Abstract

The Ring Imaging Cherenkov detectors of LHCb will use pixel Hybrid Photon Detectors to measure the spatial position of Cherenkov photons. The first six pre-production photon detectors have been tested in a beam, together with prototypes of the on-detector electronics. The tests were performed at CERN using 10 GeV/c pions together with an N₂ gas radiator as a source of Cherenkov light. With 1.1 m of radiator, around 10 photoelectrons were detected per track. The single-photon Cherenkov angle resolution was measured to be 1.66 ± 0.03 mrad, which is dominated by the pixelisation of the photon detector in the test-beam set-up. Both numbers agree with expectations.

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Keywords: Ring Imaging Cherenkov detectors; Hybrid Photon Detectors; LHCb experiment

1. Introduction

The LHCb experiment [1] will make high precision studies of CP violation and other rare phenomena in B meson decays. Particle identification (PID) in the momentum range from a few to ~100 GeV/c is essential for this physics programme. In order to provide the PID capability, two Ring Imaging Cherenkov (RICH) detectors will be employed [2]. Three different radiator materials: silica aerogel, and the fluorocarbon gases C₄F₁₀ and CF₄, will be used to produce Cherenkov light from charged particles traversing the detectors. The RICH detectors employ pixel Hybrid Photon Detectors (HPDs) to measure the spatial position of Cherenkov photons [3]. Photons with wavelengths in the range ~200–600 nm will be detected over a total active area of 2.6 m². The HPDs have a granularity of 2.5 mm × 2.5 mm, a time resolution
compatible with the LHC bunch-crossing rate of 40 MHz and an active-to-total area ratio of ~70%. In addition, the HPDs and the on-detector electronics must be radiation tolerant to a dose of up to 30 Gray/year.

HPD prototypes have been developed and subjected to extensive laboratory and beam tests [3]. Before launching the production of 550 of these detectors, a beam test was performed with the first six pre-production photon detectors and prototypes of the on-detector electronics system [4] to validate the performance. In the following sections, the photon detectors, the readout system and prototype RICH detector are briefly described, before presentation of the test-beam data.

2. The hybrid photon detector

The pixel HPD has been developed in close collaboration with industry. The HPD consists of a pixelated silicon detector anode assembly which is encapsulated inside a vacuum envelope (Fig. 1). The anode assembly is divided into 256 × 32 pixels of area 62.5 × 500 μm², with each pixel bump-bonded to a channel of the LHCbPIX1 binary readout chip [5]. Groups of eight pixels are OR-ed together in this chip, creating 1024 “super-pixels” of area 500 × 500 μm².

The HPD vacuum tube has a 7 mm-thick quartz entrance window which has a spherical surface and an S20 multi-alkali photocathode deposited on the inside. The typical quantum efficiency is around 25% at a wavelength of 270 nm. Photoelectrons emitted from the photocathode are accelerated onto the anode assembly by a 20 kV cross-focussing electron optics system. This typically produces 5000 electron–hole pairs in the silicon. The resulting image on the anode has a demagnification factor ~5 with distortions, caused by variations of the linear demagnification along the radial distance, below 10%. Such distortions can be corrected for after the data have been read out. The active diameter of the entrance window is 75 mm, giving an intrinsic active area of the HPD exceeding 80%. When the tubes are positioned in a hexagonal close-packing arrangement, the tube active-to-total surface ratio then becomes 70%.

Photoelectrons backscattered at the silicon surface of the HPD contribute to an inefficiency of the hit detection, since the energy of the primary cluster is reduced. The detection efficiency after photon conversion at the photocathode has been measured to be 85%. Ageing tests simulating 10 years of LHCb operation have shown no observable deterioration in this efficiency [6].

3. The readout electronics

The first stage of the RICH readout is the LHCbPIX1 binary pixel chip. The pixel chip is fabricated in 0.25 μm CMOS technology using a layout adapted for tolerance to ionising radiation and single event upset. The readout chip discriminates single photoelectron hits, resulting in binary data which are then read out on receipt of a trigger signal. The chip has a front-end amplifier with 25 ns shaping time and a discriminator giving a threshold of ~1500 e⁻ with a pixel-to-pixel (RMS) spread of 200 e⁻. The low noise characteristics allow detection of single photoelectrons, including those that experience charge sharing among neighbouring pixels (which can have up to a factor of ~2 lower signal amplitudes), and those that backscatter.

In the test-beam, in contrast to the final LHCb experiment, the particle arrival time and hence the trigger signal are asynchronous with respect to the supplied 25 ns clock which drives the pixel chip. The efficiency for a given photoelectron hit to pass threshold is therefore a function of the hit arrival time within the readout window. This asynchronicity causes inefficiencies in photoelectron detection that are corrected for in the subsequent analysis.

The test-beam readout scheme is shown in Fig. 2. A pair of HPDs are connected via a translator board to the so-called “Level-0” front-end readout board, mounted on-detector. The translator board is passive, and connects the HPD to the Level-0 board via a pin-grid array on the HPD and ~10 cm of Kapton cable (Fig. 3). The HPDs and Level-0 boards are mounted onto columns which also include low voltage and high voltage boards [7] to service the HPDs.

The functionality of the Level-0 board is implemented on the “PINT” chip, a radiation-tolerant fuse-link Field Programmable Gate Array (FPGA). The Level-0 board multiplexes the data from a pair of HPDs onto optical links to the off-detector (Level-1) electronics. The synchronisation of the front-end electronics and the local distribution of the clocks and triggers is achieved on the Level-0 board using the Timing, Trigger and Control Receiver ASIC (TTCvi) [8]. Following a Level-0 trigger accept, the data from a pair of pixel chips are multiplexed out at a 40 MHz rate as two 32-bit streams. The multiplexing of 32 is chosen to match the maximum average LHCb Level-0
trigger rate of 1 MHz. The 8-bit/10-bit encoding protocol [9] is used to drive the optical fibres at a 1.6 GHz rate.

For the test-beam measurements, the data from the Level-0 electronics were received by two 4-channel optical receiver S-link modules. The modules perform zero-suppression as an option, data multiplexing, buffering, formatting and re-transmission. These functions are implemented in programmable logic using an FPGA. The formatted data are then transmitted to a data acquisition PC using a custom S-link to PCI interface [10].

4. The prototype RICH detector and test-beam set-up

Beam tests with 10 GeV/c negative pions and electrons were performed at the CERN-PS in the T9 facility. A threshold Cherenkov counter in the beam was used to select pions with a small electron contamination (≤1%). The residual electron contamination was further reduced by the event selection described in Section 6. The beam acceptance was defined by two 2 x 2 cm² scintillators, positioned either side of the detector, which were used to generate the (asynchronous) trigger. Typically, ~10⁸ pions were triggered over the 2.2 s continuous spill from the accelerator. The low intensity used resulted in a negligible probability of multiple particle events. The effect of δ-rays produced in the radiator gas and vessel windows was also negligible.

The prototype RICH detector is shown schematically in Fig. 4. The prototype was large enough to allow three full-sized columns of HPDs to be mounted inside it. The columns slide into the detector housing on rails. The six pre-production HPDs used for these tests were mounted in a close-packed arrangement, identical to the one that will be used in LHCb. The six HPDs were labelled L0, L1 (left), C0 and C1 (central) and R0 and R1 (right) (see Fig. 5). A known timing problem, inherent to the test-beam set-up, affected HPDs C0 and R0. These HPDs were not considered for further analysis.
The light-tight RICH vessel contained the N₂ gas used as the Cherenkov radiator. Beam particles entered the vessel through a thin aluminium foil and passed through the centre of a mirror which was inclined at an angle of 13.4° to the beam axis. Initial alignment relative to the particle beam axis was performed using a laser. The mirror was parabolic and had a focal length of 1016 mm, a diameter of 200 mm, and a reflectivity of better than 90% over the wavelength range 225–450 nm. The mirrors in the final LHCb RICH detectors will be spherical; however, the coating was the one that will eventually be used [11]. The HPD detector plane was fixed at a distance of 1047 mm from the mirror centre, with dimensions chosen to ensure that the Cherenkov rings from N₂ could be fully contained within a single HPD. The length of radiator seen by charged particles was 1102 mm. The HPDs themselves contained in an N₂ environment, were isolated from the Cherenkov gas by a 5 mm-thick quartz plate. In position, the HPD photocathodes were 50 mm from the quartz window.

5. Simulation of the test-beam set-up

The test-beam set-up was simulated using a Monte Carlo (MC) model based on the GEANT4 software toolkit [12]. The program was used to simulate the geometry of the experimental set-up and the various physics processes, including the generation of Cherenkov light. The passage of the beam of charged particles through the detector took into account the beam divergence, beam composition and processes such as multiple scattering and decays in flight. The refractive index of nitrogen was parameterised using Sellmeir coefficients [13].

Cherenkov photons were generated at the relevant Cherenkov angle with a uniform distribution of photon energy, and the photons were then traced through the experimental set-up. Photons incident on the surface of the mirror, the quartz window of the vessel or the HPD quartz window were reflected and refracted according to the Fresnel equations. Laboratory-measured optical characteristics of the various components of the detector were used.

For photons incident on an HPD photocathode, photoelectrons were created according to the measured quantum efficiency of that HPD. Photoelectrons were assigned an energy corresponding to the electric potential between the cathode and anode of the HPD, i.e. 20 keV. The photoelectron directions were set in accordance with the laboratory-measured cross-focussing and point-spread functions (PSFs). The PSF is defined as the RMS of the photoelectron distribution at the anode, resulting from a point source of photoelectrons at the cathode. A simple model for the sharing of charge between adjacent pixels and the effect of the anode efficiency was also included in the simulation.

6. Event selection

Events triggered by a coincidence between the two scintillators, and the absence of a veto from the beam threshold Cherenkov counter, were recorded for analysis. A number of cuts were then applied to the data and used in all further analysis.

- To ensure that a well-defined Cherenkov ring could be reconstructed, at least four pixel hits were required in an event.
- In order to provide an effective veto on residual electron events, the average ring centre for all rings within a run (typically ~30 000 events) was found. The radial distance of a hit from that centre, \( r \), was then determined for each hit. Two regions were defined, namely \( r_1 < r < r_2 \) and \( r > r_2 \). The values of \( r_1 \) and \( r_2 \), 15.8 and 23.0 mm, respectively, were chosen to contain all hits from the pion rings within \( \pm 3\sigma \), while giving only a small fraction of the larger radii electron rings. Events were selected if they had at least four hits in the inner region but no more than three hits in the outer region.

After these cuts, residual electron contamination was estimated to be at the 0.1% level and there was a negligible loss of pions.

The superimposed ring images from events in a run which survive the above requirements are shown for HPD C1 in Fig. 6. For this HPD the demagnification factor was measured to be 5.64. On average ~10 hits are observed per event. The ring has a diameter of ~14 pixels, which, accounting for the HPD demagnification, corresponds to ~39 mm on the photocathode.

7. Photoelectron yields

The photon yields from the RICH detector were evaluated by counting the numbers of photons in N₂ rings, focussed onto each of the HPDs in turn. A number of
effects can cause inefficiencies in the measured number of photons, or deviations from being Poisson distributed, which necessitates the following corrections to the photon hit count:

- The asynchronous arrival of beam particles with respect to the readout clock results in an inefficiency for photoelectron detection. This inefficiency is a function of the hit arrival time within the 25 ns readout window. The effect is minimised by fine-tuning the timing of the detector; however, residual inefficiencies remain.
- Photoelectrons striking the silicon close to the edge of a pixel will result in charge sharing, causing both the struck pixel and an adjacent neighbour to register a hit.
- Two photoelectrons may strike the same pixel and, since the readout is binary, this will give rise to only a single hit.

### 7.1. Timing corrections

An inefficiency occurs due to the imperfect timing of the HPD-pixel readout. When sufficient charge is deposited in a given pixel such that the signal is above the discriminator threshold, a digital pulse is generated to signify that the pixel has been hit by a photoelectron. In the final LHCb experiment, this binary signal will be synchronously timed with respect to the 25 ns LHC bunch-crossing clock. The analogue photoelectron pulse will rise and fall within the 25 ns bunch-crossing window and will be sampled at a well-defined time. A single fine-timing adjustment of the trigger–strobe pulse will then be necessary to ensure a hit is always assigned to the bunch-crossing which triggered the event. In principle, in the absence of time jitter, time-walk and particle propagation time effects, the timing can be perfectly adjusted such that the pixel data would be fully efficient if the pulse is strobed at any time within the 25 ns beam-crossing window.

However, in the test-beam, the beam particles arrive randomly in time over the beam spill, and hence asynchronously with respect to the 25 ns clock externally supplied to the pixel chip. The pixel efficiency is then defined by a convolution of two “top-hat” timing functions; the first represents the random arrival time of the beam trigger within the 25 ns window, the second represents the variation of the relative timing of the trigger–strobe and the 25 ns clock, which can be externally adjusted to maximise the efficiency, as described above. The hit efficiency therefore has a triangular distribution as the timing delay between the trigger and clock is varied, in principle 50 ns wide, with full efficiency only occurring when the timing of the two top-hat functions exactly overlap.

Data were taken with different delays between the trigger and the 25 ns clock signals. The relative efficiency of HPD C1 as a function of this delay is shown in Fig. 7. As expected, an approximately triangular dependency with a 50 ns width is observed, however, with rounded top and edges.

In order to understand the timing distribution, a “resolution function” was used to model the charge deposition arriving at the pixel chip discriminator as a function of time. A Gaussian function representing the jitter and finite rise time of the amplifier response was used. A further refinement consists of introducing an exponential tail to model the effects of charge sharing and back-scattering, which increase the time taken for the charge collected in a given pixel to exceed the relevant threshold (time-walk). The resolution function is then given by a convolution of the terms

$$f(t) = \exp\left(-\frac{t}{\tau}\right) \otimes \frac{1}{\sqrt{2\pi}\sigma^2}\exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right)$$

(1)
where $\tau$ is the exponential decay constant used to model the effect of time-walk and $\sigma$ is the RMS of the Gaussian time spread. The readout efficiency for a photoelectron arriving at a given time, $t$, is then the integral of the resolution function between times $t$ and $t + 25\text{ns}$. Examples of the readout efficiency as a function of time are shown for HPD C1 in Fig. 8, where zero indicates the start of the readout window.

The final step required to model the triangle function of Fig. 7 is to convolute the efficiency function of Fig. 8 with a 25 ns top-hat function which represents the random arrival time of the trigger pulse within the readout window. The resulting convolution, the curve in Fig. 7, demonstrates the good agreement of the model compared to the data for HPD C1. Fits to the timing distributions of individual HPDs give resolution functions with $\sigma$ and $\tau$ values in the range 1–4 and 4–6 ns, respectively.

At the first stage of data-taking, all HPDs were timed with beam to maximise the readout efficiency, i.e. each trigger/strobe clock timing delay was adjusted to the peak setting of the triangular timing distribution. In LHCb, the timing will likewise be optimised so that the arrival of pixel signals correspond to the peak of the efficiency curve, an example of which is given in Fig. 8. The figure indicates that, in order to maximise the efficiency, the timing of the clock and trigger–strobe pulses must be optimised to within a few nanoseconds.

### 7.2. Charge sharing and double hits

The photons radiated from a 10 GeV/c pion in an N$_2$ radiator (~10) are spread over approximately 40 pixels. There is therefore a significant probability of two genuine photons giving rise to photoelectrons which strike adjacent pixels. However, the sharing of charge between adjacent pixels can also result in a single photoelectron giving rise to two (or more) pixel hits. Charge sharing results in an increase in the number of adjacent hits on the pixel chip above those that are expected from the genuine adjacent hits.

The contribution from charge sharing was extracted by combining single hits from different test-beam events into new ‘fake’ events, in which the charge-sharing contribution was removed. Given a normal event with $n$ hits, a fake event was formed by taking one hit at random from $n$ different events and combining them to form a new event. Having taken only one hit from any single event, these new fake events had no contribution from charge sharing. By comparing the fraction of events with adjacent hits in normal and fake events, the charge sharing contribution could then be determined. The probability for charge sharing, $s$, was measured for each of the HPDs and found to be in the range 2–4%. The variation between HPDs was the result of the different thresholds applied.

The fake events also allowed an estimate to be made of the probability that a photoelectron would hit a pixel which had already registered a hit from a different photon. The probability, $p$, for having such a double hit shows a linear dependence with respect to the number, $N$, of hits in the event, $p = d(N - 1)$. The coefficient $d$ was determined for each HPD and is of the order of 0.5%.

### 7.3. Extracting the photoelectron yield

The distribution of the number of observed hit pixels per event was fitted with a model in which the only free parameter was the photoelectron yield, $\mu$, namely the Poisson mean of the number of photoelectrons produced.

For a given track arrival time within the readout window, the probability $P(n; \mu, \varepsilon, s, d)$ to observe $n$ pixel hits given a photoelectron yield, $\mu$, a detection efficiency, $\varepsilon$, a charge sharing probability, $s$, and a double hit coefficient, $d$, is

$$P(n; \mu, \varepsilon, s, d) = \frac{1}{n!} \sum_{i=0}^{n} \sum_{j=0}^{\infty} P_{\mu}^{i} P_{\varepsilon}^{n-i} P_{s}^{j} P_{d}^{n-i+j(n-i+j-1)d}$$

(2)

where $P_{\mu}$ is the Poisson probability to observe $m$ pixel hits given a Poisson mean $\mu$. The first term is the probability of observing $(n - i + j)$ photoelectrons, given that a Poisson mean of $\varepsilon \mu$ is expected; the second term gives the probability of there being $i$ charge-shared hits; and the third term gives the probability that $j$ pairs of photoelectrons hit the same pixel. The latter two terms were found to be necessary in order to reproduce the distribution seen in the data.

If the track arrival time, $t$, had been measured, the above Poisson-like distribution of photoelectron hits would be multiplied by a known efficiency factor according to the curve of Fig. 8. However, since $t$ is not measured, it is necessary to take the sum of all Poisson-like distributions, multiplied by the corresponding efficiency factors

$$P(n; \mu, s, d) = \frac{1}{T} \int P(n; \mu, \varepsilon(t), s, d) \, dt$$

(3)
where $T$ is the width of the time window and the integral is taken over all times within the readout window. The efficiency factor $e$ includes the timing correction from Fig. 8, the 85% photoelectron detection efficiency and a correction to account for a small number of dead pixels.

In Fig. 9 the distribution of the number of hit pixels per event is compared to the corresponding fit, using the above model, for HPD C1. It can be seen that, given the large uncertainties in the model parameters (see Section 7.4), the model satisfactorily reproduces the distribution seen in the data. In Table 1 a summary is given of the fits for all HPDs. The last column gives the ratio of photoelectron yield from the fit to the expected value from the MC described in Section 5. The yields observed agree with the expectations for all HPDs. The determination of the errors quoted are described in the next subsection.

7.4. Systematic errors on the extraction of the photoelectron yield

Sources of uncertainty contribute to both the fitted mean number of photoelectrons, $\mu_{\text{fit}}$, and the expected mean number of photoelectrons, $\mu_{\text{exp}}$.

Five sources of uncertainty contained within the fitting process were evaluated. These were the uncertainties in the values of the fraction of charge sharing and double hits, the level of timing jitter, and the value of the time constant in the timing model. To evaluate these uncertainties, the fits to the number of hit pixels were repeated, varying the input values for each of the above sources over a range corresponding to their uncertainty. In Table 2, the resulting variation in the mean of the fit is shown, together with the range of values and uncertainties for the above effects. The systematic error on $\mu_{\text{fit}}$ is dominated by the uncertainty in the level of charge sharing.

Table 1

<table>
<thead>
<tr>
<th>HPD</th>
<th>Pixel corr.</th>
<th>$\chi^2$/ndf</th>
<th>$\mu_{\text{fit}}$</th>
<th>$\mu_{\text{exp}}$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0</td>
<td>1.00</td>
<td>21/14</td>
<td>8.7 ± 0.2</td>
<td>8.9 ± 0.6</td>
<td>0.98 ± 0.10</td>
</tr>
<tr>
<td>L1</td>
<td>0.95</td>
<td>12/16</td>
<td>10.1 ± 0.1</td>
<td>10.7 ± 0.7</td>
<td>0.94 ± 0.07</td>
</tr>
<tr>
<td>C1</td>
<td>0.85</td>
<td>30/16</td>
<td>11.5 ± 0.3</td>
<td>11.2 ± 0.6</td>
<td>1.03 ± 0.08</td>
</tr>
<tr>
<td>R1</td>
<td>0.96</td>
<td>37/16</td>
<td>10.2 ± 0.1</td>
<td>10.0 ± 0.4</td>
<td>1.02 ± 0.06</td>
</tr>
</tbody>
</table>

The table shows the dead pixel correction, the quality of the fit, $\chi^2$/ndf, the extracted photoelectron yield from the fit, $\mu_{\text{fit}}$, the expectation from the Monte Carlo, $\mu_{\text{exp}}$ and the ratio of the observed to the expected yield. The error on $\mu_{\text{fit}}$ is systematic only, as the statistical error for the fit is insignificant.

Four sources of uncertainty for the expected mean number of photoelectrons were also evaluated. These were the uncertainties in the quantum efficiency of the photocathode of each HPD, the uncertainty in the timing efficiency, the photoelectron detection efficiency, mirror reflectivity and quartz transmission. As can be seen in Table 2, the uncertainties in the expectation are much larger than those from the fitting process. The former are dominated by the contribution from the HPD characteristics—in particular, the uncertainty on the photocathode quantum efficiency. The uncertainties of the reflectivity of the mirror and the transparency of the quartz windows were small enough to be ignored in this analysis.

The final ratio of $\mu_{\text{exp}}$ to $\mu_{\text{fit}}$ and the associated error is given in Table 1.

8. Cherenkov angle distributions

The Cherenkov angle was reconstructed for the data selected using the criteria outlined in Section 6. For each selected hit, the Cherenkov angle was calculated as the angle between the reconstructed photon and beam direction. The reconstruction can be broken down into four steps: demagnification of the selected hits, photon refraction in the HPD quartz entrance window, reconstruction of the photon direction and reconstruction of the beam direction. These contributions are described in the following subsections.

8.1. Image demagnification

For each pixel hit, the photoelectron emission point on the HPD photocathode was calculated by accounting for the cross-focussing electric field of the HPD. In order to map the pixel position to a coordinate on the photocathode, a linear demagnification law in the radial position, $r_{\text{photocathode}} = 5.64 \times r_{\text{pixel}}$, was used. The demagnified image of the photocathode has a radius of ~13 pixels and is smaller than the size of the silicon anode (32 × 32 pixels). Therefore, only those pixel hits which were in the active region were considered.
8.2. Refraction in the HPD quartz

The HPD quartz entrance window refracts the incident photons. As the incident photon angle is a priori unknown, the reconstruction procedure could not correct for this effect analytically. For each coordinate on the photocathode, an iterative fit was therefore performed to find the photon impact point on the HPD entrance window. In each iteration, a candidate photon impact point on the HPD window was considered. Using this coordinate, and the assumed photon emission point in the gas radiator (see below), the angle of incidence of the photon at the quartz window was calculated. The deviation caused by the refraction was then computed and the coordinate where this candidate photon would have struck the photocathode was found. The distance between this position on the photocathode and the observed photocathode hit position (as computed from demagnified pixel hit coordinate) was subsequently minimised. The simulation indicated that this procedure gave the photon impact point to within $\mu m$.

8.3. Reconstruction of the photon direction

The photon direction (and hence the Cherenkov angle) was calculated using a quartic equation [14]. This equation is valid for spherical rather than parabolic mirrors; however, given the large focal length of the mirror used for the present tests, the difference is negligible. The equation has coefficients which depend on the photon detection point (D), the photon emission point (E) and the mirror centre-of-curvature (C), defined in Fig. 10. From the solution of this equation, the photon impact points on the mirror were calculated for each photon in the event. The photon directions were then found, assuming they were emitted half-way along the active radiator length. It will be seen that this assumption contributes minimally to the Cherenkov angle uncertainty.

8.4. Reconstruction of the beam direction

In the absence of beam tracking chambers, a method to reconstruct the beam direction from the HPD data was employed. The beam direction was found by fitting circles to the reconstructed photon impact points, projected onto the outside of the HPD entrance window. The centre of the corresponding fitted circle was assumed to be where the beam would have passed, had it been reflected by the mirror.

In order to decouple the photon reconstruction and the beam reconstruction, the beam direction was found for each hit in the event. For every hit, a circle was fitted by removing that hit and using the remaining hits. The beam direction was then calculated using the circle centre and the quartic equation described in Section 8.3. The beam direction reconstructed for each hit in all the events in a run for HPD C1 is shown for the horizontal and vertical planes in Fig. 11. The widths of single Gaussians fitted to these distributions are 1.5 and 1.1 mrad in the horizontal and vertical planes, respectively. The MC simulation was adjusted to reproduce the beam divergences seen in the data. The simulation indicates that, in both directions, the beam direction is reconstructed with an error of 0.9 mrad, implying that the underlying beam divergence is 1.2 (0.6) mrad in the horizontal (vertical) direction.

8.5. The Cherenkov angle reconstruction

The Cherenkov angle reconstructed on a hit-by-hit basis is shown for HPD C1 in Fig. 12. The data and MC simulation are in good agreement, with a mean of 19.64
(19.63 mrad) in the data (MC) and an RMS of 1.66 (1.64) mrad. The statistical errors on the RMS values are negligible. Systematic sources of uncertainty are described in Section 8.6.

The Cherenkov angle resolution can be expressed in terms of four contributions: chromatic effect, photon reconstruction, beam reconstruction and charge sharing. The photon reconstruction part can be further divided into HPD pixelisation, PSF and photon emission point effects. The contribution from each was determined from the simulation and a summary is given in Table 3. The effects were found to be largely uncorrelated and are discussed below.

As the reconstructed beam direction is computed from the fitted circle centres, in addition to the beam divergence, the HPD pixelisation, PSF and photon emission point also all contribute to the uncertainty in the beam reconstruction. The reconstruction was run on MC events using the generated photon direction instead of the reconstructed photon direction, hence separating out the contribution from the chromatic effect. The uncertainty to the Cherenkov angle from the beam reconstruction was then determined to be 0.95 mrad.

The other contributions were similarly determined by running the simulation without the relevant effects and comparing the width to that with all effects included. The resolution is dominated by the beam reconstruction and photon reconstruction contributions, with the pixelisation and PSF being the dominant contributions to the latter. The pixelisation and PSF contributions are significant owing to the compact test-beam geometry. They are not expected to be dominant in the final LHCb experiment which has a less compact optical configuration and radiators of significantly larger refractive index.
8.6. Systematic uncertainties on the Cherenkov angle resolution

The systematic errors on the Cherenkov angle resolution from various components of the test-beam system were evaluated from the simulation. A number of parameters were changed in the simulation one-by-one, in each case by an amount equivalent to the estimated error on the relevant quantity. The reconstruction was run on these MC samples and the width of the Cherenkov angle distribution compared with that from the standard simulation.

The pixel chip position within the HPD was changed by ±0.7 pixels in both “row” and “column” directions. The uncertainty originates from an estimate of the mechanical alignment uncertainty on the pixel chip position relative to the HPD axis.

As described above, the beam divergence was estimated from the distribution of the centres of the fitted circles. The average beam direction was changed by an amount equivalent to the beam divergence, in both perpendicular directions. The average beam position was moved by 50% of the window defined by the triggering scintillators (see Section 4) i.e. by ±1 cm.

A number of uncertainties in the alignment and properties of the mirror were evaluated:

- Mirror tilt angle—the tilt angle was changed to move the mean position of the Cherenkov rings by ±1 mm on the surface of the HPD. This was estimated to be the uncertainty due to the laser alignment procedure.
- Mirror position along the beam line—the mirror was moved along the beam axis in the simulation by ±2 mm, again representing the estimated uncertainty in the alignment.
- Mirror radius of curvature—this was changed in the simulation by the difference between the measured and nominal radius (±15 mm).
- Mirror focussing quality—an angular smearing effect was introduced in the MC simulation to model the surface roughness of the mirror at the measured level.

The resolutions using the modified parameters were then compared to those in the standard simulation. The errors are summarised in Table 4. Combining these uncertainties in quadrature, the total systematic uncertainty on the Cherenkov angle resolution was estimated to be ±0.03 mrad, a ~2% uncertainty compared to the measured resolution.

<table>
<thead>
<tr>
<th>Mis-alignment</th>
<th>Change in $\theta_c$ (mrad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel chip</td>
<td>0.008</td>
</tr>
<tr>
<td>Beam divergence</td>
<td>0.019</td>
</tr>
<tr>
<td>Mean beam direction</td>
<td>0.006</td>
</tr>
<tr>
<td>Mean beam position</td>
<td>0.016</td>
</tr>
<tr>
<td>Mirror tilt angle</td>
<td>0.007</td>
</tr>
<tr>
<td>Mirror position</td>
<td>0.001</td>
</tr>
<tr>
<td>Mirror radius of curvature</td>
<td>0.008</td>
</tr>
<tr>
<td>Mirror focussing quality</td>
<td>0.007</td>
</tr>
<tr>
<td>Combination in quadrature</td>
<td>0.03</td>
</tr>
</tbody>
</table>

The distribution of reconstructed Cherenkov angles was also found to agree with that from the simulation. The single-photon Cherenkov angle resolution was determined to be 1.66 ± 0.03 mrad, with the dominant contributions coming from the uncertainty in the beam direction, the HPD point-spread function and the HPD pixelisation. The latter two effects were significant due to the test-beam geometry and are not expected to be dominant in the final LHCb experiment.

In summary, the HPD complies with all requirements of the LHCb experiment.

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