Measuring thermistor resistance with very low d.c. power dissipation

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A very simple and efficient procedure for measuring thermistor resistances at very low temperatures (down to 5 mK) with d.c. bias voltages is presented. The measurements can be performed with a d.c. power dissipation in the thermistors as low as $10^{-18}$ W or less, as required by the extreme thermistor sensitivity to small heating effects. In particular this method suppresses the effects of the input current of the amplifier used for the measurement. We are using this procedure for the automatic characterization of thermistors with impedances up to $10^{10}$ Ω at temperatures as low as 10 mK. © 1996 Elsevier Science Limited

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Particle detectors based on the measurements of the increase of temperature caused by an impinging particle, are used in experiments of fundamental physics. We use bolometric detectors of large mass for double beta decay experiments and are, at present, optimizing small detectors for neutrino mass determination. The signal is read with Ge or Si thermistors thermally attached at the detector crystal.

Measurement of the high valued resistances of these semiconductor thermistors working at very low temperatures (typically ranging from 5 to 500 mK) is extremely difficult. The thermistor is sensitive to very small amounts of power injected into it, and changes its resistance as a consequence of the temperature change. Therefore, the bias current necessary to measure the thermistor resistance must result in a very low power dissipation, to reduce the measurement error to a negligible value.

The thermistor has a complex impedance, but measurement errors associated with the parasitic capacitance can be avoided by determining its resistive component at d.c.

In this paper we describe a new method to measure high valued resistances at d.c. and at very low bias level. The method suggests a way to cancel all the possible offsets of the measuring circuit, including the biasing or leakage current of the input transistors in the measurement amplifier. Thus the input transistors can be chosen for extremely low series noise at low frequency, regardless of their possible input current, allowing the use of very low d.c. bias for measurements.

This measurement procedure is not able to compensate for leakage power coming from electrical disturbances of non-static origin (i.e. r.f. sources or mechanical vibration). We use two refrigerating systems, one located a few metres below ground level, in a reinforced concrete room, the other in a Faraday cage at the Gran Sasso Underground Laboratory. Our bolometric detectors show similar behaviour in both set-ups. This fact leads us to conclude that the external pick-up disturbances are sufficiently screened in both cases.

In the following section we briefly describe thermistor properties and the problems related to the measurement of the thermistor impedance. Subsequently we illustrate the new measurement method and show the results obtained when using a remotely controlled system, able to characterize arrays of thermistors.

Thermistor properties and their measurement

The principle of operation of semiconductor thermistors can be understood by referring to the concept of 'electro-thermal feedback', which is illustrated with the help of Figure 1. The figure presents a hybrid model of the thermistor, consisting of a thermal and an electrical section. When a d.c. bias current $I_b$ is applied to the electrical section, a voltage $V_b$ is developed across the thermistor, which dissipates an electrical power $P_{el} = R_T I_b^2$. $R_T$ determines the actual thermistor temperature $T_a = P_{el}/K + T_b$ by developing a temperature gradient across the thermal conductance $K$. $T_b$ is the heat sink or base temperature. $K$ represents the thermal link between the thermistor conduction electrons,
The results show that the resistance change has a linear dependence on the d.c. bias current change.

\[ RT(T_T) = RT(T_s) - a(T_s - T) \alpha > 0 \]  

where \( K_p \) represents the physical thermal connection (glue, conductive grease, connecting wires, etc.) of the thermistor to the heat sink, while \( K_{el} \) describes the intrinsic thermal conductance inside the thermistor between the conductance electrons and the lattice. Usually, at very low temperature, the smaller, dominant conductance is \( K_{el} \).

Now, we assume a change in the bias current. The electrothermal effect that takes place can be described as follows: if the bias current \( I_{in} \) increases, \( P_e \) increases and so does the temperature. But in a semiconductor thermistor the resistance decreases as the temperature increases, hence the bias voltage and, consequently, power dissipation is also reduced, closing this electrothermal feedback loop. Thus the increase of \( P_e \) is lower than if \( R_T \) were not temperature dependent.

With the help of the circuit of Figure 1 it is possible to evaluate the effect of the bias on the thermistor resistance. We will only consider the case, of interest to us, of small values of the bias current \( I_{in} \), implying small increases of the thermistor temperature above that of the heat sink. \( R(T_T) \) will be approximated with:

\[ R(T_T) = R(T_s) - \alpha(T_s - T) \alpha > 0 \]  

where \( T_T - T_s \) is small and \( \alpha \) is the thermistor sensitivity. Assuming that \( I_{in} \) suffers a small d.c. change \( \Delta I_{in} \) from a starting value \( I_{in0} \), we obtain a thermistor temperature change \( \Delta T_T \) equal to:

\[ \Delta T_T = \frac{2I_{in0}R(T_T)}{K + \alpha I_{in0}} \Delta I_{in} \]  

(3)

hence the thermistor resistance changes by an amount:

\[ \Delta R_T = -\alpha \Delta T_T = -\frac{2\alpha I_{in0}R(T_T)}{K + \alpha I_{in0}} \Delta I_{in} \]  

(4)

In the limiting case of large \( \alpha \) or, as a more common situation, of small values of the thermal conductance \( K \), limited by \( K_{el} \) in Equation (1), we get the asymptotic approximation:

\[ \frac{\Delta R_T}{R(T_T)} \alpha \approx \frac{\Delta I_{in}}{I_{in0}} \]  

(5)

The result shows that the resistance change has a linear dependence on the d.c. bias current change.

If only d.c. bias is concerned, as seen above, the heat capacity \( C_T \) and the electrical parasitic capacitance \( C_p \) of a thermistor also shunt the thermistor would introduce an error, unless long measuring time is allowed.

The instruments so far used for this application are generally able to measure thermistor impedance only in a limited range. As an example, the Keithley 602 Electrometer is a battery powered instrument capable of measuring resistances as high as \( 10^3 \Omega \), but with d.c. bias voltages close to 1V. This happens because the input circuit of this electrometer uses MOS transistors having negligible input bias current, but high noise at low frequency. Other instruments, like the Picowatt RV-Elektronikka AVS-47, makes the measurement by balancing a bridge composed of two branches, one of which includes the thermistor. These are excited with a.c. bias (often a square wave), at frequencies of the order of \( 1 \) kHz or tens of Hz. In this case the maximum resistance measurable is of the order of several \( 1 M\Omega \), due to the effects of the electrical parasitic capacitance.

For the realization of thermal particle detectors, which work at temperatures as low as 10 mK, we use high impedance thermistors, even as high as the GΩ range. For that reason we use the new method described below to measure them with very low power dissipation and with d.c. bias. For better efficiency, the measurement set-up was complemented by a remotely controlled system to characterize and select arrays of thermistors.
A method to measure thermistor impedances at low d.c. biasing power

The measurement set-up was designed to use very low d.c. bias for the thermistor, such that the power dissipation generated would be negligible. The measurement is based on the differential scheme shown in Figure 2. A fraction $V_B$ of the voltage $V_P$ biases the thermistor $R_T$, while a voltage across the known load resistors $R_C$ is developed. The ideal amplifier $A_m$ reads the voltage $V_B$ with a gain $A$. The known values of $V_p$ and $R_C$, together with the measured value $V_B$ are used to calculate the current $I_B$, and, finally, the resistance $R_T = (V_B/I_B)$.

So far we have assumed that the parasitic voltages and currents present in Figure 2 are negligible. In an actual measurement a first consideration to be made is that errors could be introduced by the voltage $V_C$, originated by the temperature gradient across the wire connecting the reading circuit. This is at room temperature, and the thermistor is located inside the refrigeration system. These effects are actually eliminated by using a differential readout and a balanced biasing configuration. The actual measurement set-up establishes a four terminal connection, two wires for the bias and two for the readout, to eliminate the effects of the parasitic resistances. They are not indicated in Figure 2 because in the present discussion we deal with high impedances thermistors where these small parasitic resistances do not affect the measurement.

Consider now how it is possible to use very low voltage to bias the thermistor. In a real situation, the input current sources $I_{A_1}$ and $I_{A_2}$ are present at the amplifier input. If these currents are not equal (i.e. there is an offset), or if the two load resistors are not exactly equal, a voltage across the thermistor is developed and an unknown additional power is dissipated. As an example, 1pA offset current on a 1GΩ thermistor establishes 1 mV of voltage and 1 fW of power dissipation. This can be large enough to raise the thermistor temperature above the base temperature.

The method we use to avoid this and all the other possible parasitic effects is very simple. Whatever the unwanted voltage developed across the thermistor, it is possible to suppress it by adopting the following procedure. The description refers to the simplified scheme of Figure 3. $I_{OF}$ is the difference between $I_{A_1}$ and $I_{A_2}$ of Figure 2 and the voltage generator $V_P$ is split into two generators $V_{PM}$ and $V_{DEL}$. $I_{OF}$ develops a voltage across the parallel combination of $2R_C$ and $R_T$, assuming zero $V_{PM}$ and $V_{DEL}$ for the moment. We can null this voltage by adjusting the voltage generator $V_{DEL}$ until $V_{OF}$ becomes zero:

$$V_{OF} = \frac{R_T}{R_T + 2R_C} V_{DEL} + \frac{R_T 2R_C}{R_T + 2R_C} I_{OF} - 0 \tag{7}$$

The condition in Equation (7) is met when:

$$V_{DEL} = -2R_C I_{OF} \tag{8}$$

So we find that $V_{DEL}$ does not depend on the thermistor resistance. Now, after fixing the value of $V_{DEL}$, we bias the network with an arbitrary voltage $V_{PM}$, which, if applied without any offset, should lead to the value $R_{TP}$ for the thermistor. Call $R_{TP}$ the actual measured resistance: we want to prove that $R_{TP} = R_{TP}$ after correction. The output voltage $V_B$ will be:

$$V_B = \frac{R_{TP} + 2R_C}{R_{TP} + 2R_C} V_{PM} + \frac{R_T}{R_{TP} + 2R_C} V_{DEL}$$

$$+ \frac{R_{TP} 2R_C}{R_{TP} + 2R_C} I_{OF} = \frac{R_{TP}}{R_{TP} + 2R_C} V_{PM} \tag{9}$$

Figure 3 Simplified scheme of the circuit of Figure 2

where the second line follows from Equation (8): the voltage developed across the thermistor is the voltage we would have if no offset current were present. The current we are interested in, to determine the thermistor resistance value, is

$$I_B = \frac{V_{PM} - V_B}{2R_C} = \frac{V_{PM}}{R_{TP} + 2R_C} \tag{10}$$

Figure 2 Set-up for differential characterization of a thermistor. The amplifier input leakage current sources are identified as $I_{A_1}$ and $I_{A_2}$

The next step is to evaluate $V_{DEL}$ experimentally, since we do not know the value of $I_{OF}$ in the actual condition. This can be realized by adopting the procedure illustrated in the flow chart of Figure 4.

First, it is necessary to know the situation when the parasitic effects are not present. Since it is not possible to eliminate the parasitic currents, an alternative method is to short circuit both amplifier inputs to ground in the network of Figure 2. In this way we can measure the true amplifier output offset, $V_{OF}$. Once the amplifier offset is known, the thermistor is reconnected at the amplifier inputs. If a current $I_{OF}$ biases the thermistor, the voltage across it changes, hence the amplifier output voltage $V_{OAM}$ changes too. Now by varying the biasing voltage $V_P$, it is possible to get a suitable value, $V_{PAM}$, such that the amplifier output reaches the initial value $V_{OF}$, corresponding to zero voltage across the thermistor. This condition ensures that we have forced into the thermistor a current exactly equal in amplitude, but opposite in direction, to the offset current, $I_{OF}$, already present, and that the net voltage across the thermistor has become zero: the condition of Equation (8) has been realized. Due to the way the voltage $V_{DEL}$ is determined, the
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The leakage power $P_{\text{LEA}}$ equation (11) depends on the value of the thermistor impedance at base temperature. Very often the condition expressed by the first line of Equation (11) is met. As a practical example consider a thermistor having $1 \Omega$ resistance at base temperature and an amplifier with $10 \, \text{pA}$ input offset current; if $V_{\text{MIN}} = 100 \, \text{µV}$, and $A = 200$ we arc in the condition of the first line of Equation (11). The resulting maximum power $P_{\text{LEA}}$ is only $2 \times 10^{-19} \, \text{W}$, instead of $10^{-16} \, \text{W}$ that would be obtained if no correction were made.

From Equation (11) it is evident that the maximum error in the leakage power depends on many parameters. If the resistance of the thermistor is very large or very small, the power $P_{\text{LEA}}$ tends to be negligible. This means that the residual leakage power is not fixed but depends on the thermistor value at base temperature. A simple analytical analysis of the dependence of this power on the thermistor value shows that the maximum error is given by the product $I_{\text{OF}}V_{\text{MIN}}/A$, and that this maximum corresponds to a resistance of $V_{\text{MIN}}/(A_{\text{OF}})$. For the example at hand this maximum sets a leakage power of $5 \times 10^{-18} \, \text{W}$ at a resistance of $50 \, \text{kΩ}$.

An improvement of the sensitivity of the measurement can be obtained by increasing the amplifier gain or the voltage sensitivity. This means that the amplifier input transistors must be chosen to obtain a series noise at d.c. as small as possible, regardless of the value of its input current.

**Application of the method and results**

Thermistors have to be characterized at different temperatures to define their characteristics. This is done by changing the base temperature of the refrigerating system in small steps and measuring the thermistor resistance at each temperature. Time is saved if the procedure is realized automatically, with the possibility of characterizing more than one thermistor at each temperature. For this purpose we have implemented a system, controlled by a computer, capable of changing and stabilizing the base temperature and characterizing four thermistors at the same time.

The modification and stabilization of the base temperature is obtained simply by changing the power dissipation on a heater until the resistance of a thermometer reaches the desired value. The thermistor characterization system was realized based on the scheme of Figure 2 and the flowchart of Figure 4.

Figure 5 shows a simplified diagram of the set-up. The loading resistors $R_{\text{CA1(b)}}$ ($i = 1 \ldots 4$) can be connected in the network by closing the corresponding switches, which, to have the minimum leakage, are implemented using relays. Fine adjustment of the setting voltage $V_F$ is possible thanks to the partition resistors $R_T$ and/or $R_{T2}$, and $R_{T3}$. The two resistors $R_T$ established the ground reference.

Each relay of this network is controlled digitally via a remote computer control, by means of a National 8255 card and an optical link, able to suppress any electromagnetic interference and ground loop. The supply voltage $V_F$ is a GP–IB programmable HP6627A floating power supply. The actual bias voltage is read with respect to ground by setting on or off two relays connected to the two terminals of resistor $R_{T2}$, and to the HP34401A (GP–IB

![Figure 4 Flow-chart for the compensation of the power due to the preamplifier input offset currents](image-url)
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**Figure 5** Automatic system for the characterization of four thermistors at a time

![Diagram showing an automatic system for the characterization of four thermistors at a time](image)

**Figure 6** Resistance of an NTD germanium thermistor versus d.c. power dissipation when the preamplifier input offset current of about 1 pA is corrected (●), or not corrected (○).

![Graph showing resistance vs. power for an NTD germanium thermistor](image)

- Corrected
- Not corrected

programmable) multimeter. The amplifier’s output is read with the same multimeter using another relay. The preamplifier inputs can be connected to ground and the offset voltage can be adjusted remotely. The algorithm of the flow chart of **Figure 4** is realized with software which is also able to decide all the settings of the network. The thermistors are connected to the system with four wires (two for biasing and two for reading) allowing correction for the parasitic resistances of the wires and for the thermal voltage $V_T$.

Finally, **Figure 6** shows typical results of a resistance measurement, illustrating the improvement when the correcting method is applied to a neutron transmutation doped thermistor. The sample under measurement had a resistance value of about 100 MΩ at the base temperature of 10 mK. The preamplifier input offset current was found to be less than 1 pA.

**Conclusion**

Thermistors are devices whose resistive component of impedance is very sensitive to the power they dissipate. Up to now, their electrical characterization at cryogenic temperatures needed very low leakage current in the reading network. This work has proved that it is possible to cancel the effect of leakage current, and to use a very low d.c. biasing voltage on thermistors having very large impedance. The method suggests how to cancel the offset power dissipated in the thermistor due to the input current of the reading preamplifier, simply by adding an additional measurement step. The only requirement on the input transistor of the reading preamplifier is that their low-frequency noise be very low, regardless of their leakage input current. The method is currently used to characterize arrays of silicon and germanium thermistors down to 10 mK temperature in a range from a few MΩ to GΩ.

**References**


