \rangle}{\langle \mu(E) \rangle^2} = \frac{\Delta f}{n(E)}.
\]

### Table I

| Values of \( \beta \) calculated for different models all assuming lattice scattering. |

<table>
<thead>
<tr>
<th>Fluctuations in ( n )</th>
<th>( \beta )</th>
<th>Intrinsic Ge</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total ( n ) fluctuates</td>
<td>( \beta_{\text{exp}} = 1 )</td>
<td>Charge neutrality ( n = p )</td>
<td>1</td>
</tr>
<tr>
<td>Fermi distribution of ( \Delta n(E) )</td>
<td>( \beta_{\text{exp}} = 1 )</td>
<td>Equilibrium ( np = C )</td>
<td>1600</td>
</tr>
<tr>
<td>( \Delta n = \int_0^\infty \Delta n(E)dE )</td>
<td>( \beta_{\text{exp}} = 18.2 )</td>
<td>( (\Delta \sigma_n)(\Delta \sigma_p) = 0 )</td>
<td>160</td>
</tr>
<tr>
<td>( n(E) ) fluctuates</td>
<td>( \beta_{\text{exp}} = 2.75 )</td>
<td>( (\Delta \mu_n)(\Delta \mu_p) = 0 )</td>
<td>160</td>
</tr>
<tr>
<td>Total ( n ) is constant ( \mu(E) ) is constant</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Extrinsic Si or Ge | Intrinsic Ge | \( \beta_{\text{exp}} \) |
The introduction of $\Delta n$, $\Delta n(E)$ or $\Delta \mu(E)$ into the expressions for the currents leads to expressions for $\Delta V_{\text{therm}}$. It is convenient here to express the noise in the thermovoltage as a dimensionless constant, $\beta$ which relates this noise to the noise in the conductance.

$$\beta = \frac{\langle (\Delta V_{\text{therm}})^2 \rangle / \langle k(T_1 - T_2)/q \rangle^2}{\langle (\Delta V_{\text{applied}})^2 \rangle / V_{\text{applied}}^2}.$$  \hfill (9)

In this way the experimental results can be expressed independently of the contact geometry and the temperature difference used. On the same point contact one first measures the noise in the voltage across the contact by letting a constant current flow through the contact at $T_1 - T_2 = 0$. Then one applies a temperature difference and measures $\Delta V_{\text{therm}}$ at the open thermo cell.

The results are represented in table 1. Agreement between model and experiment is only found for mobility fluctuations. To this analysis one can add the case that the fluctuations in the mobilities are completely correlated. Then one finds $\beta = 0$, for intrinsic and extrinsic thermo cells. Further confirmation that the mobility fluctuates comes from measurements of the $1/f$ noise on thermo cells where simultaneously with the thermovoltage an external voltage is applied. Fig. 6 shows the influence of the applied voltage $V_a$ on the noise in the three models. The experimental results are indicated by circles. Excellent agreement exists between the experimental results and the $\Delta \mu(E)$ model. The conclusion is obvious: $1/f$ noise is a fluctuation in the mobility also for electrons in semiconductors.

4. Proposed models

In the field of $1/f$ noise we have seen a long tradition of models being proposed that are forgotten after a short time. The physics they are based upon is either too special or too artificial and sometime it is simply wrong. This tradition seems to continue. Several proposals have been made during the last two years [42, 64–77]. A few proposals, however, are so interesting that they must be mentioned here.

The McWhorter model with generation–recombination noise from surface traps is still popular, especially in work on MOSTs.

A new class of models starts from fluctuations in the temperature. Physical properties, depending on temperature then fluctuate with the same spectrum as the temperature. Two models have been worked out, one by Müller, another by J. Clarke.

Müller has a model for $1/f$ noise in p–n junctions [70–74]. This theory is not applicable to homogeneous samples. The temperature of the junction fluctuates and thereby the current through the junction. The range of time constants follows incorrectly from dividing the junction into small subjunctions. The limiting frequencies of the $1/f$ spectrum are given by the longest and shortest thermal time constant corresponding to the complete junction and the smallest subjunction.

Clarke and coworkers have a model that is applicable to homogeneous samples [42, 75–77]. They start with temperature fluctuations

$$\langle (\Delta T/T)^2 \rangle = k/C_v ,$$  \hfill (10)

where $C_v$ is the heat capacity. The noise in the resistance is then given by

$$\langle (\Delta R)^2 \rangle = (\partial R/\partial T)^2 \langle (\Delta T)^2 \rangle f .$$  \hfill (11)

The spectrum of the temperature fluctuations follows from the equation for heat diffusion. For three-dimensional diffusion the spectrum has four regions.
where the power is proportional to $\omega^0$, $\ln \omega^{-1}$, $\omega^{-1/2}$ and $\omega^{-2/3}$. For two-dimensional diffusion there are three regions in $\omega^{-1}$, $\omega^{-1/2}$ and $\omega^{-2/3}$ and for one-dimensional diffusion $\omega^{-1/2}$ and $\omega^{-2/3}$. So the best that Clarke and coworkers can obtain is a “1/f like” spectrum, as they explicitly state in an early publication. Although these temperature fluctuations do exist, they will not have 1/f spectra. With the dimensions of the samples generally used one obtains a white or rather flat spectrum in the usual frequency range. Such spectra have also been found for ionic solutions [78].

According to this theory metals with a low temperature coefficient of their resistivity would have a low 1/f noise. Clarke and Voss did experiments on thin layers of manganin where no 1/f noise was detected [42]. However, Vandamme measured contact noise on solid manganin bars [79]. The usual noise versus resistance plots were obtained with $\alpha \approx 2 \times 10^{-3}$. There is nothing special in the 1/f noise of manganin with its low temperature coefficient of resistivity.

We have seen a rather vivid discussion on noise measurements on incandescent tungsten filaments where also the problem of temperature fluctuations played an important part [80–83]. Measurements on filaments heated by ac or dc currents should discriminate between normal 1/f noise and temperature noise. The result is that the white thermal noise is increased owing to the higher temperature and that there is the usual 1/f noise although its intensity does not agree with relation (1). We would like to add two remarks to this discussion. (i) The contacts between the tungsten filament and its supports will generate noise that in this two-point arrangement will erroneously be attributed to the filament. (ii) Solid tungsten bars have the usual 1/f noise with $\alpha \approx 2 \times 10^{-3}$. This was shown by point contact measurements where Vandamme found ideal noise versus resistance plots [79].

At the end of this section on physical models I would like to remark that we need more experimental work on the physical properties of 1/f noise, just as we do on the mathematical properties. In this respect the paper by Voss and Clarke to be presented in this session is important. They experimentally treat the problem of the possible non-equilibrium character of the 1/f noise. In other words they investigate whether the current causes the noise or whether the noise is always present, the current only making it possible to measure it [84].

5. Where do we stand?

A fundamental, yet unsolved problem is: Is 1/f noise a general phenomenon always accompanying electrical conduction or must we make very specific models for each individual case. I bet on the general character.

In the study of mathematical properties of this noise we see a great activity in the field of the confrontation of theoretical models and experiments. One point is the experimental determination of statistical properties of 1/f noise, another is the discussion of possible errors introduced into the results by the experimental techniques applied.

We have now an empirical relation $\langle \Delta R/R \rangle^2 = \alpha \Delta f/N f$, implying the general character of 1/f noise. However, we cannot apply it to all inhomogeneous situations. There is good experimental evidence that the conductance fluctuations must be interpreted here as mobility fluctuations. No model has been proposed for such mobility fluctuations.

When we accept that 1/f noise demands a wide range of relaxation times, with statistical weights proportional to $\tau^{-1}$, then 1/f noise might be a specific case of a much wider problem. In the field of dielectric [85] and magnetic [86] relaxations the same set of relaxation times is required for the explanation of logarithmic decay and loss angles independent of frequency. Also in the purely mechanical problem of anelastic creep we demand this wide set of relaxation times over many decades [87]. The ad hoc explanations offered in those fields are not better than our proposals for 1/f noise. On this point we need not feel ashamed.

Acknowledgements

Writing this review was easy after the pleasant weekly discussions on the current literature that I have had with my colleagues Th.G.M. Kleinpenning, L.K.J. Vandamme and H.G.E. Beck.
References

[38] F.N. Hooge, Physica 60 (1972) 130.